

Depreciation in Use, Shifts and the Workweek of Capital

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Abstract

In this paper I examine a real business cycle (RBC) model in which firms can vary the utilization rate of capital by operating different numbers of shifts at plants. The firms' incentive to do this follows from the fact that capital is allocated to plants prior to the realization of idiosyncratic uncertainty regarding productivity at the plant level. As in standard RBC models, aggregate fluctuations in productivity drive the business cycle. During expansions firms accept the wage premium and added depreciation costs associated with second shifts, and add these shifts at their most productive plants. During recessions these additional shifts are dropped, and the least productive plants may be shut down entirely. As a result of this behavior, the model predicts that the Solow residual substantially overstates the variation in technology and that small technology shocks are significantly magnified by variation in capital's workweek.

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1. Introduction

In this paper I examine a model in which firms vary the workweek of their capital in response to aggregate technology shocks. Firms run different numbers of shifts at different plants, because in each period capital is allocated to plants prior to the realization of information about productivity at the plant level. Roughly speaking firms will operate a maximum of two shifts at their most productive plants, one shift at less productive plants, and possibly no shifts at the least productive plants. The number of shifts at individual plants and the aggregate workweek of capital both respond positively to aggregate technology shocks. As a result, part of the Solow residual reflects endogenous variation in the workweek of capital. Furthermore, because firms can make decisions on the workweek margin, technology shocks have a greater impact on aggregate output than they do in standard RBC models with a fixed number of shifts.

There is a substantial literature that discusses cyclical variation in the rate of capital utilization and the workweek of capital. Kydland and Prescott (1988) and Burnside and Eichenbaum (1996) examine real business cycle (RBC) models in which the utilization rate is a smooth margin along which firms can vary their inputs. In Kydland and Prescott's model the workweek of capital is a smoothly variable input which moves in proportion to the number of hours worked by households. In Burnside and Eichenbaum's model there are increasing marginal costs in the form of depreciation which lead to an interior solution for the utilization rate. As a result, firms increase the rate of utilization in expansions and reduce it in recessions. This endogenous variation in the utilization rate magnifies the effects of technology shocks, and provides their model with a significant propagation mechanism. It further implies that procyclical movements in productivity are partially due to changes in unmeasured factors of production, and are not interpretable as a measure of exogenous technological change.

Although both of these models have the advantage of simplicity, neither model is specific as to what the utilization margin is at the plant level: changes in the number of shifts, changes in overtime work, or changes in line speed. In this paper I specifically model all movements in utilization as changes in the number of shifts operated at individual plants. The reasons for firms varying the number of shifts at the plant level are threefold. First, the firm faces a technological constraint of running an integer number of shifts (0, 1 or 2) at each plant. Second, the cost of capital rises as it is used more intensively. In particular, when a firm chooses to operate its capital over 2 shifts instead of one, it incurs a fixed depreciation cost.¹ Third, firms face a stochastic environment in which capital must be put in place before

¹Shift premiums are another mechanism through which a fixed cost of operating additional shifts could

the productivity of each plant is known—as a result, firms will have an incentive to vary the number of shifts across plants.²

The model in this paper is empirically relevant since, in related work, Shapiro (1993) and Burnside, Eichenbaum and Rebelo (1995) show that procyclical movements in productivity are highly correlated with changes in the workweek of capital. Direct evidence suggests that the average workweek of capital in U.S. manufacturing between 1974 and 1992 was about 97 hours, or just under 14 hours per day.³ These data are sampled in the fourth quarter of each year by the U.S. Bureau of the Census. The standard deviation of the workweek over this period was approximately 3 hours per week, and was strongly procyclical. This means that, on average, capital is idle for long periods of time, but that firms either vary the number of shifts or use overtime at different rates over the business cycle. Although at finer levels of aggregation not all industries display significant time-series variation in the workweek, this behavior characterizes most of the two-digit SIC code manufacturing industries.⁴

More detailed information on the workweek of capital in the automobile industry has been studied by several authors. Aizcorbe (1992), Bresnahan and Ramey (1994), Cooper and Haltiwanger (1993) and Hall (2000) have provided detailed evidence about the margins on which output is varied at the plant level. Bresnahan and Ramey’s evidence is based on weekly data from 50 automobile plants from 1972 through 1983, while Hall’s evidence is based on weekly data from 14 plants of the Chrysler Corporation from 1990 through 1994. While the data studied by these authors differs somewhat, both papers find that changes within the choice set of one, two or three shifts account for almost none of the workweek variation at the plant level. Almost all variation in the workweek is due to shutdown and startup, between zero shifts and the ‘normal’ number of shifts. The normal number of shifts varies substantially by plant. The implications of this evidence for aggregate fluctuations is unclear, since the automobile industry may not be representative of the entire manufacturing

emerge. Shapiro (1995) has argued that one reason for idle capital is that there is a substantial premium for work at night.

²Cooley, Hansen and Prescott (1995) examine a model in which firms choose to operate one shift or shut down—the mechanism they use is idiosyncratic productivity shocks at the plant level along with a minimum scale of labor used in production. Campbell (1998) examines a model in which the idiosyncratic component of the plant level shocks is persistent and in which technology shocks are embodied in capital. Rather than leading to shutdowns and startups, this model leads to the exit and entry of plants. In a related paper, Hall (2000) describes the problem faced by producers of automobiles. He argues that nonconvexities in firms’ cost functions at the plant level cause production bunching at plants facing low demand for their output.

³I am grateful to Joseph Beaulieu of the Board of Governors of the Federal Reserve System for providing me with data on the workweek of capital in U.S. manufacturing. The data, at the 4-digit industry level are fixed-employment weighted averages of the plant level data. At higher levels of aggregation they are weighted by the value of installed capital.

⁴Several industries are referred to as continuous processors, since, for technological reasons, they operate most efficiently on a 24-hour basis.

sector, never mind other sectors. It remains to be seen which type of nonconvexity is most relevant in determining the behavior of the typical U.S. manufacturing industry.

In the next section I describe the model. I outline the necessary conditions for competitive equilibrium and show that these define a unique bounded path within the neighborhood of a unique nonstochastic steady state. In the third section I calibrate numerical versions of the model which I simulate in order to explore the different implications of the model for aggregate fluctuations. In the fourth section I make concluding remarks.

2. A Model of the Workweek

In this section I describe the model which can be viewed as an extension of the indivisible labor model described by Hansen (1985) and Rogerson (1988). Their labor market structure facilitates my characterization of a competitive equilibrium to a decentralized economy.⁵

2.1. The Household's Problem

Households own the firms, and make decisions regarding consumption and leisure. A household's instantaneous utility is given by

$$\ln(C_t) + \eta \ln(L_t)$$

where C_t is the household's time- t consumption and L_t is the household's effective time- t leisure. As in Hansen (1985) and Rogerson (1988), the household's consumption possibilities set is restricted so it may work full time, or not at all. Each time period is divided into two shifts each of length h . Contracts between households and firms commit the household to (i) work h hours on the first shift with probability N_{1t} , (ii) work h hours on the second shift with probability N_{2t} , with the contract excluding the possibility of working both shifts.⁶ Hence, the probability of not working is $1 - N_{1t} - N_{2t}$. As in Hansen (1985) and Rogerson (1988) the contracts are traded, so that they, in effect, provide complete unemployment insurance to households.

The household's effective leisure is given by

$$L_t = [2 - \mathbf{1}_{\{1\}} - \mathbf{1}_{\{2\}}(1 + \phi)] h,$$

where $\eta > 0$, $\phi > 0$, and $\mathbf{1}_{\{j\}}$ is an indicator function taking on the value 1 if the household works during shift j . The assumption that $\phi > 0$ implies that households prefer to work in

⁵The difficulty in setting up a social planner's problem for this model arises from the nonconvexity in production. It is easy to deal with this nonconvexity in a decentralized model.

⁶Contracts will always have this feature given the structure of preferences described below.

the first shift rather than the second, other things equal.⁷

Given the nature of the contracts described above, the expected utility of a household, up to a constant, is given by

$$U(C_t, N_{1t}, N_{2t}) = \ln(C_t) - \eta_1 N_{1t} - \eta_2 N_{2t}$$

where $\eta_1 = \eta \ln(2 - \phi)$ and $\eta_2 = \eta \ln[(2 - \phi)/(1 - \phi)] > \eta_1$. Because the contracts are traded and households are *ex-ante* identical, they will maximize the discounted stream of their expected utility subject to a common budget constraint.

I assume that households own the capital stock and rent it to firms at a competitively determined rate (net of depreciation), R_t .⁸ The household's problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln(C_t) - \eta_1 N_{1t} - \eta_2 N_{2t}]$$

subject to

$$C_t + K_{t+1} - K_t \leq W_{1t} N_{1t} h + W_{2t} N_{2t} h + R_t K_t + \Pi_t, \quad t = 0, 1, \dots$$

where $0 < \beta < 1$, W_{1t} is the real wage rate on the first shift, W_{2t} is the real wage rate on the second shift and Π_t are profits of the firms at time t .

The first order conditions for the consumer's problem are standard:

$$\eta_j = W_{jt} h / C_t, \quad j = 1, 2. \quad (2.1)$$

$$1 = E_t(\beta C_t / C_{t+1})(1 + R_{t+1}) \quad (2.2)$$

$$C_t + K_{t+1} - K_t = W_{1t} N_{1t} h + W_{2t} N_{2t} h + R_t K_t + \Pi_t. \quad (2.3)$$

An immediate consequence of the consumer's first order conditions is that the wages in the two sectors are a constant ratio of one another. Specifically, $W_{2t}/W_{1t} = \eta_2/\eta_1$.⁹ Henceforth I will use the notation $W_t = W_{1t}$ and let $W_{2t} = \psi W_t$ where $\psi = \eta_2/\eta_1$.

2.2. The Firm's Problem

Firms are identical. Each firm consists of a uniform unit-mass continuum of plants, indexed by location $i \in [0, 1]$. Within each firm, plants are subject to idiosyncratic risk, so that for

⁷Of course, no household will sign a contract where it works both shifts since this would imply $-\phi h$ hours of effective leisure.

⁸In other words, the firm pays $R_t + \delta_t$ to the household for each unit of capital rented.

⁹Shapiro (1995) argues that this premium is about 25%. Here I treat η_2/η_1 as a free parameter, whose implications are explored in section 3.

an individual plant, the level of labor augmenting technology is given by $X_i = X \exp(\epsilon_i)$.¹⁰ The idiosyncratic shock $\epsilon_i \sim \text{iid}(0, \sigma_\epsilon^2)$ is assumed to be i.i.d. across time, plants and firms. I denote the cumulative distribution function of ϵ by $F(\epsilon)$, and its density function by $f(\epsilon)$. It is analytically convenient, but not necessary, to make the further assumption that ϵ is uniformly distributed on the interval $(-\bar{\epsilon}, \bar{\epsilon}) = (-\sqrt{3}\sigma_\epsilon, \sqrt{3}\sigma_\epsilon)$. For the moment I assume that X_i is a stationary stochastic process.

Within plant i , on shift j , one worker produces $hk_{ij}^\alpha X_i^{1-\alpha}$ units of output, where k_{ij} is the capital-labor ratio at plant i , on the j th shift. So, output at plant i across two shifts is given by

$$Y_i = hK_i^\alpha X_i^{1-\alpha} (N_{i1}^{1-\alpha} + N_{i2}^{1-\alpha})$$

where N_{ij} is employment on the j th shift at plant i . Notice that capital is allocated to plants and cannot be varied by shift, although it might be idle during a given shift.

Capital's depreciation rate depends on its time in use. As in Greenwood, Hercowitz and Huffman (1988) and Burnside and Eichenbaum (1996) I assume that the depreciation rate of a unit of capital at plant i is given by

$$\delta_i = \delta_0 + \delta U_i^\gamma,$$

where $U_i \in \{0, h, 2h\}$ represents its time in use, $\delta_0 \geq 0$, $\delta > 0$ and $\gamma \geq 1$. When $\delta_0 > 0$ there is a 'rust' component to depreciation, as suggested by Basu and Kimball (1997).

The timing of firms' decisions relative to the resolution of uncertainty in the economy is described as follows:

1. At the beginning of the period aggregate uncertainty is resolved as X is revealed. Firms rent capital, K , from households.
2. Without knowledge of ϵ_{it} at different plants firms allocate K across plants.¹¹ Since ϵ_i is i.i.d. the plants are *ex-ante* identical, so the firm will allocate capital evenly across plants: $K_i = K$ for all i .
3. After capital is in place, the idiosyncratic shocks, ϵ_i , are observed.
4. Firms then choose U_i and N_{ij} in order to maximize their profits. The firm's choice of

¹⁰Since the firm's problem is static I will, at times, suppress time subscripts to economize on notation.

¹¹In this sense, capital is clay for a period, once it is allocated to a plant. Making capital more clay-like, by making plant level allocations of capital less easy to reverse, would be an attractive extension of the model. However, it would substantially complicate the solution of the model, by introducing an infinite number of state variables. To avoid this difficulty, I pursue the model with the assumption described here.

U_i is constrained by the fact that

$$U_i = \begin{cases} 0 & \text{if } N_{i1} = N_{i2} = 0 \\ 2h & \text{if } N_{i1} > 0 \text{ and } N_{i2} > 0 \\ h & \text{otherwise.} \end{cases}$$

The firm's problem is:

$$\max_{K, \{N_{i1}, N_{i2}\}_{i \in [0,1]}} Y - Wh[N_1 + (1 + \psi)N_2] - (R + \bar{\delta})K$$

$$\text{subject to } N_j = \int_0^1 N_{ij} di, \quad \bar{\delta} = \int_0^1 \delta_i di,$$

$$Y = hK^\alpha \int_0^1 X_i^{1-\alpha} (N_{i1}^{1-\alpha} + N_{i2}^{1-\alpha}) di, \quad (2.4)$$

and

$$\delta_i = \begin{cases} \delta_0 & \text{if } N_{i1} = N_{i2} = 0 \\ \delta_0 + \delta(2h)^\gamma & \text{if } N_{i1} > 0 \text{ and } N_{i2} > 0 \\ \delta_0 + \delta h^\gamma & \text{otherwise.} \end{cases} \quad (2.5)$$

It is convenient to decompose the firm's problem into two parts: (i) the choice of N_{ij} for all i, j , given K , and (ii) the choice of K .

The firm faces a non-convexity in choosing N_{ij} because it must incur the fixed cost $\delta h^\gamma K$ to run the first shift, and the additional fixed cost $\delta h^\gamma (2^\gamma - 1)K$ to operate a second shift. Of course, since $\psi > 1$, if the firm chooses to run the plant for one shift it will be the first shift. Assuming that a plant is operated during shift j , the optimal level of employment during that shift will, as usual, equate the marginal product of labor to the real wage. So, if $N_{ij} > 0$, it is given by

$$N_{ij} = (1 - \alpha)^{1/\alpha} K X_i^{(1-\alpha)/\alpha} W_j^{-1/\alpha}. \quad (2.6)$$

Now consider the relative profitability of running the plant for different numbers of shifts. If the firm runs the plant for no shifts, its profit from that plant is given by

$$\pi_0 = -(R + \delta_0)K. \quad (2.7)$$

Using, (2.6), if the firm runs the plant for one shift, its profit from that plant will be

$$\pi_1 = \alpha (1 - \alpha)^{(1-\alpha)/\alpha} hK (X_i/W)^{(1-\alpha)/\alpha} - \delta h^\gamma K + \pi_0 \quad (2.8)$$

while its profit from running the plant for two shifts will be

$$\pi_2 = \alpha (1 - \alpha)^{(1-\alpha)/\alpha} [1 + \psi^{(\alpha-1)/\alpha}] hK (X_i/W)^{(1-\alpha)/\alpha} - \delta(2h)^\gamma K + \pi_0 \quad (2.9)$$

The firm will run at least one shift as long as $\pi_1 - \pi_0 > 0$, or equivalently

$$\epsilon_i > \nu_1 = \frac{\alpha}{1-\alpha} \ln(\delta h^\gamma) + \ln(W/X) - \kappa. \quad (2.10)$$

where

$$\kappa = \frac{1}{1-\alpha} \ln[\alpha^\alpha (1-\alpha)^{1-\alpha} h^\alpha]. \quad (2.11)$$

The firm will prefer two shifts to one whenever $\pi_2 - \pi_1 > 0$, or equivalently

$$\epsilon_i > \nu_2 = \nu_1 + \ln \psi + \frac{\alpha}{1-\alpha} \ln(2^\gamma - 1). \quad (2.12)$$

So, the firm's decisions regarding the optimal number of shifts to operate at a plant are determined by threshold levels of the idiosyncratic plant-level productivity shock.

Given K , the profits generated by the firm can be calculated by integrating across the distribution of ϵ :

$$\pi = \alpha (1-\alpha)^{(1-\alpha)/\alpha} h K (X/W)^{(1-\alpha)/\alpha} [g(\nu_1) + \psi^{(\alpha-1)/\alpha} g(\nu_2)] - (R + \bar{\delta})K \quad (2.13)$$

where

$$\bar{\delta} = \delta_0 + \delta h^\gamma [2^\gamma - F(\nu_1) - (2^\gamma - 1) F(\nu_2)]$$

and

$$g(\nu) = \int_\nu^{\bar{\epsilon}} e^{(1-\alpha)\epsilon/\alpha} f(\epsilon) d\epsilon.$$

Notice that the ν_j , $j = 1, 2$, are not functions of K .¹²

Given that (2.13) is linear in K , the firm's first order condition for capital determines the rental rate of capital as

$$R = \alpha (1-\alpha)^{(1-\alpha)/\alpha} h (X/W)^{(1-\alpha)/\alpha} [g(\nu_1) + \psi^{(\alpha-1)/\alpha} g(\nu_2)] - \bar{\delta}. \quad (2.14)$$

Using (2.14) and (2.13) we can see that the firm makes zero profits in equilibrium.

2.3. Equilibrium

In this section I characterize the conditions that govern competitive equilibrium. Using a first-order log-linear approximation, I show that the difference equation governing the solution to the necessary conditions has a unique bounded solution in the neighborhood of the nonstochastic steady state.

¹²The firm's profits are $\int_0^1 (\pi_0 1_{\{\epsilon_i < \nu_1\}} + \pi_1(i) 1_{\{\nu_1 \leq \epsilon_i < \nu_2\}} + \pi_2(i) 1_{\{\epsilon_i \geq \nu_2\}}) di$. Given the assumption that the ϵ_i are i.i.d. draws from the uniform distribution, a simple change of variables allows this to be written as $\int_{-\bar{\epsilon}}^{\nu_1} \pi_0 d\epsilon + \int_{\nu_1}^{\nu_2} \pi_1(\epsilon) f(\epsilon) d\epsilon + \int_{\nu_2}^{\bar{\epsilon}} \pi_2(\epsilon) f(\epsilon) d\epsilon$. This leads to the expression in (2.13).

Given the expression for the firm's profits the household's budget constraint can be written as the resource constraint

$$C_t + K_{t+1} - (1 - \bar{\delta}_t)K_t = Y_t \quad (2.15)$$

The rest of the economy is described by the definitions of the threshold values of the idiosyncratic shocks

$$\nu_{1t} = \frac{\alpha}{1 - \alpha} \ln(\delta h^\gamma) + \ln(W_t/X_t) - \kappa, \quad (2.16)$$

$$\nu_{2t} = \nu_{1t} + \ln \psi + \frac{\alpha}{1 - \alpha} \ln(2^\gamma - 1), \quad (2.17)$$

the demands for labor

$$N_{1t} = (1 - \alpha)^{1/\alpha} K_t X_t^{(1-\alpha)/\alpha} W_t^{-1/\alpha} g(\nu_{1t}), \quad (2.18)$$

$$N_{2t} = (1 - \alpha)^{1/\alpha} K_t X_t^{(1-\alpha)/\alpha} (\psi W_t)^{-1/\alpha} g(\nu_{2t}), \quad (2.19)$$

the household's first order condition for labor supply

$$\eta_1 = W_t h / C_t, \quad (2.20)$$

the definition of depreciation

$$\bar{\delta}_t = \delta_0 + \delta h^\gamma [2^\gamma - F(\nu_{1t}) - (2^\gamma - 1) F(\nu_{2t})], \quad (2.21)$$

the household's first order condition for capital

$$1 = E_t(\beta C_t / C_{t+1})(1 + R_{t+1}), \quad (2.22)$$

where

$$R_t = \alpha (1 - \alpha)^{(1-\alpha)/\alpha} h (X_t / W_t)^{(1-\alpha)/\alpha} [g(\nu_{1t}) + \psi^{(\alpha-1)/\alpha} g(\nu_{2t})] - \bar{\delta}_t, \quad (2.23)$$

the output of all first shifts

$$Y_{1t} = (1 - \alpha)^{(1-\alpha)/\alpha} h (X_t / W_t)^{(1-\alpha)/\alpha} K_t g(\nu_{1t}), \quad (2.24)$$

the output of all second shifts

$$Y_{2t} = (1 - \alpha)^{(1-\alpha)/\alpha} h [X_t / (\psi W_t)]^{(1-\alpha)/\alpha} K_t g(\nu_{2t}), \quad (2.25)$$

and total output

$$Y_t = Y_{1t} + Y_{2t}. \quad (2.26)$$

A solution to (2.15)–(2.26) constitutes a competitive equilibrium. To see that the dynamics of the competitive equilibrium are stable in the neighborhood of a nonstochastic steady

state it is useful to consider the linearized stochastic difference equations that result from the system of necessary conditions. In the appendix I show that the nonstochastic steady state is unique, and that the linearized necessary conditions can be reduced to a system of two first-order difference equations in C_t , K_t and X_t :

$$\frac{C}{Y}\hat{c}_t = \frac{1-\alpha}{\alpha} \left(1 + \frac{K}{Y}\Delta\right) (\hat{x}_t - \hat{c}_t) + \left[1 + (1-\bar{\delta})\frac{K}{Y}\right] \hat{k}_t - \frac{K}{Y}\hat{k}_{t+1}, \quad (2.27)$$

$$E_t\hat{c}_{t+1} - \hat{c}_t = \beta\frac{1-\alpha}{\alpha}(R + \bar{\delta})E_t(\hat{x}_{t+1} - \hat{c}_{t+1}), \quad (2.28)$$

where $\hat{c}_t = (C_t - C)/C$, $\hat{k}_t = (K_t - K)/K$, $\hat{x}_t = X_t - 1$ and C , Y , K , $\bar{\delta}$, and R are the steady state values, respectively, of C_t , Y_t , K_t , $\bar{\delta}_t$ and R_t . The constant $\Delta = \delta h^\gamma[f(\nu_1) + (2^\gamma - 1)f(\nu_2)]$, where ν_1 and ν_2 are the steady state values of ν_{1t} and ν_{2t} . In the appendix I show that this system has a unique bounded solution which is stable in the neighborhood of the steady state.

3. Quantitative Exploration of the Model

In this section I explore the quantitative properties of the model. To do this I first calibrate parameter values. Then using the linearized model I present impulse response functions. Finally, the laws of motion implied by the linearized model are used to compute the moments of aggregate time series which are compared to the same moments in U.S. data.

Although the model described in the previous section assumed that X_t is a stationary process, it is straightforward to modify the model to allow for growth in X_t . Here I will assume that X_t follows the law of motion

$$\ln(X_t) = \mu + \ln(X_{t-1}) + \xi_t, \quad (3.1)$$

with $\xi_t \sim \text{iid}(0, \sigma_\xi^2)$.

In choosing the parameter values, I tried to closely match certain key features of U.S. data over the period 1974–1992.¹³ These include the means of the output-capital ratio, the share of investment in output, the fraction of time devoted to market work, the depreciation rate, the growth rate of output, the real interest rate, and the workweek of capital, along with the variance of output. The baseline parameter values are given in Table 1.

3.1. The Behavior of Firms

The behavior of firms can be understood by reference to Figure 1. For a period in which $X = K = 1$, Figure 1 illustrates profits at the plant level associated with operating zero, one or two shifts, for different levels of the idiosyncratic shock, ϵ .

¹³This is the sample over which the workweek of capital data from the Bureau of the Census are available.

If the firm runs zero shifts it simply loses the fixed cost $-(R + \delta_0)K$, so the first curve is a straight line below zero. If the plant operates one shift, it generates profits given by equation (2.8) which is an increasing, exponential function of ϵ minus the fixed cost of running one shift. On the other hand, if the plant operates two shifts, it generates profits given by (2.9), which is a proportionally higher exponential function of ϵ , minus a larger fixed depreciation cost.

The figure illustrates the existence of threshold values, ν_1 and ν_2 , that determine the optimal numbers of shifts associated with particular values of ϵ . Since ϵ is assumed to be uniformly distributed, these thresholds may lie outside the domain of its distribution, $(-\bar{\epsilon}, \bar{\epsilon})$, i.e. ν_1 might be less $-\bar{\epsilon}$ and ν_2 might be greater than $\bar{\epsilon}$. In that case all plants would run one shift.

Further intuition can be gained by considering the cost curves for two plants, as in Figure 2. Figure 2 shows the marginal and average cost curves, and the total profit curve, at individual plants with different values of the idiosyncratic productivity shock. Imagine that a *plant* had to produce a given amount of output Y given some amount of capital, K . Let the curve $C_1(Y)$ describe the lowest cost at which this output could be produced using only one shift:

$$C_1(Y) = \min_{N_1} WhN_1 + (R + \delta_1)K \text{ s.t. } Y = hK^\alpha (X_i N_1)^{1-\alpha},$$

where $\delta_1 = \delta_0 + \delta h^\gamma$. The cost function can be derived by solving the constraint for N_1 , and substituting this into the expression for cost to obtain

$$C_1(Y) = AY^{1/(1-\alpha)} + (R + \delta_1)K,$$

where $A = (W/X_i)(hK)^{-\alpha/(1-\alpha)}$.

Let $C_2(Y)$ describe the lowest cost at which Y units of output could be produced using two shifts:

$$\begin{aligned} C_2(Y) &= \min_{N_1, N_2} Wh(N_1 + \psi N_2) + (R + \delta_0 + \delta(2h)^\gamma)K \\ &\text{s.t. } Y = hK^\alpha X_i^{1-\alpha} (N_1^{1-\alpha} + N_2^{1-\alpha}), \end{aligned}$$

where $\delta_2 = R + \delta_0 + \delta(2h)^\gamma > \delta_1$. Notice that it is optimal to set the relative marginal products on the two shifts $(N_2/N_1)^{-\alpha}$ equal to ψ , the relative wage. The constraint can then be used to solve for N_1 and N_2 so that

$$C_2(Y) = AY^{1/(1-\alpha)} (1 + \psi^{(\alpha-1)/\alpha})^{\alpha/(\alpha-1)} + (R + \delta_2)K,$$

Notice that the variable part of $C_2(Y)$ is everywhere less than the variable part of $C_1(Y)$ since $\psi > 0$ and $\alpha/(\alpha - 1) < 0$. In this model, the reason $C_2(Y)$ is not everywhere below $C_1(Y)$,

is because at low levels of output it will not be profitable to absorb the fixed depreciation cost, $(\delta_2 - \delta_1)K$, of operating the second shift. The firm's actual cost curve will be $C(Y) = \min\{C_1(Y), C_2(Y)\}$.

The graphs in Figure 2 have been drawn in such a way that it is optimal to operate one shift at the plant with a low value of ϵ , and two shifts at the plant with a high value of ϵ . It is also possible for a plant to have such a low level of idiosyncratic productivity that it simply doesn't produce, and takes a loss from the fixed cost of 'rust' depreciation. In the baseline parameterization of the model this happens with zero probability.

When a good aggregate shock hits the economy, this moves some one-shift plants into the two-shift category. The intuition for this result is obtained by reconsidering equations (2.16) and (2.17), which imply that ν_1 and ν_2 are proportional to W_t/X_t . The direct impact of a higher level of technology, X_t , on ν_{1t} and ν_{2t} is negative. However, there is also an offsetting indirect effect through the positive impact of the technology shock on the real wage, W_t . Whether ν_{1t} and ν_{2t} rise or fall depends on the combination of these effects. It turns out that in the short-run the combined effect is negative, because the real wage rises by less than the level of technology upon the impact of the shock. In the long-run W_t/X_t , ν_{1t} and ν_{2t} return to their steady state values.

3.2. Impulse Response Functions

Figure 3 illustrates the impulse response functions of several key variables in the model. These measure the response, in percent, of each variable to a one percent innovation in the level of aggregate technology. Given (3.1) this innovation permanently raises the level of technology. As in the indivisible labor model with permanent shocks to technology, this permanent rise in productivity causes the household to increase its lifetime consumption. Consumption rises from below to reach its permanently higher level because the marginal product of capital and the interest rate lie above their long-run values throughout the transition to the new steady state.

Also as in the indivisible labor model, there are substitution and income effects on labor supply. The substitution effects are driven by the real wage (marginal product of labor) which rises directly due to the technology shock, and indirectly due to capital accumulation. These effects dominate the downward income effect on labor supply that is driven by the increase in household's lifetime consumption.

The workweek of capital responds to the technology shock because the shock affects the threshold values at which it is efficient to run one or two shifts, ν_1 and ν_2 . Recall that the expressions (2.10) and (2.12) show that ν_1 and ν_2 move in proportion to W/X . The technology shock raises X directly which tends to decrease the threshold values at which a

plant is sufficiently productive to be operated. However, a 1 percent technology shock also raises W . Notice that at the plant level the marginal product of labor is given by

$$MPN = (1 - \alpha)hK^\alpha X_i^{1-\alpha} N_i^{-\alpha}.$$

Hence, because of the direct effect of the technology shock, W rises by $1 - \alpha$ percent. It also rises by α percent of any response of the capital stock. We know that in the long-run the capital stock increases by 1 percent, and that there are no long-run labor supply effects, so that in the long-run W rises by 1 percent. Thus, there is no long-run effect on ν_1 and ν_2 .

However, it turns out that even in the short-run technology shocks have almost no effect on the intensive margin, i.e. on how much labor is used at each plant that is operating. Most of the effect on total labor hours is due to variation at the extensive margin, i.e. how many shifts are operating. Hence, the very short-run effect of the shock on the marginal product of labor is dominated by the $X^{1-\alpha}$ term. As a result, in the very short-run W/X falls by about α percent.

The short-run decline of ν_{1t} and ν_{2t} , in response to a positive technology shock, implies that the workweek of capital increases. In the baseline parameterization this implies that the number of plants running two shifts rises, but, since ν_{1t} lies below the domain of ϵ , the number of plants running at least one shift remains unchanged at 100 percent. Consequently, the utilization rate of the firm's capital increases. In the baseline parameterization the workweek of capital increases by 0.8 percent in the impact period and remains above its steady-state level for several periods. This increase reflects a 5 percent increase in the number of plants operating second shifts.

As mentioned above, the increase in overall employment is almost all at the extensive margin. It is distributed across shifts in the following way: a small drop in the number of workers on each first shift, and a sharp rise in the number of workers on second shifts. This result can be explained as follows. The demand for first-shift labor at a plant, according to (2.6), is proportional to $(K/X_i)(X_i/W)^{1/\alpha}$. This rises in response to a shock if capital converges faster to its new long run steady state value than $W^{1/\alpha}$ does. In the baseline parameterization of the model the opposite is the case, but the result is sensitive to several of the model's parameters. There are other parameterizations in which employment rises at all plants, in addition to more shifts being added.

Figure 3 also shows that there is a significant response of output to the shock. Output goes up by more than 1.35 percent in the impact period. Output goes up rapidly (measured in percentage terms) across all second shifts, while output goes up by less than its long-run level across all first shifts. This pattern of output changes occurs for two reasons. At the impact of the shock, output rises at all operating plants because productivity is higher. But

output goes up more across all second shifts because more of these are operated. After the impact period, as the number of second shifts being operated actually declines (recall that v_{2t} converges back to its steady state value), this effect dominates the accumulation of capital, and total output across second shifts falls. However, the accumulation of capital causes a continued increase in output across all first shifts. In the long-run about 80 percent of the increase in output comes from first shifts and 20 percent from second shifts, as these are the steady state shares of each type of shift in total output.

In contrast to Burnside and Eichenbaum (1996), the impulse-response functions in this model do not display any hump-shape. In other words, the maximal response of all variables occurs in the impact period. This is because a key feature of their model is missing from the current one. Because employment at the plant level can respond immediately to aggregate and plant level shocks, the output response is maximized in the impact period. In their model, employment could not adjust instantaneously to shocks, and was set one period in advance. As a result, the maximal response occurred in the second period, producing a hump shape. Adding this sort of feature to the current model would be a straightforward extension.

3.3. Statistical Properties of the Model

Table 2 presents several first and second moments of key time series. The table indicates the implications of the model, as well as the corresponding statistics from U.S. data. The sample of U.S. data is from 1974–1992, the period over which the workweek of capital data are available. I convert these data into workdays, and thus measure the utilization rate in terms of the hours per day that capital is in use. The standard U.S. macroeconomic time series data are described in more detail in Burnside and Eichenbaum (1996). The measure of consumption is real per capita consumption of nondurables and services plus an imputed service flow from the stock of consumer durables. Investment is gross fixed investment by the private and public sectors, plus purchases of consumer durables. Output is the sum of consumption and investment, government purchases of goods and services, plus net exports.¹⁴ Hours worked was measured by total hours worked from the establishment survey. The capital stock was measured by the sum of all private and government capital.

Good results in the third column of Table 2, labeled ‘Baseline Model,’ should not be regarded as surprising, as the parameters of the model were chosen so that it would closely replicate the output-capital ratio, the average depreciation rate of capital, the average growth rate of output and the variance of output. Like many RBC models, the model does reasonably

¹⁴An alternative would have been to measure Y as $C + I$, but this would still have left the difficulty of which measure of the capital stock to use.

well at mimicking the relative volatilities of consumption and output as well as investment and output. The model underpredicts the volatility in hours, which, in the model, arises entirely from changes in the level of employment. Although the model roughly matches the mean workday of capital, it does not do well in terms of the volatility of the workday, whose standard deviation is less than two-thirds the size of its counterpart in the data.¹⁵ Increased volatility in employment and the workday could be obtained by increasing the variance of the aggregate shocks. However, this would come at the cost of additional, counterfactual, output volatility.

As in Burnside and Eichenbaum (1996), and Burnside, Eichenbaum and Rebelo (1993) there is endogenous measurement error in the Solow residual. Because the workweek of capital responds to shocks to the level of technology, the Solow residual partly reflects the direct impact of shocks to technology and partly reflects changes in the workweek (see Figure 3). If the growth rate of the Solow residual is computed in a standard way as

$$\Delta \ln(S_t) = (1 - \alpha)^{-1} [\Delta \ln(Y_t) - \alpha \Delta \ln(K_t) - (1 - \alpha) \Delta \ln(N_t)]$$

then the model implies that it will be equal to the sum of the innovation to technology and an endogenous measurement error, e_t (see the Appendix):

$$\Delta \ln(S_t) = \mu + \xi_t + e_t.$$

As it turns out, the baseline parameterization of the model predicts that the standard deviation of $\Delta \ln(S_t)$ is 1.03 percent, when the standard deviation of the true shocks to technology is $\sigma_\xi = 0.7$ percent. The error, e_t , is not very volatile, with a standard deviation of 0.33 percent, but is almost perfectly correlated with ξ_t , which explains the magnitude of the impact on the variance of the Solow residual.

The next columns shows that increasing the amount of idiosyncratic uncertainty by 50% (in terms of its standard deviation, σ_ϵ) increases the mean workday of capital, but its standard deviation actually declines significantly. It has little impact on the volatility of hours worked. The effect on the volatility of the workday can be understood by considering its definition $\bar{U}_t = 2h - [F(\nu_{1t}) + F(\nu_{2t})]h$. In the baseline parameterization, and the parameterization with $\sigma_\epsilon = 0.15$, $F(\nu_1) = 0$ at the steady state value of ν_1 implying that there are no plants which shut down completely. In this case the workday is $\bar{U}_t = [2 - F(\nu_{2t})]h$. Assuming ϵ is drawn from the uniform distribution, we have the further result that

$$\bar{U}_t = \left(\frac{3}{2} - \frac{\sqrt{3}\nu_{2t}}{6\sigma_\epsilon} \right) h.$$

¹⁵The workweek of capital in my data set corresponds to the manufacturing sector, whereas the model is a model of the entire economy. Since manufacturing is a relatively volatile sector this finding may not be too troubling.

Although the volatility of ν_{2t} goes up with the increase in σ_ϵ , it does not go up as much as σ_ϵ , so the volatility of the workweek actually drops.

Decreasing the premium for work at night, determined by the parameter ψ , to a tiny constant raises the workday by about 2 hours, since the cost of running second shifts decreases. But this leaves the volatility of the workday and employment almost unchanged.

Finally, there are the parameters of the depreciation rate, δ_0 , δ and γ . The model was parameterized by letting $h = 0.5$, so that $\delta_1 = \delta_0 + \delta \left(\frac{1}{2}\right)^\gamma$ while $\delta_2 = \delta_0 + \delta$. For a fixed value of γ , lowering the parameter δ lowers the fixed costs of moving from zero to one shift, and of moving from one to two shifts, in exact proportion. This is expected to lead to an increase in the workweek of capital. Interestingly, lowering δ to about 0.035 from 0.047 leaves the mean depreciation rate of capital almost unchanged because the increase in the workweek offsets the direct effect of the smaller δ . The workday increases by about 2.4 hours, and its standard deviation rises slightly. The volatility of the growth rates of output, consumption, employment, and investment all fall slightly.

Leaving δ unchanged and decreasing γ , on the other hand, increases the workweek and increases the depreciation rate if one shift is operated, while leaving unchanged the depreciation rate if two shifts are operated. So, this means that lowering γ unambiguously raises the depreciation rate of capital. With $\gamma = 1.31$ rather than 1.55, the volatility of the growth rates of output, consumption, employment, and investment all fall. The workday becomes longer by over 3 hours on average but is no more volatile than before, while the average depreciation rate of capital is unrealistically high.

Another experiment is one which lowers γ and δ at the same time, to leave δ_1 roughly unchanged, but making δ_2 lower. With $\delta = 0.044$ and $\gamma = 1.46$ the mean depreciation rate rises, because the workday rises by almost 2 hours. However, the standard deviation of the workday is almost unchanged.

Finally, small changes in the parameter δ_0 have almost no impact on the results, because the probability of shutting down a plant completely in the baseline model is zero.

3.4. Drawbacks of the Model

Clearly the baseline model and its variants face systematic difficulties in matching key features of the aggregate data: the volatility of hours worked and the volatility of the workweek of capital. This may be due to several features of the baseline parameterization of the model:

1. Shutdowns never happen. Shutdowns occur when σ_ϵ is approximately doubled. However, even with a significant increase in σ_ϵ the volatility of the workweek is not much higher than it is with $\sigma_\epsilon = 0.10$. In fact, the volatility of the workday as a function

of σ_ϵ is plotted in Figure 4. The discontinuities at $\sigma_\epsilon \approx 0.055$ and $\sigma_\epsilon \approx 0.185$ are explained as follows. For $\sigma_\epsilon < 0.055$ all plants run one shift in the nonstochastic steady state. As a result, the linearized model implies no volatility in the workday.¹⁶ For $0.055 < \sigma_\epsilon < 0.185$ a fraction of the plants operates second shifts in nonstochastic steady state, but there are no shutdowns. For $\sigma_\epsilon > 0.185$ there are shutdowns as well. So, apart from the points of discontinuity, the volatility of the workday is decreasing in σ_ϵ .

2. Overtime is not modeled. This is probably important in limiting the model's ability to match the volatility of hours worked simultaneously with the volatility of output.
3. At the micro level, the model assumes that plant level productivity shocks are i.i.d. Although this is analytically convenient, it causes the model to miss a key feature of the data, which is that plant shift patterns are persistent. If the data from the automobile industry are representative of the rest of the economy, this persistence might better be captured by modeling shocks as coming from tastes, or by modeling the vintages of different plants.
4. The model may be too restrictive in forcing the 'rust' depreciation to be positive. If firms actually perform maintenance when capital is idle, then the effective cost of being idle for a period may be negative. In fact, parameterizations of the model in which $\delta_0 < 0$ perform better on several dimensions.
5. As suggested by Betancourt and Clague (1981), there may be economies of scale in terms of how duration of use affects maintenance costs. They argue that the depreciation cost of running extra shifts is probably small. In the extreme, if this was incorporated into the current model, then the important margin would become the zero-two shift margin, and no firm would operate only one shift. Their argument suggests that if fixed costs exist, they are associated with some other aspect of production other than the depreciation rate of capital.

Ongoing work should investigate alternative models which address these issues. The most difficult issues to deal with are likely the second and third, but Hall's (2000) work suggests that these are key elements in explaining the nature of the workweek, at least in the automobile industry. Modeling persistent idiosyncratic taste shocks or productivity shocks will add a layer of analytical difficulty to the model which is currently absent. The

¹⁶An exact solution to the model, which might, for example, assume normality of the aggregate technology shock would assign some small positive probability to shutdowns and second shifts, but this probability would be negligible because of the relative size of σ_ξ and σ_ϵ .

main difficulty is in keeping track of the distribution of state variables across plants through time. The other issues can be satisfactorily dealt with in models with i.i.d. idiosyncratic uncertainty.

4. Conclusions

In this paper I have examined a model of time-varying capital utilization in which the margin along which firms can vary utilization is the number of shifts they operate at each plant. Since shifts come in fixed lengths, operating additional shifts at a plant requires that a firm absorbs additional fixed depreciation costs associated with the increased operating time of the plant. This nonconvexity in the firm's cost function means that with a distribution in productivity levels across plants which is observed after the allocation of capital, there are threshold values of productivity defining whether the firm will run zero, one or two shifts at each plant.

The implications of the model are several. First, the model implies that small technology shocks are propagated to a much greater degree than in a standard RBC model. The firm's ability to use its utilization margin implies that output will be more volatile than the process driving the aggregate level of technology. This finding is consistent with the findings of Burnside and Eichenbaum (1996). The advantage of the current model is that it provides micro-foundations for the utilization margin at the plant level.

Second, I show that the model predicts substantial variation in the Solow residual not directly due to technology shocks, but, instead, due to endogenous variation in the rate of utilization. Standard measures of technology shocks based on the Solow residual would overstate the magnitude of these shocks by almost 50% given my baseline calibration.

Third, while the model can match the volatility of output, for a wide range of parameter values it does not appear to be able to replicate the amount of variation in aggregate hours or the amount of variation in the workweek of capital. One aspect of this is that plants rarely shut down. I argue that this is likely due to specific features of the model; for example, the fact that the model abstracts from the ability of firms to use overtime, and the fact that the nature of fixed costs may be quite different than what I have assumed.

Fourth, the model does not allow for persistence in the cross-sectional distribution of the workweek across plants. This is a simple function of the i.i.d. nature of the idiosyncratic shocks in the model. While the i.i.d. assumption is analytically convenient, it will need to be discarded in future work in order to rationalize a key feature of the data.

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A. Appendix

A.1. Uniqueness of the Nonstochastic Steady State

In the nonstochastic steady state of the competitive equilibrium defined above, the necessary conditions (2.15)–(2.26) must hold for constant values of the variables that appear in the equations. The steady state value of X_t is normalized to $X = 1$. Notice that (2.22) implies

$$R = \beta^{-1} - 1. \quad (\text{A.1})$$

Hence, (2.23) implies

$$\beta^{-1} - 1 = \alpha (1 - \alpha)^{(1-\alpha)/\alpha} h W^{(\alpha-1)/\alpha} [g(\nu_1) + \psi^{(\alpha-1)/\alpha} g(\nu_2)] - \bar{\delta}, \quad (\text{A.2})$$

with, from (2.16), (2.17), and (2.21)

$$\nu_1 = \frac{\alpha}{1 - \alpha} \ln(\delta h^\gamma) + \ln(W) - \kappa, \quad (\text{A.3})$$

$$\nu_2 = \nu_1 + \ln \psi + \frac{\alpha}{1 - \alpha} \ln(2^\gamma - 1), \quad (\text{A.4})$$

$$\bar{\delta} = \delta_0 + \delta h^\gamma [2^\gamma - F(\nu_1) - (2^\gamma - 1) F(\nu_2)] \quad (\text{A.5})$$

where

$$g(\nu) = \int_\nu^{\bar{\epsilon}} e^{(1-\alpha)\epsilon/\alpha} f(\epsilon) d\epsilon. \quad (\text{A.6})$$

Together (A.2)–(A.6) imply a nonlinear equation $G(W) = \beta^{-1} - 1$ which has a unique solution, W .¹⁷ To see this, notice that

$$\begin{aligned} G'(W) &= \frac{\alpha - 1}{\alpha} \alpha (1 - \alpha)^{(1-\alpha)/\alpha} h W^{-1/\alpha} [g(\nu_1) + \psi^{(\alpha-1)/\alpha} g(\nu_2)] + \\ &\quad \alpha (1 - \alpha)^{(1-\alpha)/\alpha} h W^{-1/\alpha} [g'(\nu_1) + \psi^{(\alpha-1)/\alpha} g'(\nu_2)] + \\ &\quad + \delta h^\gamma [f(\nu_1) + (2^\gamma - 1) f(\nu_2)] W^{-1}. \end{aligned}$$

Using (A.3), (A.4) and (2.11) notice that

$$\begin{aligned} g'(\nu_1) &= -e^{(1-\alpha)\nu_1/\alpha} f(\nu_1) \\ &= -\delta h^\gamma W^{(1-\alpha)/\alpha} \alpha^{-1} (1 - \alpha)^{(\alpha-1)/\alpha} h^{-1} f(\nu_1) \end{aligned}$$

and that

$$\begin{aligned} g'(\nu_2) &= -e^{(1-\alpha)\nu_2/\alpha} f(\nu_2) \\ &= -\delta h^\gamma (\psi W)^{(1-\alpha)/\alpha} \alpha^{-1} (1 - \alpha)^{(\alpha-1)/\alpha} h^{-1} (2^\gamma - 1) f(\nu_2). \end{aligned}$$

¹⁷ $G(W)$ is the right-hand side of (A.2) written as a function of W .

Hence

$$G'(W) = \frac{\alpha - 1}{\alpha} \alpha (1 - \alpha)^{(1-\alpha)/\alpha} hW^{-1/\alpha} [g(\nu_1) + \psi^{(\alpha-1)/\alpha} g(\nu_2)] < 0.$$

We also know that as $W \rightarrow 0$, $\nu_j \rightarrow -\infty$, $F(\nu_j) \rightarrow 0$, $g(\nu_j) \rightarrow E[e^{(1-\alpha)\epsilon/\alpha}]$ and $\bar{\delta} \rightarrow \delta_0 + \delta(2h)^\gamma$. Hence $\lim_{W \rightarrow 0} G(W) = \infty$. Also as $W \rightarrow \infty$, $\nu_j \rightarrow \infty$, $F(\nu_j) \rightarrow 1$, $g(\nu_j) \rightarrow 0$ and $\bar{\delta} \rightarrow \delta_0$. Hence $\lim_{W \rightarrow \infty} G(W) = -\delta_0$. Given that $G'(W) < 0$ for all $W > 0$, $\lim_{W \rightarrow 0} G(W) = \infty$ and $\lim_{W \rightarrow \infty} G(W) = -\delta_0$, there must be a value of $0 < W < \infty$ for which (A.2) holds.

Once the nonstochastic steady state value of W is determined, the other steady state values are easily obtained. The steady state value of C is given, from (2.20) as $C = Wh/\eta_1$. We have ν_1 , ν_2 , and $\bar{\delta}$, given by (A.3), (A.4), and (A.5). From (2.24) and (2.25), $Y_1/K = (1 - \alpha)^{(1-\alpha)/\alpha} hW^{(\alpha-1)/\alpha} g(\nu_1)$ and $Y_2/K = (1 - \alpha)^{(1-\alpha)/\alpha} h(\psi W)^{(\alpha-1)/\alpha} g(\nu_2)$. Of course this implies $Y/K = Y_1/K + Y_2/K$. Then from (2.15) we have $K = C/[(Y/K) - \bar{\delta}]$. Finally, $N_1 = (1 - \alpha)^{1/\alpha} KW^{-1/\alpha} g(\nu_1)$, and $N_2 = (1 - \alpha)^{1/\alpha} K(\psi W)^{-1/\alpha} g(\nu_2)$.

A.2. The Linearized Model

The next step is to linearize the system of equilibrium conditions (2.15)–(??), (2.20)–(2.26).¹⁸ I will define $\hat{c}_t = (C_t - C)/C$, $\hat{y}_t = (Y_t - Y)/Y$, $\hat{k}_t = (K_t - K)/K$, $\hat{\delta}_t = \bar{\delta}_t - \bar{\delta}$, $\hat{\nu}_{jt} = \nu_{jt} - \nu_j$, $\hat{w}_t = (W_t - W)/W$, $\hat{R}_t = R_t - R$, and $\hat{y}_{jt} = (Y_{jt} - Y_j)/Y_j$, and $\hat{x}_t = X_t - 1$, where Z generically represents the nonstochastic steady state value of the variable Z_t .

The resource constraint (2.15) becomes

$$\frac{C}{Y} \hat{c}_t = \hat{y}_t + (1 - \bar{\delta}) \frac{K}{Y} \hat{k}_t - \frac{K}{Y} \hat{\delta}_t - \frac{K}{Y} \hat{k}_{t+1}. \quad (\text{A.7})$$

The expressions for the cutoff values (2.16) and (2.17) become

$$\hat{\nu}_{1t} = \hat{\nu}_{2t} = \hat{w}_t - \hat{x}_t. \quad (\text{A.8})$$

The first order condition for labor supply, (2.20), becomes

$$\hat{w}_t = \hat{c}_t. \quad (\text{A.9})$$

The depreciation rate, (2.21), becomes

$$\hat{\delta}_t = -\delta h^\gamma f(\nu_1) \hat{\nu}_{1t} - \delta h^\gamma (2^\gamma - 1) f(\nu_2) \hat{\nu}_{2t}. \quad (\text{A.10})$$

The Euler equation for capital, (2.22), becomes

$$E_t \hat{c}_{t+1} - \hat{c}_t = \beta E_t \hat{R}_{t+1}. \quad (\text{A.11})$$

¹⁸I have eliminated the equations (2.18) and (2.19) because (i) these determine N_{1t} and N_{2t} in terms of the other variables, and (ii) N_{1t} and N_{2t} do not appear in any of the other equations.

The expression for R_t , (2.23), becomes

$$\begin{aligned}\hat{R}_t &= \frac{1-\alpha}{\alpha} \alpha (1-\alpha)^{(1-\alpha)/\alpha} hW^{(\alpha-1)/\alpha} [g(\nu_1) + \psi^{(\alpha-1)/\alpha} g(\nu_2)] (\hat{x}_t - \hat{w}_t) \\ &\quad + \alpha (1-\alpha)^{(1-\alpha)/\alpha} hW^{(\alpha-1)/\alpha} [g'(\nu_1) \hat{\nu}_{1t} + \psi^{(\alpha-1)/\alpha} g'(\nu_2) \hat{\nu}_{2t}] - \hat{\delta}_t.\end{aligned}\quad (\text{A.12})$$

The expressions for output, (2.24), (2.25), and (2.26) become

$$\hat{y}_{jt} = \frac{1-\alpha}{\alpha} (\hat{x}_t - \hat{w}_t) + \hat{k}_t + [g'(\nu_j)/g(\nu_j)] \hat{\nu}_{jt} \quad (\text{A.13})$$

and

$$\hat{y}_t = (Y_1/Y) \hat{y}_{1t} + (Y_2/Y) \hat{y}_{2t}. \quad (\text{A.14})$$

The system (A.7)–(A.14) represents a system of ten equations in ten unknowns. We reduce this system by eliminating the variables \hat{y}_{1t} , \hat{y}_{2t} , $\hat{\nu}_{1t}$, $\hat{\nu}_{2t}$ and \hat{w}_t , along with the five equations (A.13), (A.8) and (A.9).

Notice that we then have a system of five equations

$$\frac{C}{Y} \hat{c}_t = \hat{y}_t + (1-\bar{\delta}) \frac{K}{Y} \hat{k}_t - \frac{K}{Y} \hat{\delta}_t - \frac{K}{Y} \hat{k}_{t+1}. \quad (\text{A.15})$$

$$\hat{\delta}_t = -\delta h^\gamma [f(\nu_1) + (2^\gamma - 1) f(\nu_2)] (\hat{c}_t - \hat{x}_t). \quad (\text{A.16})$$

$$E_t \hat{c}_{t+1} - \hat{c}_t = \beta E_t \hat{R}_{t+1}. \quad (\text{A.17})$$

$$\begin{aligned}\hat{R}_t &= \frac{1-\alpha}{\alpha} \alpha (1-\alpha)^{(1-\alpha)/\alpha} hW^{(\alpha-1)/\alpha} [g(\nu_1) + \psi^{(\alpha-1)/\alpha} g(\nu_2)] (\hat{x}_t - \hat{c}_t) \\ &\quad + \alpha (1-\alpha)^{(1-\alpha)/\alpha} hW^{(\alpha-1)/\alpha} [g'(\nu_1) + \psi^{(\alpha-1)/\alpha} g'(\nu_2)] (\hat{c}_t - \hat{x}_t) - \hat{\delta}_t.\end{aligned}\quad (\text{A.18})$$

$$\hat{y}_t = \frac{1-\alpha}{\alpha} (\hat{x}_t - \hat{c}_t) + \hat{k}_t + \left[\frac{Y_1 g'(\nu_1)}{Y g(\nu_1)} + \frac{Y_2 g'(\nu_2)}{Y g(\nu_2)} \right] (\hat{c}_t - \hat{x}_t). \quad (\text{A.19})$$

In the previous section we showed that

$$g'(\nu_1) = -\delta h^\gamma W^{(1-\alpha)/\alpha} \alpha^{-1} (1-\alpha)^{(\alpha-1)/\alpha} h^{-1} f(\nu_1) \quad (\text{A.20})$$

$$g'(\nu_2) = -\delta h^\gamma (\psi W)^{(1-\alpha)/\alpha} \alpha^{-1} (1-\alpha)^{(\alpha-1)/\alpha} h^{-1} (2^\gamma - 1) f(\nu_2). \quad (\text{A.21})$$

Using these results and (A.16), we can rewrite (A.18) as

$$\begin{aligned}\hat{R}_t &= \frac{1-\alpha}{\alpha} \alpha (1-\alpha)^{(1-\alpha)/\alpha} hW^{(\alpha-1)/\alpha} [g(\nu_1) + \psi^{(\alpha-1)/\alpha} g(\nu_2)] (\hat{x}_t - \hat{c}_t) \\ &= \frac{1-\alpha}{\alpha} (R + \bar{\delta}) (\hat{x}_t - \hat{c}_t).\end{aligned}\quad (\text{A.22})$$

We proceed to eliminate \hat{R}_t and $\hat{\delta}_t$ from our system of equations and obtain

$$\frac{C}{Y} \hat{c}_t = \hat{y}_t - \frac{K}{Y} \delta h^\gamma [f(\nu_1) + (2^\gamma - 1) f(\nu_2)] (\hat{x}_t - \hat{c}_t) + (1-\bar{\delta}) \frac{K}{Y} \hat{k}_t - \frac{K}{Y} \hat{k}_{t+1} \quad (\text{A.23})$$

$$E_t \hat{c}_{t+1} - \hat{c}_t = \beta \frac{1-\alpha}{\alpha} (R + \bar{\delta}) E_t (\hat{x}_{t+1} - \hat{c}_{t+1}) \quad (\text{A.24})$$

$$\hat{y}_t = \left\{ \frac{1-\alpha}{\alpha} - \left[\frac{Y_1 g'(\nu_1)}{Y g(\nu_1)} + \frac{Y_2 g'(\nu_2)}{Y g(\nu_2)} \right] \right\} (\hat{x}_t - \hat{c}_t) + \hat{k}_t. \quad (\text{A.25})$$

In the last step, we can eliminate \hat{y}_t from the system of equations. In doing this it is useful to note that the expressions for Y_1/K and Y_2/K from the previous section, and the expressions (A.20) and (A.21) imply that

$$\begin{aligned} \frac{Y_1 g'(\nu_1)}{Y g(\nu_1)} &= -\frac{1}{\alpha} \frac{K}{Y} \delta h^\gamma f(\nu_1) \\ \frac{Y_2 g'(\nu_2)}{Y g(\nu_2)} &= -\frac{1}{\alpha} \frac{K}{Y} \delta h^\gamma (2^\gamma - 1) f(\nu_2). \end{aligned}$$

Hence, defining

$$\Delta = \delta h^\gamma [f(\nu_1) + (2^\gamma - 1) f(\nu_2)]$$

we can rewrite (A.23) as

$$\frac{C}{Y} \hat{c}_t = \frac{1-\alpha}{\alpha} \left(1 + \frac{K}{Y} \Delta \right) (\hat{x}_t - \hat{c}_t) + \left[1 + (1-\bar{\delta}) \frac{K}{Y} \right] \hat{k}_t - \frac{K}{Y} \hat{k}_{t+1} \quad (\text{A.26})$$

and repeat (A.24) as

$$E_t \hat{c}_{t+1} - \hat{c}_t = \beta \frac{1-\alpha}{\alpha} (R + \bar{\delta}) E_t (\hat{x}_{t+1} - \hat{c}_{t+1}). \quad (\text{A.27})$$

To determine stability we can write these two equations as a first-order system of difference equations in the vector $z_t = (\hat{k}_t \quad \hat{c}_t)'$:

$$E_t z_{t+1} = W z_t + R E_t \hat{x}_{t+1} + Q \hat{x}_t,$$

where

$$\begin{aligned} W &= \begin{pmatrix} \frac{Y}{K} + (1-\bar{\delta}) & (1-1/\alpha)(Y/K + \Delta) - \frac{C}{K} \\ 0 & [1 + \beta(1/\alpha - 1)(R + \bar{\delta})]^{-1} \end{pmatrix} \\ R &= \begin{pmatrix} 0 \\ \beta(R + \bar{\delta}) / [\alpha/(1-\alpha) + \beta(R + \bar{\delta})] \end{pmatrix} \\ Q &= \begin{pmatrix} (1/\alpha - 1)(Y/K + \Delta) \\ 0 \end{pmatrix} \end{aligned}$$

There is a unique bounded path associated with this system, whenever one of the eigenvalues of W is greater than 1 in absolute value while the other is less than 1. The eigenvalues are simply the diagonal elements of W . But, by inspection we can easily see that the second diagonal element of W , $[1 + \beta(1/\alpha - 1)(R + \bar{\delta})]^{-1}$, is less than 1. The first diagonal element is greater than 1 since $Y/K > \bar{\delta}$ in order for consumption to be positive in the steady state.

A.3. The Solow Residual

We have the result that $N_{i1} > 0$ for $\epsilon_i \geq \nu_1$ and $N_{i2} > 0$ for $\epsilon_i \geq \nu_2$. Let $\bar{N}_j = N_{ij}$ for $\epsilon_i = \nu_j$. Then (2.6) implies that

$$N_{ij} = \exp \left[\frac{1-\alpha}{\alpha} (\epsilon_i - \nu_j) \right] \bar{N}_j. \quad (\text{A.28})$$

Since $N_j = \int_0^1 N_{ij} di$, we obtain

$$N_j = g(\nu_j) \exp \left(-\frac{1-\alpha}{\alpha} \nu_j \right) \bar{N}_j. \quad (\text{A.29})$$

Using (A.28) and (A.29) we can rewrite output, given by (2.4), as

$$Y = hK^\alpha X^{1-\alpha} [N_1^{1-\alpha} g(\nu_1)^\alpha + N_2^{1-\alpha} g(\nu_2)^\alpha].$$

This implies that the Solow residual

$$S \equiv \left(\frac{Y}{K^\alpha N^{1-\alpha}} \right)^{1/(1-\alpha)} \propto X \left[\left(\frac{N_1}{N} \right)^{1-\alpha} g(\nu_1)^\alpha + \left(\frac{N_2}{N} \right)^{1-\alpha} g(\nu_2)^\alpha \right]^{1/(1-\alpha)}.$$

Using (2.18) and (2.19) we can rewrite this expression as

$$S \propto X [g(\nu_1) + \psi^{(\alpha-1)/\alpha} g(\nu_2)]^{1/(1-\alpha)} [g(\nu_1) + \psi^{-1/\alpha} g(\nu_2)]^{-1}.$$

TABLE 1

BASELINE PARAMETER VALUES

β	0.99	η	1.326
α	0.371	δ_0	0.001
μ	0.0025	δ	0.047
σ_ξ	0.007	γ	1.555
σ_ϵ	0.1	ϕ	0.05

Note: The parameters are defined in the text. Note that $\eta_1 = \eta \ln(2 - \phi) \approx 0.889$, and $\psi = \eta_2/\eta_1 = 1 - \ln(1 - \phi)/\ln(2 - \phi) \approx 1.077$.

TABLE 2

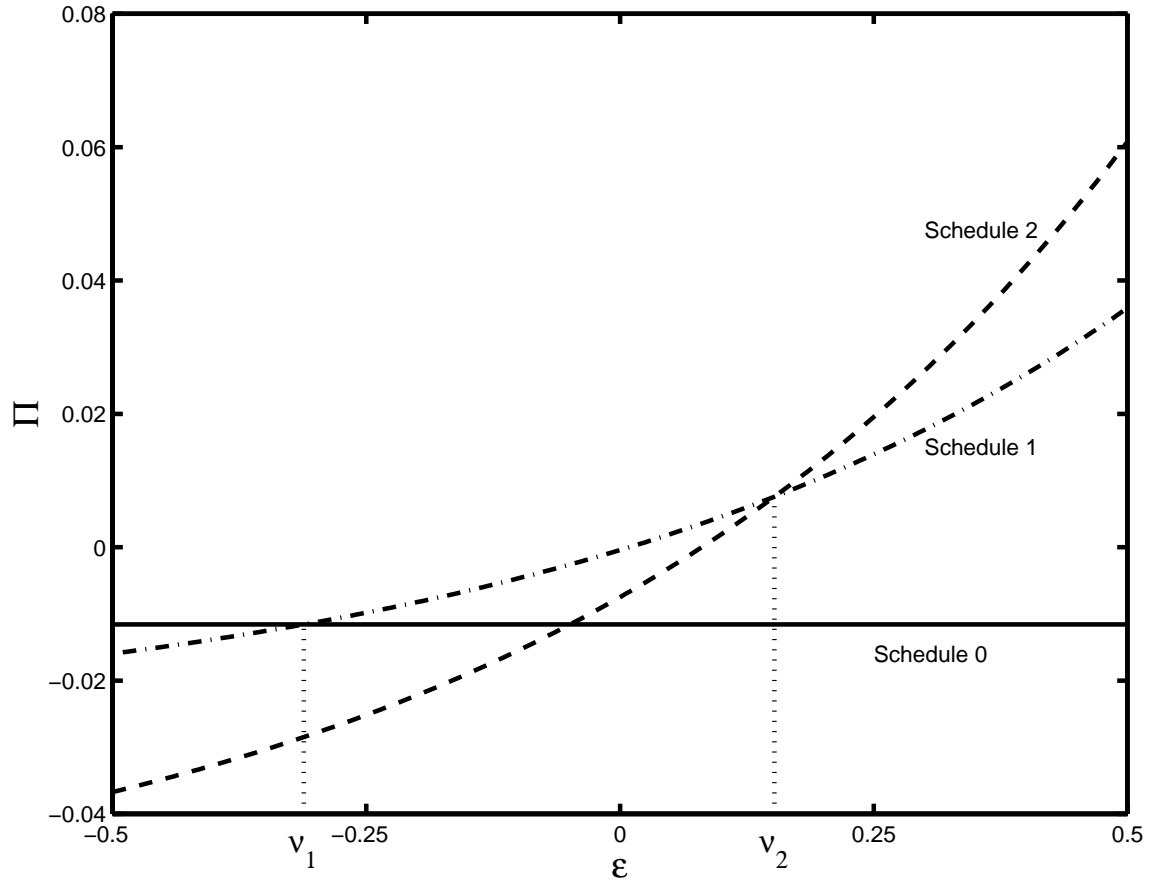
PROPERTIES OF THE MODEL

Statistic	U.S. Data	Baseline Model	$\sigma_\epsilon = 0.15$	$\psi \approx 1$	$\delta = 0.035$	$\gamma = 1.31$	$\delta = 0.044$ $\gamma = 1.46$
Means							
I_t/Y_t	0.25	0.26	0.27	0.28	0.26	0.28	0.27
K_t/Y_t	10.6	10.3	9.8	9.0	10.6	8.1	9.4
$\bar{\delta}_t$	0.020	0.023	0.025	0.028	0.022	0.033	0.026
$\Delta \ln(Y_t)$	0.0026	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025
Workday	13.9h	14.3h	15.0h	16.3h	16.7h	17.4h	16.0h
Standard Deviations							
$\Delta \ln(Y_t)$	0.94%	0.97%	0.88%	0.93%	0.92%	0.89%	0.93%
$\Delta \ln(C_t)$	0.43%	0.47%	0.43%	0.46%	0.45%	0.45%	0.46%
$\Delta \ln(I_t)$	2.67%	2.40%	2.13%	2.20%	2.30%	2.03%	2.22%
$\Delta \ln(N_t)$	0.77%	0.47%	0.43%	0.48%	0.46%	0.43%	0.45%
Workday	0.39h	0.24h	0.18h	0.24h	0.27h	0.24h	0.25h

Note: The U.S. data are from 1974Q1–1992Q4, the period for which the workweek of capital data are available. The standard macroeconomic aggregates are described in more detail in Burnside and Eichenbaum (1996) and in the text. Statistical properties of the model were obtained using a simulated sample of 10000 observations. Note also that $\psi \approx 1$ corresponds to $\phi \approx 0$.

FIGURE 1

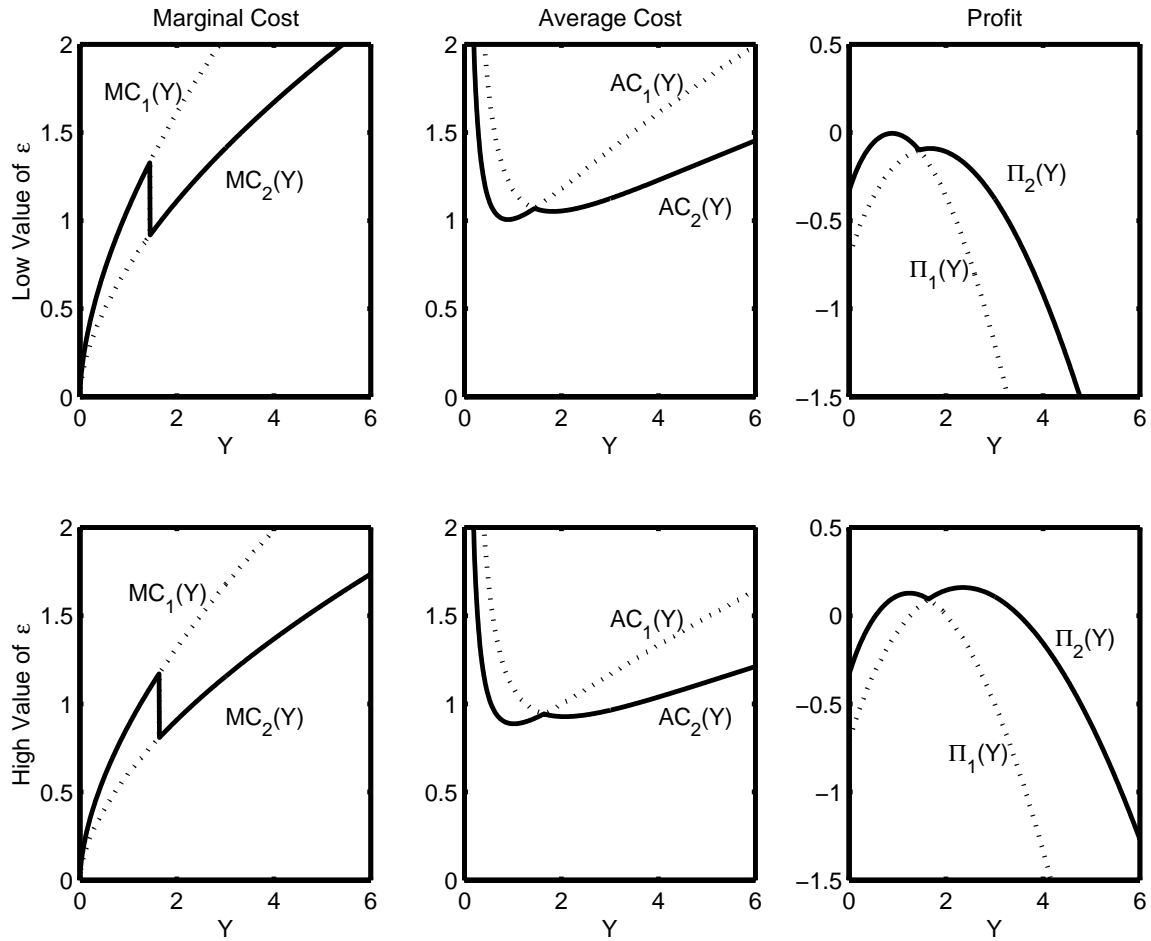
MAXIMAL PROFITS FOR DIFFERING NUMBERS OF SHIFTS AS A
FUNCTION OF THE SIZE OF THE IDIOSYNCRATIC SHOCK



Note: The schedules are drawn under the assumption that $K = X = 1$. Schedule x represents maximized profits conditional on the firm running x shifts. For $\varepsilon < v_1$ the firm shuts down both shifts. For $\varepsilon > v_2$, the firm runs two shifts. For $v_1 < \varepsilon < v_2$, the firm runs only the first shift.

FIGURE 2

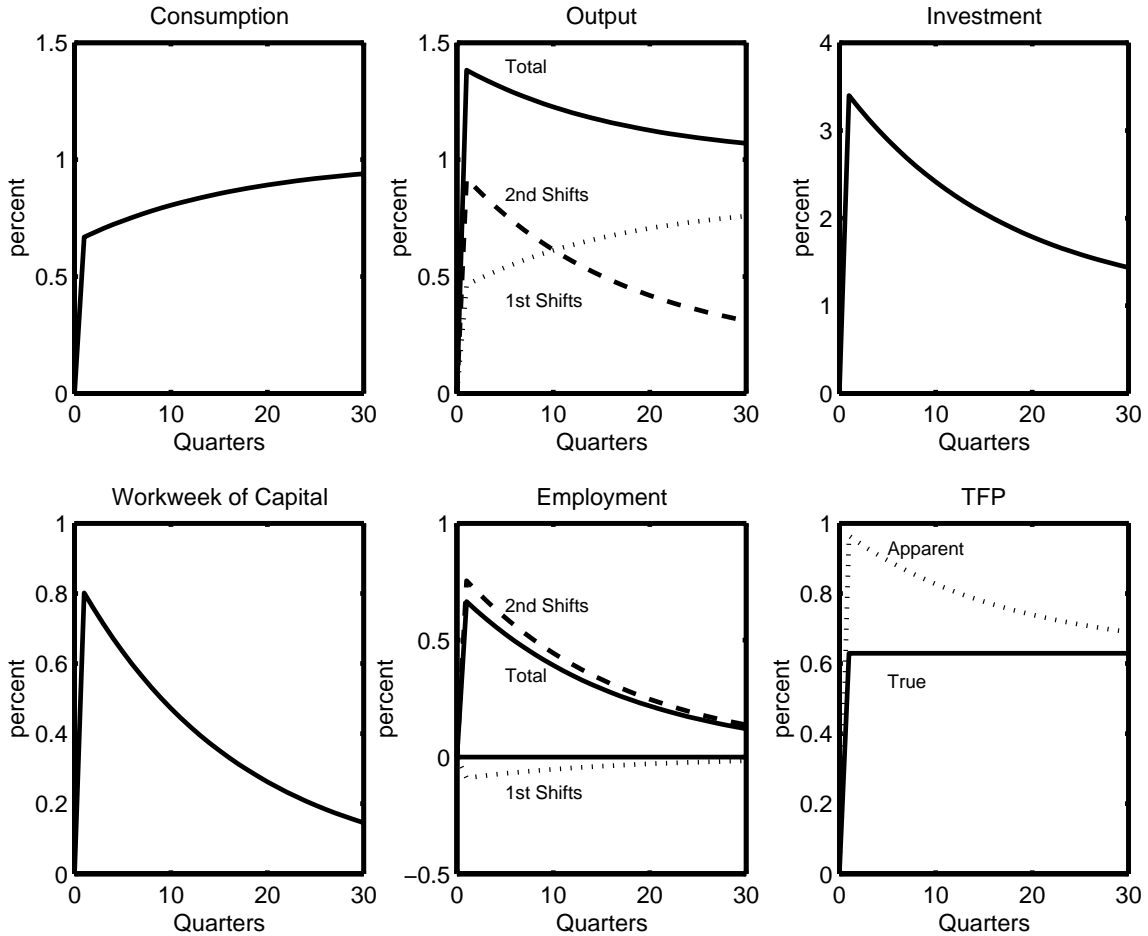
COST AND PROFIT FUNCTIONS AT THE PLANT LEVEL



Note: MC_i , AC_i and Π_i are the marginal cost, average cost and profit functions assuming i shifts are used to produce Y units of output. The solid lines represent the actual marginal cost, average cost and profit curves. The point of discontinuity in the marginal cost diagram is the level of output at which the firm would find it profitable to start using two shifts to produce its output, rather than one. To the left of this level of output, in any diagram, the dotted line is what cost or profit would be if, suboptimally, two shifts were used. To the right of this level of output, in any diagram, the dotted line is what cost or profit would be if, suboptimally, one shift was used.

FIGURE 3

IMPULSE RESPONSE FUNCTIONS



Note: Each figure shows the response of a variable, measured in units of its percentage deviation from its steady state value, to a 1 percent shock to the aggregate level of technology. In the output and employment diagrams the breakdown of the changes in output and employment between first and second shifts is also indicated. The TFP diagram indicates the change in true total factor productivity and the change in apparent TFP if a simple aggregate Cobb-Douglas production function is used to measure it.

FIGURE 4

IDIOSYNCRATIC UNCERTAINTY AND THE VOLATILITY OF THE WORKDAY

