

# Assessing New Approaches to Fiscal Sustainability Analysis\*

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The term fiscal sustainability has many definitions, though it almost always refers to the fiscal policies of a government or the public sector. One concept of sustainability relates to *solvency*, the ability of the government to service its debt obligations in perpetuity without explicit default. Rarely does fiscal sustainability analysis reach the conclusion that a government is insolvent unless it is already obvious to all concerned that it is insolvent.

Another concept of fiscal sustainability relates to the government's ability to maintain its *current* policies while remaining solvent. With this concept, fiscal sustainability analysis has broader scope. It can discuss the types and consequences of fiscal and monetary policy adjustments needed in order to avoid insolvency in the future. Even more broadly, fiscal sustainability analysis has, at times, encompassed discussions centered on the optimality of policy rather than its mere feasibility.

One of the shortcomings of fiscal sustainability analysis is that it often does not take into account the effects of uncertainty. As we will see below, most of the simple analytical tools for assessing sustainability uses the modeling framework of perfect foresight. This can lead to misleading conclusions about the sustainability of policy that are of the yes or no variety. Either policy is deemed sustainable or it is not. But little is said about the probability with which a government might become insolvent, or the risks to fiscal sustainability.

Recently the literature on fiscal sustainability has expanded in a number of interesting ways. Several methods for bringing uncertainty into the analysis have been proposed, with Barnhill and Kopits (2003), International Monetary Fund (IMF, 2003), Xu and Ghezzi (2003) and Mendoza and Oviedo (2004a) being prominent among them. The World Bank's Latin

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America and Caribbean region commissioned several case studies of fiscal sustainability that apply these new methods: da Costa, Silva and Baghdassarian (2004), Mora (2004) and Mendoza and Oviedo (2004b). My purpose is to explain and assess these new approaches to fiscal sustainability analysis. Part of this assessment involves examining the fruitfulness of their application in the case studies.

In order to complete my assessment, I keep in mind the following tasks for which an analyst might use fiscal sustainability analysis:

1. Estimation of the government's ability to borrow.
2. Prediction of the onset of fiscal crises.
3. Assessment of the fiscal risks associated with contingent liabilities.
4. Assessment of the prior fiscal policy record, and discussion of future policy choices.

I show that each new approach to fiscal sustainability analysis, by bringing uncertainty into the picture, strengthens the ability of the analysis to deal with one of these tasks. For example, Mendoza and Oviedo's (2004a) paper develops a theoretical framework in which explicit government borrowing limits can be calculated. Xu and Ghezzi's (2003) paper develops a method for pricing government debt and calculating fair yield spreads. As a byproduct of their analysis, default probabilities at different horizons are calculated. So their method has unique potential to predict crises. Only one method—the one proposed by Barnhill and Kopits (2003)—is designed to deal with the issue of valuation of contingent liabilities. And, as I argue below, the IMF's (2003) approach appears to be specifically designed for aiding policy dialogue with government counterparts. Typically, however, each approach becomes highly specialized to the task at hand. Thus, these new approaches require the analyst to think very clearly about how he wants to use fiscal sustainability analysis before he sets out to perform it.

Each method has some merit—in the dimensions discussed above—obtained by relaxing problematic assumptions made in simple fiscal sustainability analysis. On the other hand, other assumptions have to be made, and I find important problems with each method that suggest that we should not expect *major* improvements in the accuracy of estimated debt ceilings, forecasting power, contingent liability valuation, or policy analysis from simple implementation of these methods. Methods that are designed to produce—or rely on—forecasts

of the timing of fiscal crises, such as Xu and Ghezzi's (2003), typically make very specific assumptions regarding the dynamics of foreign exchange reserves, and the government's primary balance. These assumptions are similar to those upon which standard analysis is based, and their realism can be questioned. None of the methods proposed deals adequately with how government finances are affected by real exchange rate movements, and vice versa; not even the method proposed by Mendoza and Oviedo (2004a), which at least addresses the issue. As I mentioned above, Barnhill and Kopits' (2003) approach involves the valuation of contingent liabilities, but it is highly specialized to oil revenue (a contingent asset), and I question its general applicability. Finally, while the IMF's (2003) approach is the most flexible, it also the most atheoretical. It involves out-of-sample simulations of the government budget that do not take account of possible endogeneity issues. To be fair, while none of the methods deals with endogeneity in a perfectly satisfactory way, this reflects the magnitude of the analytical problems that need to be overcome.

Despite these criticisms, each method, when backed by sound analytical thinking on the part of an applied economist, can provide insight into fiscal policy issues. These benefits and pitfalls are brought out by an examination of the case studies.

Da Costa, Silva and Baghdassarian's (2004) case study focuses on Brazil. Probably the most successful part of their case study is where they implement Xu and Ghezzi's (2003) model using high frequency Brazilian asset market data. Interestingly, their analysis indicates a 20 percent default probability at a one year horizon and a roughly 90 percent probability at a 7 year horizon. In and of themselves, these are interesting results, but they highlight the fact that there is a gap in the literature at the moment. No studies to date have systematically studied how Xu and Ghezzi's methods would have performed out of sample in historical data, so the degree of confidence we can have in these probability estimates is unknown. Additionally, the authors implement Barnhill and Kopits' (2003) model for Brazil, but they do not identify any significant sources of contingent liabilities, and their results seem to offer little input into policy discussions. They also implement a "structural" variant of the IMF's approach. Unfortunately the model's out of sample performance is poor.

Mora's (2004) case studies of Colombia and Costa Rica using Mendoza and Oviedo's (2004a) approach are interesting. The approach associates a government's ability to borrow with its ability to cut spending in a crisis. Colombia's current level of debt can be rationalized

within his framework if one assumes that in the worst case scenario, Colombia could reduce primary spending to about 8.5 percent of GDP. But, in the sample of data he presents, the lowest level of government spending observed was 10.3 percent of GDP. This illustrates an inherent difficulty with the approach: to use the model quantitatively one must identify parameters pertaining to worst case scenarios that may never have been observed. The results are useful, however, in that they illustrate why Costa Rica could maintain more debt than Colombia: its growth rate is faster and its revenues less cyclically sensitive. Mora also implements a variant of the IMF approach, but it is less successful.

Finally, Mendoza and Oviedo's (2004b) case studies provide the most interesting implementation of their method. They study Brazil, Colombia, Costa Rica and Mexico. Rather than using their method to estimate debt limits for these countries, they turn their method on its head. Interpreting the maximal quantities of debt that these countries have accumulated in the past as their debt limits, the authors use their model to infer the relative abilities of the respective governments to cut spending in the face of a crisis. Since each of these countries has borrowed up to between 50 and 55 percent of GDP at some point in the past, but the tax bases of the four countries are quite different, the conclusion is that the four countries have different minimal expenditure levels. Mendoza and Oviedo rightly point out that the competing hypothesis—which would emerge from standard fiscal sustainability considerations—that the four countries have sufficiently different average expenditure levels to explain their debt limits is not supported by the data. Nonetheless, there is no sense in which their results can be tested: the minimal expenditure levels of the four countries are unobservable, and it is not clear that previously observed levels of debt correspond to upper bounds on borrowing.

The first section of this report reviews the models and assumptions that lie behind traditional fiscal sustainability analysis. The second section reviews, in turn, the four proposed enhancements to fiscal sustainability analysis. The third section examines the case studies. The fourth section provides concluding remarks.

# 1. Traditional Fiscal Sustainability Analysis

## 1.1. The Government Budget Constraint

Fiscal sustainability analysis is based on the public sector or *government budget constraint*,<sup>1</sup> which is an identity:

$$\text{net issuance of debt} = \text{interest payments} - \text{primary balance} - \text{seigniorage}. \quad (1.1)$$

The net issuance of debt is gross receipts from issuing new debt minus any amortization payments made in the period.

The identity, (1.1), can be expressed in mathematical notation as

$$B_t - B_{t-1} = I_t - X_t - (M_t - M_{t-1}). \quad (1.2)$$

Here the subscript  $t$  indexes time, measured in years,  $B_t$  is the stock of public debt at the end of period  $t$ ,  $I_t$  is interest payments,  $X_t$  is the primary balance (revenue minus noninterest expenditure), and  $M_t$  is the monetary base at the end of period  $t$ , all measured in units of local currency (pesos).

The right-hand side of (1.2) is a flow, while the left-hand side is the change in a stock. When working with actual data, measurement issues arise from the definitions of these flows and stocks. In particular, how the government's outstanding debt obligations are valued, and how debt service is divided into amortization and interest become key questions. These issues are dealt with in more detail in Burnside (2003a), but they are not of particular concern to the discussion in this report, so they will not be discussed further.

In (1.2) debt should be net of any comparable assets and interest payments should be net of any receipts. So that (1.2) holds, the primary balance should include all flows that affect the debt level, such as privatization revenue or the costs associated with financial sector bailouts. Alternatively, these items, which are often accounted for *below the line* in official documents, may be added to (1.2) as separate items.

Furthermore, the analysis must fix on a particular definition of the government or public sector. By including seigniorage revenue (the change in the monetary base) in (1.2), we have implicitly defined the public sector to include the central bank. It is conventional, though not necessary, to define the public sector as the consolidation of the central government, state and local governments, state-owned nonfinancial enterprises and the central bank.

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<sup>1</sup>This section is closely based on two chapters in a forthcoming World Bank handbook on fiscal sustainability: Burnside (2003a) and Burnside (2003b).

In this report I do not specifically address some current issues in fiscal sustainability analysis: (i) the net worth of the government, (ii) broad approaches to contingent liabilities, and (iii) accurate measurement of the contribution of nonfinancial public sector enterprises to the primary balance. While these are all important issues they are not broadly addressed by the four methodological papers being discussed here, nor by the case studies.

The flow budget constraint is used to derive the lifetime government budget constraint, which plays a crucial role in simple assessments of fiscal sustainability. Before deriving the lifetime budget constraint, it is useful to make several assumptions.

*Assumption Set 1.* Assume that (i) time is discrete, (ii) debt has a maturity of one period, (iii) debt is real (in other words, its face value is indexed to the price level) and (iv) debt issued at date  $t - 1$  pays a real interest rate,  $r_{t-1}$ .

With these assumptions (1.2) can be rewritten as

$$b_t = (1 + r_{t-1})b_{t-1} - x_t - \sigma_t, \quad (1.3)$$

where  $b_t = B_t/P_t$  is the end-of-period  $t$  stock of real debt,  $x_t = X_t/P_t$  is the real primary surplus and  $\sigma_t = (M_t - M_{t-1})/P_t$  is the real value of seigniorage revenue. Rearranging (1.3) we have

$$b_{t-1} = (1 + r_{t-1})^{-1}b_t + (1 + r_{t-1})^{-1}(x_t + \sigma_t). \quad (1.4)$$

If (1.4) is updated to period  $t$ , and is used to substitute out  $b_t$  on the right-hand side of (1.4), we obtain

$$b_{t-1} = (1+r_{t-1})^{-1}(1+r_t)^{-1}b_{t+1} + (1+r_{t-1})^{-1}(x_t + \sigma_t) + (1+r_{t-1})^{-1}(1+r_t)^{-1}(x_{t+1} + \sigma_{t+1}). \quad (1.5)$$

The same procedure could be used to recursively substitute out  $b_{t+1}$ ,  $b_{t+2}$ , etc., until after several iterations we would obtain

$$b_{t-1} = b_{t+j}/R_{t-1}^{t+j-1} + \sum_{i=0}^j (x_{t+i} + \sigma_{t+i})/R_{t-1}^{t+i-1}, \quad (1.6)$$

where  $R_t^{t+i} = (1 + r_t)(1 + r_{t+1}) \cdots (1 + r_{t+i})$ .

If we impose the condition

$$\lim_{j \rightarrow \infty} b_{t+j}/R_{t-1}^{t+j-1} = 0 \quad (1.7)$$

then we obtain what is frequently called the government's *lifetime budget constraint*:

$$b_{t-1} = \sum_{i=0}^{\infty} (x_{t+i} + \sigma_{t+i})/R_{t-1}^{t+i-1}. \quad (1.8)$$

Intuitively, the lifetime budget constraint states that the government finances its initial debt by raising seigniorage revenue and running primary surpluses in the future, whose present value is equal to its initial debt holdings.<sup>2</sup> The lifetime budget constraint could also be derived under the assumption that all debt is nominal, rather than real, in which case we would replace (1.8) with

$$B_{t-1} = \sum_{i=0}^{\infty} (X_{t+i} + \Delta M_{t+i}) / N_{t-1}^{t+i-1}, \quad (1.9)$$

where  $N_t^{t+i} = (1 + n_t)(1 + n_{t+1}) \cdots (1 + n_{t+i})$  and  $n_t$  is the nominal interest rate on debt issued at date  $t$ .

## 1.2. The Steady-State Fiscal Sustainability Condition

The most basic tool used in fiscal sustainability analysis uses a steady state version of the lifetime budget constraint, (1.8), with stocks and flows expressed as fractions of GDP. Let  $y_t$  be real GDP, and define  $\bar{b}_t = b_t/y_t$ ,  $\bar{x}_t = x_t/y_t$  and  $\bar{\sigma}_t = \sigma_t/y_t$ . Given this notation we can rewrite (1.8) as

$$\bar{b}_{t-1} = \sum_{i=0}^{\infty} (\bar{x}_{t+i} + \bar{\sigma}_{t+i})(y_{t+i}/y_{t-1}) / R_{t-1}^{t+i-1}. \quad (1.10)$$

Imagine a steady state in which (i) the primary surplus as a fraction of GDP is a constant  $\bar{x}$ , (ii) seigniorage as a fraction of GDP is a constant  $\bar{\sigma}$ , (iii) real GDP grows at a constant rate  $g$ , so that  $y_t/y_{t-1} = 1 + g$ , and (iv) the real interest rate,  $r_t$ , is a constant  $r$ . In this case, (1.10) reduces to

$$\bar{b}_{t-1} = \sum_{i=0}^{\infty} \left(\frac{1+g}{1+r}\right)^{i+1} (\bar{x} + \bar{\sigma}). \quad (1.11)$$

Assuming that  $r > g$ , (1.11) reduces to

$$\bar{b}_{t-1} = \bar{b} \equiv (\bar{x} + \bar{\sigma}) \frac{1+g}{r-g}. \quad (1.12)$$

The condition (1.12) is often used to make simple assessments of fiscal sustainability. The first step, typically, is to make assumptions about “reasonable” values of  $\bar{\sigma}$ ,  $r$  and  $g$ . These assumptions might be based on historical trends or long-term projections. If one were to also make an assumption about  $\bar{x}$ , this would imply a value for  $\bar{b}$ . If  $\bar{b} > \bar{b}_{t-1}$ , where  $\bar{b}_{t-1}$  would be the current debt-to-GDP ratio, one might conclude that the government’s finances were sustainable. Alternatively, (1.12) could be solved for  $\bar{x}$ , given the other assumptions:

$$\bar{x} = \frac{r-g}{1+g} \bar{b}_{t-1} - \bar{\sigma}. \quad (1.13)$$

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<sup>2</sup>Burnside (2003b) discusses the appropriate interpretation of (1.7).

Now, instead of setting  $\bar{x}$  equal to some historical average or projection, one could determine the value that  $\bar{x}$  would need to take in the future in order to sustain the initial debt level  $\bar{b}_{t-1}$ .

A by-product of the steady-state condition is that if (1.12) is satisfied, then the government's debt-to-GDP ratio remains constant over time at its initial level. It should be pointed out that this is not a necessary condition for fiscal sustainability. There are many paths for debt that satisfy (1.10). The constant debt-to-GDP ratio emerges because of the steady state assumption. Absent a steady state, the debt-to-GDP ratio does not need to be constant in order for sustainability to hold.

### 1.3. Other Simple Forms of Fiscal Sustainability Analysis

The steady-state fiscal sustainability condition is somewhat limiting in that it assumes  $\bar{x}_t$ ,  $\bar{\sigma}_t$ ,  $g_t$  and  $r_t$  are all constant. Alternatively, a variation on the steady-state condition involves assuming that  $\bar{x}_t$ ,  $\bar{\sigma}_t$ ,  $g_t$  and  $r_t$  evolve along deterministic paths  $\{\bar{x}_{t+j}, \bar{\sigma}_{t+j}, g_{t+j}, r_{t+j}\}_{j=0}^{\infty}$ . Notice that (1.3) can be rewritten as

$$\bar{b}_t = \frac{1 + r_{t-1}}{1 + g_t} \bar{b}_{t-1} - \bar{x}_t - \bar{\sigma}_t \quad (1.14)$$

and that (1.7) can be rewritten as

$$\lim_{j \rightarrow \infty} \bar{b}_{t+j} y_{t+j} / R_{t-1}^{t+j-1} = 0. \quad (1.15)$$

The practitioner assumes something about the paths for  $\bar{x}_t$ ,  $\bar{\sigma}_t$ ,  $g_t$  and  $r_t$  and uses (1.14) to determine the implied path of  $\bar{b}_t$ . If this path appears to violate (1.15), the practitioner might conclude that the government's finances are unsustainable.

Typically the deterministic paths chosen by the practitioner will approach a steady state. For example, suppose that the economy is initially in a deep recession. It might be reasonable to assume that the primary balance will be below some steady state value for a while and that the real growth rate will be above some steady state value for a while. Also, if a small country is currently weathering a rise in international interest rates, it might be useful to assume that  $r_t$  is temporarily above some long-run level. If the economy is assumed to reach a steady state by some finite date  $T > t$ , then the practitioner can make his assessment based on whether the steady state condition, (1.12), will hold at that date.

#### 1.4. Shortcomings of Simple Fiscal Sustainability Analysis

Even in its modified form, presented in the previous section, simple fiscal sustainability analysis has obvious shortcomings. Most important among these is that it is based on forward looking analysis that takes no account of uncertainty. It assumes that  $\bar{x}_t$ ,  $\bar{\sigma}_t$ ,  $g_t$  and  $r_t$  evolve along deterministic paths. As a result, it tends to provide information about how big the average primary balance might need to be in the future in order to ensure sustainability. On the other hand, it provides little guidance as to how a variety of shocks might affect the government's ability to service its debt. It also is silent as to how the government might have to adjust policy in order to remain solvent. It also says nothing about the possible limitations on government borrowing that might arise from macroeconomic volatility. Traditional analysis also leaves out the possibility of default. It only asks how big the primary balance must be to avoid default, rather than assessing the likelihood of a default.

Recent innovations in fiscal sustainability analysis are designed to address these shortcomings. Barnhill and Kopits (2003) adopt a *Value-at-Risk* methodology which is meant to address how macroeconomic volatility and contingent liabilities affect fiscal sustainability. In essence, this method can be thought of as measuring the variance of the right-hand side of (1.10). The IMF (2003) proposes a number of variations on simple fiscal sustainability analysis. Primarily these involve formal stochastic modeling of the processes for  $\bar{x}_t$ ,  $\bar{\sigma}_t$ ,  $g_t$  and  $r_t$ , and Monte Carlo experiments in which these processes are simulated and the corresponding  $\bar{b}_t$  series is computed. This allows probability distributions for the path of  $\bar{b}_t$  to be derived. IMF (2003) also discusses crisis prediction, but this is done outside the formal confines of fiscal sustainability analysis. Xu and Ghezzi (2003) model the probability of default. As we will see their approach resembles the IMF approach, in that it involves modeling the stochastic processes that enter into the government budget constraint. However, as we will see they look at a concept of sustainability that is more closely related to liquidity than pure solvency. Finally, Mendoza and Oviedo (2004a) show how the stochastic behavior of revenue flows affects the average ability of a government to borrow. In particular, they explain how the simple approach to fiscal sustainability would fail to predict any distinction between the abilities to borrow of governments facing different levels of macroeconomic volatility.

## 1.5. A Few Words About Existing Stochastic Methods

Before moving to the next section, where the recent innovations in fiscal sustainability analysis are discussed further, it is worth saying a few words about existing techniques that explicitly recognize the stochastic nature of a government's finances. Most of these relate in one way or another to Hamilton and Flavin's (1986) analysis, which was applied to the US government's debt. To understand their approach, consider, again the government's budget constraint given by (1.2). To write this as (1.3) we made two assumptions that Hamilton and Flavin avoid, namely the single period maturity assumption and the real debt assumption. Suppose, instead, that we maintain the definitions of  $b_t$ ,  $x_t$  and  $\sigma_t$ , but also define  $i_t = I_t/P_t$  and  $\pi_t = P_t/P_{t-1} - 1$ . Then (1.2) can be rewritten as

$$b_t - \frac{1}{1 + \pi_t} b_{t-1} = i_t - x_t - \sigma_t \quad (1.16)$$

Now define  $r$  as the *average ex-post* real interest rate on government debt and add  $(1+r)b_{t-1}$  to both sides of (1.16):

$$b_t + \frac{r + \pi_t + r\pi_t}{1 + \pi_t} b_{t-1} = (1+r)b_{t-1} + i_t - x_t - \sigma_t \quad (1.17)$$

Notice that there is a sense in which, on average,  $i_t$  should equal  $(r + \pi_t + r\pi_t)b_{t-1}/(1 + \pi_t)$ . Hence, it makes sense to define an error term  $v_t = i_t - (r + \pi_t + r\pi_t)b_{t-1}/(1 + \pi_t)$  and rewrite (1.17) as

$$b_t = (1+r)b_{t-1} - x_t - \sigma_t + v_t. \quad (1.18)$$

Hamilton and Flavin (1986) also note that if  $b_t$  is defined as the real *market value* of government debt at the end of period  $t$ , the error term  $v_t$  will include the capital loss (if positive) or gain (if negative) from any changes in the market value of long-term debt. In addition the error term captures asymmetry of timing, within the year, between the issuance of debt, and the flows that are debt creating or reducing.

Forward iteration on (1.18) implies that

$$b_t = (1+r)^{-j} b_{t+j} + \sum_{i=1}^j (1+r)^{-i} (x_{t+i} + \sigma_{t+i} - v_{t+i}). \quad (1.19)$$

Thus it is natural to impose the condition  $\lim_{j \rightarrow \infty} (1+r)^{-j} b_{t+j} = 0$  to obtain the equivalent of the government's lifetime budget constraint. Hamilton and Flavin's test is of a weaker hypothesis, that

$$\lim_{j \rightarrow \infty} (1+r)^{-j} E_t b_{t+j} = 0. \quad (1.20)$$

Their null hypothesis is that (1.20) holds, and this is tested against a specific alternative:

$$\lim_{j \rightarrow \infty} (1+r)^{-j} E_t b_{t+j} = a(1+r)^t \quad (1.21)$$

with  $a \neq 0$ . Under the alternative hypothesis

$$b_t = E_t \sum_{i=1}^{\infty} (1+r)^{-i} (x_{t+i} + \sigma_{t+i} - v_{t+i}) + a(1+r)^t. \quad (1.22)$$

Defining  $\eta_t = -E_t \sum_{i=1}^{\infty} (1+r)^{-i} v_{t+i}$  and  $z_t = x_t + \sigma_t$ , (1.22) can be written as

$$b_t = E_t \sum_{i=1}^{\infty} (1+r)^{-i} z_{t+i} + a(1+r)^t + \eta_t. \quad (1.23)$$

If  $b_t$  were a price, then the term  $a(1+r)^t$  would look like a bubble, if the sequence of  $z_t$ 's represented fundamentals determining the price. Thus, the first test that Hamilton and Flavin perform is based on Diba and Grossman (1984) and Hamilton and Whiteman's (1985) critique of Flood and Garber's (1980) test for bubbles in prices. The test proposed by Hamilton and Flavin can be summarized as follows.

1. Assume that  $\eta_t$  is stationary and test for a unit root in  $z_t$ . If one rejects the presence of a unit root in  $z_t$ , then one can assume that  $E_t \sum_{i=1}^{\infty} (1+r)^{-i} z_{t+i}$  is also stationary, and one may proceed to the second step.

2. Test for a unit root in  $b_t$ . If one cannot reject the presence of a unit root, then (1.20) is rejected in favor of (1.21). Otherwise (1.20) is not rejected.

The second test proposed by Hamilton and Flavin is based on a regression, where it is assumed that forecasts of  $v_{t+i}$  and  $z_{t+i}$  are rationally formed by projecting them on current and lagged values of  $z_t$  and lagged values of  $b_t$ . In this case (1.23) can be written as

$$b_t = \beta_0 + \sum_{i=1}^{k_b} \gamma_i b_{t-i} + \sum_{i=0}^{k_z} \kappa_i z_{t-i} + a(1+r)^t + \varepsilon_t \quad (1.24)$$

where  $\varepsilon_t$  is a white noise error term. The constants  $k_b$  and  $k_z$  and the parameters  $\gamma_i$ ,  $i = 1, \dots, k_b$  and  $\kappa_i$ ,  $i = 1, \dots, k_z$  are determined as functions of  $r$  and the coefficients in the rules for rationally forecasting  $v_t$  and  $z_t$ . A test for fiscal sustainability, in this case, is simply a test of whether  $a = 0$  or not.

These formal statistical tests are much more complex than the simple methods described earlier. The question is whether they are more useful for fiscal sustainability analysis. They have the obvious advantage of not requiring the steady-state assumption, nor do they involve choosing arbitrary deterministic paths for  $g_t$ ,  $r_t$ ,  $\bar{x}_t$  and  $\bar{\sigma}_t$ . But these tests do have a major

drawback, apart from the obvious standard criticisms of unit root tests. This drawback is that they are based on historical data and are, therefore, fundamentally backward looking. While they may be useful in identifying violations of fiscal sustainability in historical data, they say little about whether *future* surpluses will be sufficient to service the *current* stock of debt.

## 2. New Approaches to Fiscal Sustainability Analysis

### 2.1. Barnhill and Kopits' Value-at-Risk Approach

Barnhill and Kopits (B&K, 2003) model the government as if it were a firm. Their goal is to measure the government's net worth as an explicitly stochastic process, and to assess the probability of this net worth becoming negative, which is interpreted as the probability of government default. In this sense, their model represents a significant extension to the type of fiscal sustainability analysis described above in which it is assumed that the government always satisfies its flow budget constraint.

**The Basics of Value-at-Risk** As mentioned above, B&K adopt a value-at risk (VaR) methodology to address the ways in which macroeconomic volatility and contingent liabilities affect fiscal sustainability. Before describing how they model VaR in the context of the government budget constraint, a more general discussion of VaR methods is in order. These methods have mostly been used in finance to assess the potential losses to a portfolio of assets. To take a simple example, suppose that an investor holds a portfolio of assets summarized by a vector  $q$  of quantities held, and a vector  $p$  of prices. So the value of the portfolio is  $v = p'q$ . The value of the portfolio conditional on all currently available information is, say, given by some cumulative probability distribution function  $F(v)$ . The VaR is defined as the difference between the initial value of the portfolio,  $v_0$ , and the value of the portfolio at some arbitrarily chosen quantile of the distribution function  $F$ . Say the 10% quantile is chosen, and that  $\underline{v}$  is the 10% quantile of the distribution; i.e.  $F(\underline{v}) = 0.1$ . Then a statement about VaR would be: with a 90 percent level of confidence the VaR is less than or equal to  $v_0 - \underline{v}$ . Precisely what the value at risk is equal to depends on the chosen quantile, the shape of the distribution  $F$ , and, of course, this depends on the underlying distribution of the price vector  $p$ .

Simple results emerge from the assumption that asset returns are normally distributed.

Suppose, for example, that the initial price vector is  $p_0$  and that the quantities,  $q$ , are treated as fixed. The value of the portfolio can then be written as  $v = v_0 + v_0(w'r)$  where  $w$  is the vector of initial portfolio weights with  $i$ th element  $q_i p_{0i}/(q'p_0)$  and  $r$  is a vector of net asset returns with  $i$ th element  $r_i = p_i/p_{0i} - 1$ . Suppose that  $r$  is normally distributed with mean zero and covariance matrix  $\Sigma$ . Then  $v$  is normally distributed with mean  $v_0$  and variance equal to  $\sigma_v^2 = v_0^2(w'\Sigma w)$ . If we wanted to compute the VaR at a  $100 \times \alpha$  percent degree of confidence, we would set  $\underline{v} = w_0 + \sigma_v z_{1-\alpha}$ , where  $z_{1-\alpha}$  is the  $1 - \alpha$  quantile of the standard normal distribution. Hence, the VaR would be less than or equal to  $-z_{1-\alpha} \sqrt{w'\Sigma w} v_0$ .<sup>3</sup>

**Value-at-Risk Applied to the Government's Balance Sheet** More generally, of course, it may not be appropriate to make the normality assumption. In addition, the value in question may not be that of a portfolio of assets. Indeed, this is the case when the VaR method is applied to the government budget by B&K, who model something they call the net worth of the government. Roughly speaking their measure of net worth compares the value of the government's outstanding debt to the present value of the net flows with which that debt will be serviced. At first, such a definition may seem unintuitive given that, in theory, the government's lifetime budget constraint must be satisfied along all equilibrium paths. In other words, there is a sense in which theory appears to imply that B&K's concept of net worth should always equal 0. An explanation is in order.

B&K exclude "discretionary expenditure, occasional levies and seigniorage" revenue from the net budget flows. Their intention, in doing so, appears to be to exclude changes in the government budget that are specifically (desperately?) used in order to close the government budget constraint. For this reason, there is no reason for the net worth measure to be zero. One might imagine a decomposition of the nominal primary balance  $X_t$  into  $X_t^p$ , a more planned component of the primary balance, and  $X_t^r$ , a residual component. B&K's concept of net worth is based on the difference between the present value of the future realizations of  $X_t^p$  and the government's initial debt. A second conceptual difference between B&K's approach and the lifetime budget constraint, as described above, is that it is an ex-ante assessment of the government's finances. So it compares the expected present value of the future realizations of  $X_t^p$  to the current market value of the government's debt.

We will let  $W_t$  represent B&K's concept of net worth at the end of period  $t$ . Given the

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<sup>3</sup>Generally speaking  $\alpha$  is set to a value close to 1, so that  $z_{1-\alpha}$  is negative. This implies that the value at risk is positive.

discussion above, it corresponds to:

$$W_t = E_t \left( \sum_{i=1}^{\infty} X_{t+i}^p / N_t^{t+i} \right) - B_t, \quad (2.1)$$

where  $B_t$  is the market value of the government's outstanding debt at the end of period  $t$ .

B&K model the flows and discount factors that appear on the right hand side of (2.1) as functions of the future paths of real output, interest rates, the exchange rate, commodity prices and the domestic price level. These variables, in turn, are treated as exogenous. Given specific stochastic processes for the exogenous variables, repeated long (but finite) sample Monte Carlo simulations can be used to approximate  $\sum_{i=1}^{\infty} X_{t+i}^p / N_t^{t+i}$ . The mean of  $\sum_{i=1}^{\infty} X_{t+i}^p / N_t^{t+i}$  across these simulations is the estimate of  $E_t(\sum_{i=1}^{\infty} X_{t+i}^p / N_t^{t+i})$ . If necessary, the simulations can be used to value the government's outstanding debt,  $B_t$ .<sup>4</sup>

Conditional on information available at time  $t$ ,  $W_t$  is a single value. On the other hand, conditional on the same information,  $W_{t+1}$  is a random variable, whose distribution can also be characterized using the simulations. Let the 5 percent quantile of the distribution of  $W_{t+1}$  be denoted  $W^*$ . B&K call  $W^*$  the government's *risk-adjusted net worth*, and  $W_t - W^*$  is their definition of the VaR.

**Implementing the VaR Approach** While this description sounds reasonable enough, the devil is in the details. How does a practitioner actually apply the VaR approach? Unfortunately, B&K provide only a little guidance.

For example, an important question is: Which components of the government budget belong in  $X_t^p$ ? The answer to this question is less than obvious. Unfortunately, in their one illustration of their method—a case study Ecuador—B&K sidestep this issue, because they do not attempt this decomposition. Instead, they measure only the following contributions to net worth:<sup>5</sup>

1. the government's financial liabilities, net of any financial assets,
2. revenue from oil reserves,

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<sup>4</sup>This step is necessary if there are no readily available data on the market value of the government's debt. To calculate the market value, one would use the term structure of current rates to compute the present value of future government debt service already contracted.

<sup>5</sup>Conceptually, some of the items listed above can be thought of as components of  $B_t$ . Others can be thought of as contributions to  $E_t(\sum_{i=1}^{\infty} X_{t+i}^p / N_t^{t+i})$ .

3. net contingent liabilities stemming from public pensions, possible banking crises, and natural disasters, and
4. the government's real assets.<sup>6</sup>

B&K also model something they call the *residual primary balance*, which will be discussed below.

B&K's simulations are based on modeling 7 stochastic process for (i) the Ecuadoran sucre/US dollar exchange rate, (ii) the interest rate on sucre-denominated Ecuador government bonds, (iii) the spread on dollar-denominated Ecuadoran government bonds over comparable US government bonds, (iv) the interest rate on US government bonds, (v) the seasonally-adjusted consumer price index, (vi) seasonally-adjusted real nonoil GDP, and (vii) the spot price of West Texas Intermediate crude oil in US dollars.

To value financial liabilities and assets one needs to know the stream of payments associated with each contract. Given this information, it is straightforward to price the assets if one knows the entire term structure of a country's discount bonds. For example, suppose there is an outstanding bond which promises to pay  $D_{t+i}$  dollars at each period in the future. The market value of this bond at the end of time  $t$  is given by  $\sum_{i=1}^{\infty} D_{t+i}/(1 + n_{t,i})^i$ , where  $n_{t,i}$  is the yield to maturity on an  $i$ -period discount bond issued at time  $t$ . Obviously to value the government's outstanding debt requires knowledge of its structure (i.e. knowledge of the outstanding stream of  $D_t$ s), as well as the yields on the discount bonds. If the latter are not available directly from financial market data, they can often be estimated using a model of the term structure. To simulate the value of the government's outstanding debt one period ahead requires that the discount yields one period ahead be simulated. B&K spend little time detailing how the simulations of the 7 stochastic processes are used to accomplish this, although more information is provided in another paper by Barnhill and Maxwell (2002), but in principal it is not difficult to imagine how this would be done.

To value oil revenue, B&K compute the present value of the revenue stream. The revenue stream is the product of the quantity of barrels produced (which is modeled deterministically) and the difference between the oil price (one of the stochastic processes) and the production cost per barrel (also modeled deterministically). Few details of the deterministic processes

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<sup>6</sup>The fourth category is problematic, in that the government's real assets only help it retire its debt to the extent that it reaps the return to them. Often this will not be the case, and the market value of the assets will exceed their inside value to the government.

for production and cost are given. In addition, B&K appear to compute the present value using a constant discount factor. More appropriately, they would use the same discount rates that are used to value government debt.

It is not clear how B&K value the other contingent assets and liabilities included in their measure of net worth. They measure the value of Ecuador's biodiverse resources, state-owned enterprises, El Niño, banking crises and the social security imbalance. The value of each liability is modeled, but what it is not clear from their discussion is how these values depend on the simulated stochastic processes described earlier.

Real assets are measured at book value. This would imply that their value would not change from one period to the next. However, in the end, B&K measure net worth in US dollars. Hence, the value of these real assets varies with the sucre/dollar exchange rate.

Finally, B&K calculate the present value of the residual primary balance. The residual primary balance is the difference between revenue (net of pension contributions, oil revenue and the surplus of state owned enterprises) and primary expenditure (net of pension benefits paid). The excluded items, of course, have been included elsewhere in the measure of net worth. The residual primary balance term can be thought of as measuring  $E_t(\sum_{i=1}^{\infty} X_{t+i}^r/N_t^{t+i})$ . It would appear that B&K model  $X_t^r$  as a constant times the simulated value of nominal GDP. It is not clear how they discount the flows to compute a present value.

B&K present two measures of net worth. One is, roughly speaking, consistent with (2.1) because it excludes the residual balance term. The other includes  $E_t(\sum_{i=1}^{\infty} X_{t+i}^r/N_t^{t+i})$  in the measure of net worth. Their conclusions are not sharply affected by the choice of net worth concept.

**Summary Discussion** Clearly, B&K's method extends fiscal sustainability analysis into interesting territory. Nonetheless, it is problematic in several ways. First, their paper does not serve as a useful guidebook on the use of VaR methods. The average operational economist at an international organization will not be able to read their paper and apply their method to a randomly selected country. The problem is not the technical level of the analysis. Rather, it is the lack of detail about precisely how to implement the method.

Second, and more importantly, however, there are several conceptual problems.

- It is less than clear that the net worth concept is related to default probabilities.

B&K present tables in which they provide summary statistics for the distribution of  $W_{t+1}$ , including the mean, the standard deviation and the minimum and maximum values, the 5 percentile ( $W^*$ ). They also present the probability with which  $W_{t+1} < 0$ . They describe this as a default probability. However, there is good reason to question whether this is a proper interpretation. First, seigniorage revenue and the residual primary balance are usually excluded from the analysis. Second, even when the residual primary balance is included, it is assumed not to adjust to the government's other fiscal outcomes. Presumably if the government felt that default was becoming likely it would adjust the residual primary balance accordingly. B&K's simulations do not allow for this possibility.

- The simulations do not take into account the relationship between default risk and interest rates. The interest rate processes are strictly exogenous and are fitted to historical data.
- It is probably fair to say that the only items in the balance sheet receiving serious treatment in B&K's case study are the government's financial assets and liabilities, its revenue from oil, and the contingent liabilities in the public pension system. This may be quite reasonable for Ecuador, whose oil reserves are valued at about 4 times its GDP. For other countries, however, it would be important to more carefully model some of the items that B&K relegate to the residual primary balance.
- It could be argued that the main purpose of fiscal sustainability analysis is to point out fiscal policy imbalances, their possible consequences, the possible policy responses and their likely effects. It is unclear how B&K's method helps the practitioner do a better job except, perhaps, in pointing out imbalances that show up stochastically.

## 2.2. Xu and Ghezzi's Fair Spread Model

Like B&K, Xu and Ghezzi (X&G, 2003) model the flows in the government budget as stochastic processes in order to estimate default probabilities. These default probabilities can then be mapped into a term structure model to compute the fair spreads that would apply to the sovereign debt of a particular country. So while the precise application and details of the models are different, at a broader conceptual level they are quite similar.

**Modeling the Government Budget** There are two differences between X&G’s approach and the model we described in earlier sections. The most important of these distinctions is that X&G consider a version of the government’s flow budget constraint that makes explicit the distinction between the government’s debt, denominated in various currencies, and its treasury reserves. The other distinction is that X&G use a continuous time model. While this is notationally convenient, and allows them to derive analytical results, we can illustrate the principles behind their method using a model in discrete time.

Let the stock of domestic currency debt—measured in local currency units (LCUs)—be denoted by  $D_t^d$ , while the stock of foreign currency debt—measured in dollars—is denoted  $D_t^f$ . Let  $C_t$  represent treasury reserves measured in LCUs.<sup>7</sup> Let  $S_t$  represent the exchange rate, measured in LCUs per dollar. Let  $n_t^d$  represent the interest rate on domestic currency debt,  $n_t^f$  the interest rate on foreign currency debt, and  $n_t^c$  is the interest rate on treasury reserves.

Although X&G include treasury reserves as an asset, they exclude seigniorage from their specification of the government budget constraint. Given this, the discrete time analog to their government budget constraint is

$$\Delta D_t^d + S_t \Delta D_t^f - \Delta C_t = n_t^d D_{t-1}^d + n_t^f S_t D_{t-1}^f - n_t^c C_{t-1} - X_t. \quad (2.2)$$

where  $X_t$  is the primary balance, as before.<sup>8</sup> It is easy to see that (2.2) corresponds to (1.2) when all assets and liabilities are rolled over period-by-period, seigniorage is excluded, interest rates are set at the beginning of the period, and the government’s net debt comes in three flavors.<sup>9</sup>

X&G choose to rewrite the budget constraint by defining all stocks and flows relative to nominal GDP. Let  $Y_t$  represent nominal GDP during period  $t$ . Define total debt as  $D_t = D_t^d + S_t D_t^f$  and let  $\bar{d}_t = D_t/Y_t$ ,  $\bar{d}_t^d = D_t^d/Y_t$ ,  $\bar{d}_t^f = S_t D_t^f/Y_t$ ,  $\bar{c}_t = C_t/Y_t$  and  $\bar{x}_t = X_t/Y_t$ . Then (2.2) can be rewritten as

$$\Delta \bar{d}_t - \Delta \bar{c}_t = \frac{n_t^d - \mu_t}{1 + \mu_t} \bar{d}_{t-1}^d + \frac{(1 + \delta_t)n_t^f + \delta_t - \mu_t}{1 + \mu_t} \bar{d}_{t-1}^f + \frac{\mu_t - n_t^c}{1 + \mu_t} \bar{c}_{t-1} - \bar{x}_t, \quad (2.3)$$

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<sup>7</sup>More naturally, treasury reserves would be modeled as a foreign currency asset rather than as a local currency asset. X&G show that this has no significant implications if an interest parity condition holds.

<sup>8</sup>In formulating (2.2) we have assumed that interest rates on debt outstanding at the end of period  $t - 1$  are determined at time  $t$ . For this reason we use the time- $t$  subscript on the interest rate, rather than the time- $t - 1$  subscript as in earlier sections.

<sup>9</sup>X&G actually use a further breakdown of the government’s local currency debt into fixed rate and floating rate debt. While this distinction may be important in practice, it is not important in illustrating their method. Here we assume that all local currency debt is issued at fixed rates.

where  $\mu_t = Y_t/Y_{t-1} - 1$  is the growth rate of nominal GDP and  $\delta_t = S_t/S_{t-1} - 1$  is the rate of depreciation of the exchange rate.

In their simplest example, X&G make the following simplifying assumptions: (i) the gross debt-to-GDP ratio is constant, so that all changes in the net debt-to-GDP ratio are reflected in treasury reserves ( $\Delta \bar{d}_t = 0$ ,  $\bar{d}_t = \bar{d}$ ), (ii) local and foreign currency debt are constant fractions of the overall debt stock ( $\bar{d}_t^d = \omega \bar{d}_t = \omega \bar{d}$  and  $\bar{d}_t^f = (1 - \omega) \bar{d}_t = (1 - \omega) \bar{d}$ ), and (iii) the interest rate on treasury reserves is the same as the growth rate of nominal GDP ( $n_t^c = \mu_t$ ).<sup>10</sup> With these assumptions (2.3) becomes

$$\Delta \bar{c}_t = -\frac{1}{1 + \mu_t} \left\{ (n_t^d - \mu_t) \omega + [(1 + \delta_t) n_t^f + \delta_t - \mu_t] (1 - \omega) \right\} \bar{d} + \bar{x}_t, \quad (2.4)$$

At this stage, X&G make assumptions about the stochastic processes governing  $\bar{x}_t$ ,  $n_t^d$ ,  $n_t^f$ ,  $\delta_t$  and  $\mu_t$ . These are:

- Nominal GDP is a geometric Brownian motion with drift. In discrete time, the natural analog is to assume that nominal GDP growth is given by  $\mu_t = \mu + \sigma_\mu \epsilon_t^\mu$ , where  $\epsilon_t^\mu$  is a white noise with unit variance.
- The exchange rate is a geometric Brownian motion with drift. In discrete time, the natural analog is to assume that the depreciation rate is given by  $\delta_t = \delta + \sigma_\delta \epsilon_t^\delta$ , where  $\epsilon_t^\delta$  is a white noise with unit variance.
- The primary balance as a share of GDP is the increment of a Weiner process. In discrete time, the natural analog is to assume that  $\bar{x}_t = \bar{x} + \sigma_x \epsilon_t^x$ , where  $\epsilon_t^x$  is a white noise with unit variance.
- The foreign currency interest rate is constant,  $n_t^f = n^f$ ;
- The domestic currency interest rate is given by  $n_t^d = n^d + \sigma_d \epsilon_t^\delta$ ; and
- The processes  $\epsilon_t^\mu$ ,  $\epsilon_t^\delta$  and  $\epsilon_t^x$  are mutually independent.

The second last assumption implies that shocks to the rate of depreciation are reflected in local currency interest rates. If domestic and foreign currency debt were equally risky, and an interest parity condition held, one might expect that  $\sigma_d$  should be equal to 1, but X&G do not impose this.

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<sup>10</sup>It is not clear what justifies the last assumption, though it is useful for eliminating  $c_t$  from the right-hand side of (2.3).

With these 6 assumptions, (2.4) the law of motion of  $\Delta\bar{c}_t$  is fully specified. It is worth noting that  $\Delta\bar{c}_t$  is iid, given that the processes for  $\mu_t$ ,  $\bar{x}_t$ ,  $\delta_t$  and  $n_t^d$  are each iid, and that it can be written as

$$\Delta\bar{c}_t = \xi + \epsilon_t, \quad (2.5)$$

where  $\xi$  is a function of the various model parameters and  $\epsilon_t$  is a white noise process whose distribution is a function of the model parameters and the parameters governing the  $\epsilon^\mu$ ,  $\epsilon^\delta$  and  $\epsilon^x$  processes. Given this law of motion, if we start at date 0, with some amount of reserves  $\bar{c}_0 > 0$ , the probability of reserves being exhausted at date  $t$  is easily computed, and is given by

$$\Pr(\bar{c}_t \leq 0) = \Pr(\epsilon_1 + \epsilon_2 + \dots + \epsilon_t \leq -\bar{c}_0 - t\xi). \quad (2.6)$$

If  $\epsilon_t$  were approximately normally distributed with variance  $\sigma^2$ , then  $\Pr(\bar{c}_t \leq 0) \cong \Phi[-(\bar{c}_0 + t\xi)/\sigma t^{1/2}]$  where  $\Phi$  is the cdf of the standard normal distribution. X&G would interpret this as the probability that the government is in default by date  $t$  and they would use this probability to assess the fair pricing of the government's debt.

**Extending Xu and Ghezzi's Approach** X&G recognize that one of their assumptions, that the gross debt-to-GDP ratio is constant over time, is quite unrealistic. In their model, a country absorbs all fiscal imbalances by running down its reserves. This seems highly unlikely in practice.

Another problem with their model is that it takes interest rates as exogenous, yet it has implications for the pricing of debt. Of course, this is inconsistent. The interest rates, themselves, ought to be consistent with the pricing implied by the model.

To deal with these issues X&G modify their model. The first modification involves changing the assumption about debt being constant and reserves absorbing all fiscal imbalances. The second modification involves pseudo-endogenization of the interest rates in the model.

To model the government's debt dynamics, X&G replace their earlier assumption with a two phase model. In the first phase,  $C_t$  is assumed to grow *deterministically* at the rate  $\mu$ , the average growth rate of nominal GDP. As a consequence, the debt-to-GDP ratio does most of the work of absorbing the effects of shocks. With this assumption, notice that  $\Delta\bar{c}_t = (\mu - \mu_t)\bar{c}_{t-1}/(1 + \mu_t)$ , so we can rewrite the government budget constraint, (2.3), as

$$\Delta\bar{d}_t = \frac{n_t^d - \mu_t}{1 + \mu_t} \bar{d}_{t-1}^d + \frac{(1 + \delta_t)n_t^f + \delta_t - \mu_t}{1 + \mu_t} \bar{d}_{t-1}^f + \frac{\mu - n_t^c}{1 + \mu_t} \bar{c}_{t-1} - \bar{x}_t. \quad (2.7)$$

X&G again use the assumption that local and foreign currency debt are constant fractions of the overall debt stock ( $\bar{d}_t^d = \omega \bar{d}_t$  and  $\bar{d}_t^f = (1 - \omega) \bar{d}_t$ ) but they replace their assumption about  $n_t^c$  by assuming that an interest parity condition holds. The analog to that condition here is that  $1 + n_t^c = (1 + r_t^f)(1 + \delta_t)$ , where  $r^f$  is the external risk free rate.<sup>11</sup> With these additional assumptions (2.7) becomes

$$\Delta \bar{d}_t = \frac{n_t^d \omega + [(1 + \delta_t)n_t^f + \delta_t](1 - \omega) - \mu_t \bar{d}_{t-1}}{1 + \mu_t} \bar{d}_{t-1} + \frac{\mu - (1 + \delta_t)r_t^f - \delta_t}{1 + \mu_t} \bar{c}_{t-1} - \bar{x}_t. \quad (2.8)$$

What is less clear about X&G's assumptions is how interest rates evolve. They describe the following model for the average interest rate on foreign currency debt,  $n_t^f$ . The average interest rate is given by

$$n_t^f = z_t^f i_t^f + (1 - z_t^f) n_{t-1}^f, \quad (2.9)$$

where  $z_t^f$  is the proportion of foreign debt that is newly issued and  $i_t^f$  is the marginal interest rate on new debt. The process for  $z_t^f$  is

$$z_t^f = 1 - (1 - 1/m_{t-1}^f)(\bar{d}_{t-1}/\bar{d}_t), \quad (2.10)$$

where  $m_t^f$  is the average maturity of the debt at the end of period  $t$ . The longer the maturity of existing debt the less new debt has to be issued. On the other hand if the debt stock is growing more new debt has to be issued. The average maturity of the debt evolves according to

$$m_t^f = z_t^f \tau_t^f + (1 - z_t^f)(m_{t-1}^f - 1), \quad (2.11)$$

where  $\tau_t^f$  is the average maturity of newly issued debt. To close the model one must make assumptions about  $\tau_t^f$ , which X&G do by parameterizing it. One must also solve for  $i_t^f$ , the marginal interest rate on new debt. X&G state that the marginal interest rate can be computed using the baseline model, in the following way. Given  $\bar{c}_{t-1}$  and  $\bar{d}_{t-1}$  as initial conditions, use the model (2.4) to solve for the default probability at each horizon and the implied interest rate at each horizon. Use these implied interest rates and an assumption about the maturity structure of newly issued debt to compute the marginal interest rate,  $i_t^f$ . Of course, proceeding in this way is inconsistent with their new model, since  $\bar{c}_t$  does not evolve in the same manner in the new model as it does in the baseline model. In fact, in the new model, since  $C_t$  grows exogenously at the rate  $\mu$ , as long as  $\mu$  is positive treasury reserves are never exhausted and default never occurs.

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<sup>11</sup>The rates  $r^f$  and  $n^f$  need not correspond to one another.

They also argue that the domestic currency interest rate can be solved for using the foreign currency interest rate, but again, they do not provide the details. Once  $i_t^f$  reaches some exogenously specified threshold level it is assumed that the government can no longer borrow.

In the second phase, the government is no longer able to borrow. In fact, X&G assume that the government cannot issue any new debt in the second phase, so existing debt is gradually amortized. In this case,  $\bar{d}_t$  will evolve in a way that is entirely dictated by the structure of the debt in existence upon entry to the second phase, by exogenous changes in the exchange rate, and exogenous changes in nominal GDP. The evolution of  $\bar{c}_t$  is then governed by (2.3) given the evolution of  $\bar{d}_t$ , which is mechanical once the second phase is entered.

**Summary Discussion** Like Barnhill and Kopits, X&G extend fiscal sustainability analysis into interesting territory with their model of fair spreads. To be fair, it is arguable that they did not intend their model to be used in fiscal sustainability analysis. Rather they are more interested in working out a model that can either explain or interpret the spreads on sovereigns in emerging markets. As a tool for fiscal sustainability analysis, however, their approach is problematic.

- Their paper is technically demanding, which means that—although they provide most of necessary detail for implementing their method—operational economists are unlikely to use their model in large numbers.
- There is no particular reason to assume that reserve exhaustion is the point at which default occurs. Many countries default, or suffer a currency crisis, well before their reserves are exhausted. Even when reserves are exhausted, it is not clear, from a theoretical standpoint, why a government would not simply borrow more in order to replenish the stock of reserves. X&G do not grapple with this issue.
- The interest rates in X&G’s model are not adequately endogenized. They use their baseline model to calculate the interest rates that apply in their expanded model. But, because reserves evolve completely differently in the two models, these interest rates are no more appropriate than exogenously specified interest rates would be.

- As with B&K’s approach, it is unclear how X&G’s fair spread model is useful to a practitioner whose main job is to point out fiscal policy imbalances, discuss their possible consequences, the possible policy responses and their likely effects.

### 2.3. Mendoza and Oviedo’s Model of the Tormented Insurer

Mendoza and Oviedo (M&O, 2004a) construct an explicitly stochastic model to explain the apparent inability of governments in emerging markets to borrow as much as governments in the industrial countries. The deterministic framework, outlined in Section 1, would explain a government’s ability to borrow in terms of its average, or steady state, primary balance, the real interest rate it faces, and the economy’s real growth rate. Volatility would play no role in the explanation. M&O present cross-sectional evidence on roughly 50 countries that indicates a negative association between public debt-to-GDP ratios and (i) the volatility of government revenue as well as (ii) the volatility of GDP growth. Although their evidence does not control for knowledge lenders may have about the ability of the governments in these countries to repay their debt, this suggests that the deterministic approach is inadequate. Their model has important implications for fiscal sustainability analysis because it suggests that steady-state analysis could seriously overstate the sustainability of debt levels in emerging markets.

M&O present two models that explicitly recognize the fact that governments operate in a stochastic environment where the path of revenue is uncertain. Governments in emerging markets also face financial frictions such as sudden stops (rapid capital flow reversals that prevent the government from borrowing) and liability dollarization (the denomination of debt mainly in foreign currency), that affect fiscal sustainability. M&O’s models capture the essence of these financial frictions, although they do so in somewhat different ways.

**Model 1** In the first model the government is assumed to issue all of its debt in dollar-denominated securities, although the currency of denomination plays no important role in the model. The government’s budget constraint is given by:

$$B_t^f = (1 + n_{t-1}^f)B_{t-1}^f - (T_t^f - G_t^f), \quad (2.12)$$

where  $B_t^f$  is the stock of dollar debt,  $n_t^f$  is the dollar interest rate on debt contracted at date  $t$ ,  $T_t^f$  is tax revenue (in dollars) and  $G_t^f$  is primary spending (in dollars). Like B&K and X&G, seigniorage revenue is omitted from the analysis.

To convert the model to one in which debt is expressed as a fraction of GDP, we can define  $\bar{b}_t = S_t B_t^f / (P_t y_t)$ ,  $\bar{t}_t = S_t T_t^f / (P_t y_t)$  and  $\bar{g}_t = S_t G_t^f / (P_t y_t)$ , where  $S_t$  is the local currency per dollar exchange rate,  $P_t$  is the GDP deflator and  $y_t$  is real GDP. We can also define the growth rate of real output,  $\gamma_t = y_t / y_{t-1} - 1$ , the depreciation rate  $\delta_t = S_t / S_{t-1} - 1$ , the inflation rate  $\pi_t = P_t / P_{t-1} - 1$ , and  $r_t = (1 + n_{t-1}^f)(1 + \delta_t) / (1 + \pi_t) - 1$  and rewrite (2.12) as

$$\bar{b}_t = \frac{1 + r_t}{1 + \gamma_t} \bar{b}_{t-1} - (\bar{t}_t - \bar{g}_t). \quad (2.13)$$

At this stage M&O make a number of assumptions about the model: (i) the real local currency (peso) ex-post interest rate on government debt is constant, i.e.  $r_t = r$  for all  $t$ , and (ii) the growth rate of output is constant, i.e.  $\gamma_t = \gamma$  for all  $t$ . Under these assumptions, the lifetime budget constraint derived from (2.13) is just a restatement of (1.10) with seigniorage excluded from revenue:

$$\bar{b}_{t-1} = \sum_{i=0}^{\infty} (\bar{t}_{t+i} - \bar{g}_{t+i}) [(1 + \gamma) / (1 + r)]^{i+1}. \quad (2.14)$$

M&O make further assumptions: (iii)  $\bar{t}_t$  is stochastic, and is bounded from below, i.e.  $\bar{t}_t \geq \underline{t}$  for all  $t$ , (iv) as long as debt remains below some upper bound  $\bar{b}$ , the level of government spending is constant, i.e.  $\bar{g}_t = \bar{g}$ ; (v) if the level of debt reaches  $\bar{b}$  primary spending is permanently cut to a lower level,  $\bar{g}_t = \underline{g}$ .

The third assumption captures the notion that government revenue is volatile. In the fourth assumption  $\bar{g}$  is thought of as the normally desired level of government spending. In the fifth assumption  $\underline{g}$  is thought of as the lowest politically feasible level of government spending. The upper bound for debt,  $\bar{b}$ , is endogenously determined by the law of motion of  $\bar{t}_t$ , and the size of  $\underline{g}$ . To see why, consider that M&O assume that the government never defaults. In this way their analysis is identical to that discussed in Section 1, in that it assumes (2.14) is satisfied for all possible realizations of  $\{\bar{t}_t\}$ . Given that revenue is stochastic it is always possible—though highly unlikely—that the government could face a long sequence of bad revenue shocks. In particular, one could imagine an infinitely long sequence of realizations of  $\bar{t}_{t+i} = \underline{t}$ ,  $\forall i \geq 0$ . In this situation the right-hand side of (2.14) is maximized if  $\bar{g}_{t+i}$  is minimized for  $i \geq 0$ , i.e. if  $\bar{g}_{t+i} = \underline{g}$ ,  $\forall i \geq 0$ . Substitution of  $\bar{t}_{t+i} = \underline{t}$  and  $\bar{g}_{t+i} = \underline{g}$  into (2.14) we see that the maximal level of debt that can be credibly sustained by the government irrespective of the future realizations of  $\bar{t}_t$  is

$$\bar{b}^* = (1 + \gamma)(\underline{t} - \underline{g}) / (r - \gamma). \quad (2.15)$$

Given this model, it is clear that with a sufficiently bad sequence of  $\bar{t}_t$ s the borrowing limit binds. The government, then, switches to the less politically desirable spending level,  $\underline{g}$ , in order to avoid default along all possible future paths of  $\bar{t}_t$ . In applying the model to fiscal sustainability analysis there is a clear lesson. The level of debt that the government can sustain is *not* determined by the steady state, or average, values of  $\bar{t}_t$  and  $\bar{g}_t$ . Suppose these are denoted  $\bar{t}$  and  $\bar{g}$ , respectively. Standard fiscal sustainability analysis would suggest that  $(1 + \gamma)(\bar{t} - \bar{g})/(r - \gamma)$  is the amount of debt the government can sustain. Here the average primary balance determines the sustainable debt level. In M&O's model the relevant primary balance is the one the government would run if it suffered its worst revenue realization and could not borrow. Since  $\underline{t} - \underline{g}$  could be much lower than  $\bar{t} - \bar{g}$  the sustainable debt level could be much lower than standard analysis would suggest.

**Model 2** The second model put forward by M&O is a more complex and detailed general equilibrium model. Some of the details of this model are less important than others. For our purposes, the main object of interest is, again, the government budget constraint. As in the first model, the government borrows only in dollars and the flow budget constraint is given by (2.12)

The second model has more natural specifications of government revenue and expenditure. Measured in local currency, revenue is given by

$$T_t = \tau(P_t^T y_t^T + P_t^N y_t^N), \quad (2.16)$$

where  $\tau$  is the tax rate,  $P_t^T$  and  $P_t^N$  represent prices of traded and nontraded goods, and  $y_t^T$  and  $y_t^N$  are endowments of traded and nontraded output. The prices of the goods are determined endogenously, while the endowments are treated as exogenous stochastic processes.

Measured in local currency primary expenditure is given by

$$G_t = P_t^T g_t^T + P_t^N (g_t^N + v_t), \quad (2.17)$$

where  $g_t^T$  and  $g_t^N$  represent government purchases of traded and nontraded goods, while  $v_t$  represents transfers, measured in units of nontraded goods.  $P_t^T$  and  $P_t^N$  represent prices of traded and nontraded goods, while  $y_t^T$  and  $y_t^N$  are endowments of traded and nontraded output. The prices of the goods are determined endogenously, while the endowments are treated as exogenous stochastic processes.

Of course,  $T_t^f = T_t/S_t$  and  $G_t^f = G_t/S_t$ , where  $S_t$  is the local currency per dollar exchange rate. If, as is standard, we assume that purchasing power parity holds for traded goods and that the foreign price of traded goods is 1,  $P_t^T = S_t$ . Furthermore, if we define the relative price of nontraded goods as  $p_t^N = P_t^N/S_t$ , then  $T_t^f = \tau(y_t^T + p_t^N y_t^N)$ , while  $G_t^f = g_t^T + p_t^N(g_t^N + v_t)$ . With these results, and, with the further assumption that  $n_t^f = n^f$  for all  $t$ , the budget constraint, (2.12), can be rewritten as

$$B_t^f = (1 + n^f)B_{t-1}^f - [\tau(y_t^T + p_t^N y_t^N) - g_t^T - p_t^N(g_t^N + v_t)]. \quad (2.18)$$

To complete the second model, M&O's imagine optimizing households who have standard time-separable preferences defined over traded and nontraded consumption goods as well as the government's traded and nontraded purchases. They receive the endowments of the two goods in each period, pay taxes to the government, and make consumption and savings decisions in order to maximize lifetime utility subject to their budget constraints.

Subject to its budget constraint, the government, in turn, chooses plans for the  $g_t^T$  and  $g_t^N$  to maximize an objective function that resembles household utility, although it discounts future utility somewhat differently than households.<sup>12</sup> Importantly, the government is not a Ramsey planner. It does not take into account how its choices of  $g_t^T$  and  $g_t^N$  will affect the behavior of households. Rather it takes the  $c_t^T$  and  $c_t^N$  sequences as given. Transfers are assumed to be constant in units of nontraded goods:  $v_t = v$  for all  $t$ . The tax rate is set to satisfy the lifetime budget constraint in a nonstochastic steady state, with an arbitrary initial debt level for the government. In the second model there is no political economy story to justify a lower bound on government spending. Instead, M&O put lower bounds on  $g_t^T$  and  $g_t^N$  by specifying preferences such that there are subsistence levels of each, denoted  $\underline{g}^T$  and  $\underline{g}^N$ , respectively. The government sets  $g_t^T > \underline{g}^T$  and  $g_t^N > \underline{g}^N$  to avoid households obtaining infinitely negative utility.

Given the first order conditions for the household and government optimization problems, and given the market clearing conditions, the model can be solved for the household decisions,  $c_t^T$  and  $c_t^N$ , the government decisions,  $g_t^T$  and  $g_t^N$ , and the equilibrium relative price,  $p_t^N$ , as functions of  $y_t^T$  and  $y_t^N$  and the initial debt levels. Shocks to endogenous endowments of  $y_t^T$  and  $y_t^N$  give rise to endogenous variation in  $p_t^N$ ,  $g_t^N$  and  $g_t^T$  and the government's primary balance:  $X_t^f = \tau(y_t^T + p_t^N y_t^N) - g_t^T - p_t^N(g_t^N + v)$ .

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<sup>12</sup>See Mendoza and Oviedo (2004a) for a more detailed discussion of the specification of discount factors for the household and government.

The model implies an upper bound on the government's debt level:

$$B^{f*} = \underline{X}^f / n^f \text{ where } \underline{X}^f = \min\{\tau(y_t^T + p_t^N y_t^N) - g_t^T - p_t^N(g_t^N + v)\}. \quad (2.19)$$

Here, the minimization is over all possible states of the world possible in the model.<sup>13</sup> It is clear that debt can never exceed this limit, because if it did, the government could conceivably face an infinitely long sequence of  $X_t^f$  realizations equal to  $\underline{X}^f$  and would have to default on its debt. Since this is ruled out, debt is bounded from above by  $B^{f*}$ . M&O write the debt limit in terms of  $\min\{\tau(y_t^T + p_t^N y_t^N) - \underline{g}^T - p_t^N(\underline{g}^N + v)\}$ . Since  $g_t^T$  and  $g_t^N$  do not get arbitrarily close to their lower bounds, however,  $\min\{\tau(y_t^T + p_t^N y_t^N) - \underline{g}^T - p_t^N(\underline{g}^N + v)\} > \underline{X}^f$  and leads to a less binding upper bound on debt.

The lesson for fiscal sustainability analysis, in this case, is that the distribution of the shocks to the endowments, and the resulting distribution of the primary balance, puts an upper bound on the level of debt that the government can sustain without default. We can imagine two governments faced with economies in which the means of the endowments are identical, but one of the economies has more variable endowments. It is likely that the two economies will have similar mean values of  $X_t^f$ , but the latter economy will likely have a wider distribution for  $X_t^f$ , a lower value of  $\underline{X}^f$ , and, therefore, a tighter limit on its ability to borrow.<sup>14</sup> Naive fiscal sustainability analysis would conclude that the two countries had similar abilities to borrow.

**Discussion** M&O's models are interesting in that unlike B&K and X&G they are aimed directly at the question of fiscal sustainability. Both models show that standard steady-state fiscal sustainability analysis can lead to misleading conclusions about a government's ability to borrow. When default is ruled out, the volatility of the primary balance becomes an important detriment to borrowing capacity. This is an important lesson for practitioners.. Nonetheless, there are significant problems with both models.

In the first model, the most important factors determining a country's ability to borrow are (i) the lower bound of the distribution of revenue,  $\underline{t}$ , and (ii) the lowest politically feasible expenditure level,  $\underline{g}$ . But in a world similar to the one imagined in the first model, one

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<sup>13</sup>We have described the debt limit in dollar units. If the model had steady state growth in the endowments (and in the subsistence levels of  $g_t^T$  and  $g_t^N$ ) the appropriate debt limit would be expressed in terms of debt relative to trend GDP, the primary balance would be expressed in similar units, and the appropriate constant term would be  $(1 + \gamma)/(n^f - \gamma)$ .

<sup>14</sup>This argument is a little loose because, of course, the distribution of  $X_t^f$  depends not only on the distribution of the endowments but also on the distribution of the endogenous variables  $p_t^N$ ,  $g_t^T$  and  $g_t^N$ .

would observe lots of variation in tax revenue, but only occasional changes in expenditure coinciding with crisis events. Once one considers the empirical evidence, this emphasis on revenue volatility seems misplaced. In many emerging markets primary expenditure varies more than government revenue. For example, Burnside and Mescheryakova (2003)—who study fiscal policy in Mexico in the period 1980–2003—show that the standard deviation of the cyclical components of federal tax revenue, federal nontax revenue and revenue from public sector enterprises were 0.7, 1.1 and 0.7 percent of GDP, respectively (see Table 1). The standard deviations of the cyclical components of total revenue and primary expenditure were, respectively, 1.1 percent of GDP and 1.6 percent of GDP.<sup>15</sup>

TABLE 1  
CYCLICAL PROPERTIES OF PUBLIC SECTOR REVENUE AND  
EXPENDITURE IN MEXICO (1980–2003)

Cyclical component of	Std. Dev.		Correlation with GDP
	%	× share of GDP	
Revenue	4.2	1.1	0.23
Federal tax revenue	6.5	0.7	0.56
Federal nontax revenue	21.2	1.1	-0.11
Revenue of PSEs	7.5	0.7	0.01
Primary expenditure	7.4	1.6	0.61

Source: Burnside and Mescheryakova (2003).

<sup>15</sup>To understand Table 1 in more detail some notes on methodology may be helpful. Each revenue and expenditure series in Table 1 was first converted to constant pesos using the GDP deflator. Each series was then detrended using the Hodrick-Prescott filter to obtain its “cyclical component” which can be interpreted as a percentage deviation from trend. The column marked “Std. Dev. %” provides the standard deviation of each of these series. Clearly, *federal nontax revenue* is by far the most volatile, but it is a small fraction of overall revenue. The next column, marked “× share of GDP” multiplies the standard deviation of each series by its average size as a percentage of GDP. In a sense, this rescales the volatility numbers so that we can understand their importance in contributing to the volatility of the primary balance. The final column indicates the correlation between each series and the cyclical component of GDP.

In the Mexican case, the volatility of primary expenditure is not something driven entirely by crisis events, as Model 1 would suggest. This is clearly indicated by Figures 1a and 1b which illustrate data on revenue and expenditure, expressed as percentages of GDP and detrended levels.

Talvi and Végh (2000) relate the volatility of revenue to the volatility of the tax base and use this to try to explain procyclical fiscal policy in emerging markets. But as their own evidence shows, government purchases are more procyclical than revenue in most Latin American economies. Mexico is no exception as Table 1 and Figure 1b clearly indicate. Model 1 is silent on this aspect of fiscal policy.

Even if Model 1 were modified to capture the volatility of primary expenditure, it would attribute the volatility in the government's debt stock entirely to changes in the primary balance. To see this consider (2.13), which describes the evolution of the debt-to-GDP ratio in Model 1. If we substitute M&O's assumptions that  $r_t$  and  $\gamma_t$  are constant into (2.13) we find that

$$\Delta \bar{b}_t = \frac{r - \gamma}{1 + \gamma} \bar{b}_{t-1} - \bar{x}_t, \quad (2.20)$$

where  $\bar{x}_t$  is the primary balance as a fraction of GDP. In reality, if one used (2.20) in an accounting exercise to explain changes in government debt in an emerging market, one would be left needing a very big residual on the right-hand side. The reason is straightforward: ex-post real interest rates and growth rates are not constant in emerging markets. In fact, they are highly volatile. Mexico is a case in point, as Figure 2 indicates. It plots  $r_t$  under the assumption that the relevant dollar interest rate is the US Fed Funds rate, as well as  $\gamma_t$ , the growth rate of real GDP. Both vary substantially over time, though the dominant variation is in  $r_t$ . If we rewrite (2.13) as

$$\Delta \bar{b}_t = \frac{r_t - \gamma_t}{1 + \gamma_t} \bar{b}_{t-1} - \bar{x}_t, \quad (2.21)$$

the  $(r_t - \gamma_t)\bar{b}_{t-1}/(1 + \gamma_t)$  term often ends up explaining a substantial fraction of the changes in government debt.<sup>16</sup>

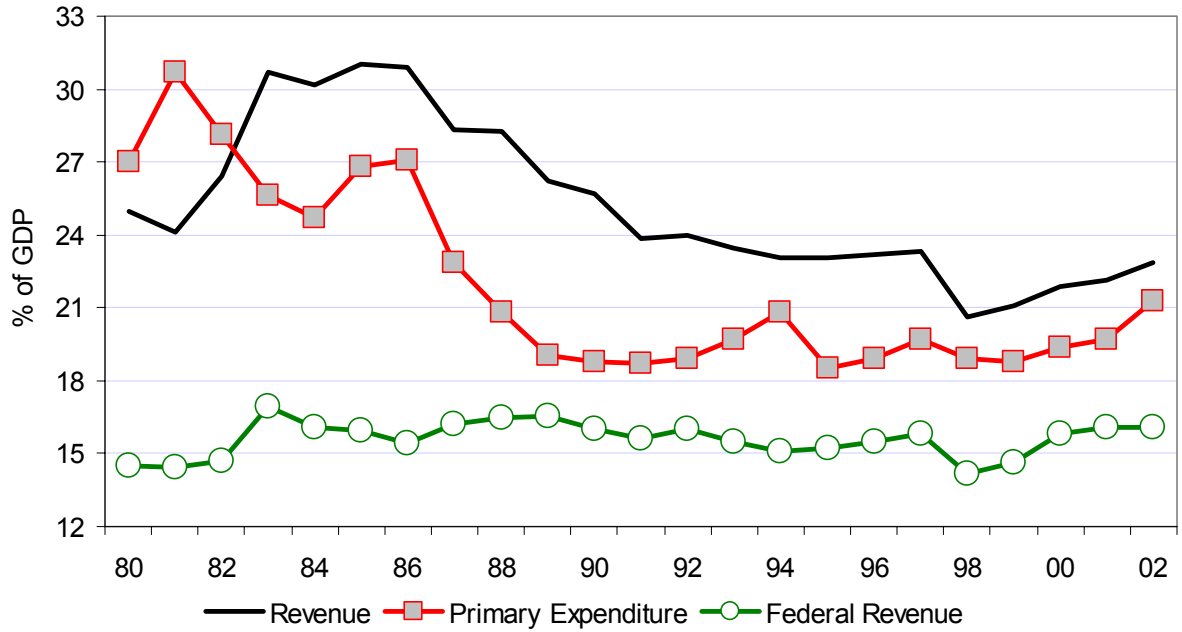
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<sup>16</sup>One should be careful here to note that (2.21) was derived under the assumption that all government debt is dollarized. A more complicated formula applies when some of the debt is denominated in local currency.

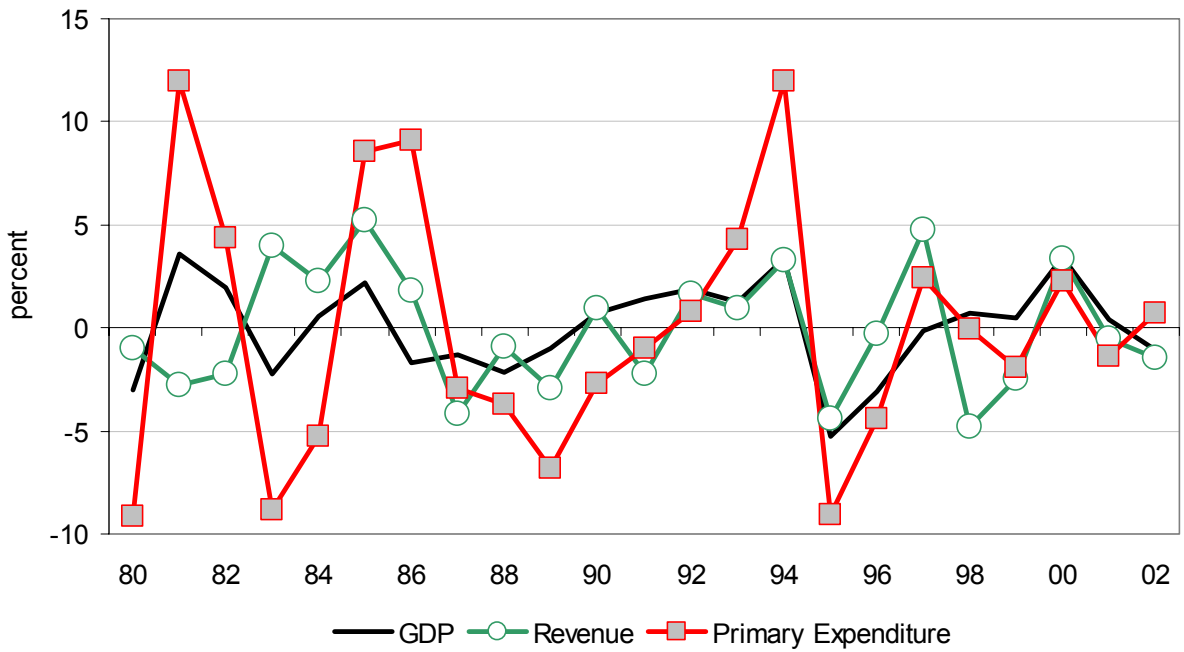
FIGURE 1

PUBLIC SECTOR REVENUE AND EXPENDITURE IN MEXICO

(a) As a Percentage of GDP



(b) Cyclical Components Defined by the Hodrick-Prescott Filter

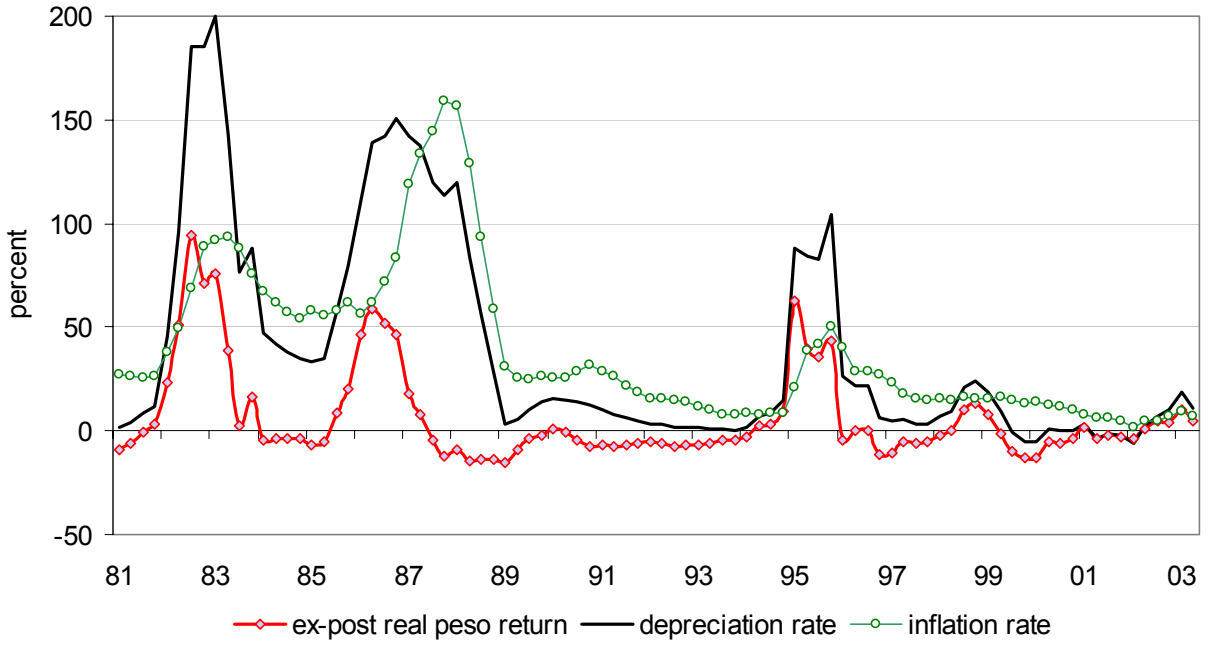


Source: Author's calculations and Burnside and Mescheryakova (2003).

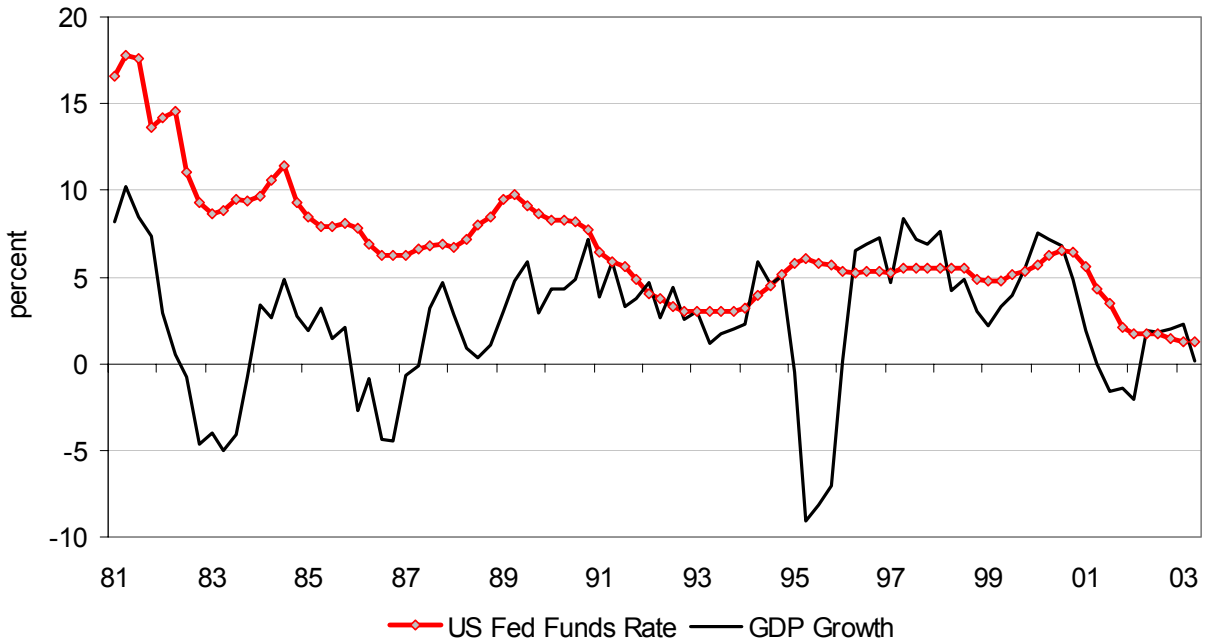
FIGURE 2

EX-POST REAL PESO RETURNS AND GDP GROWTH (AND THEIR COMPONENTS) IN MEXICO

(a) The Ex-Post Peso Return, Peso Depreciation and Inflation



(b) The US Fed Funds Rate and the Growth Rate of Mexico's Real GDP



Source: Author's calculations and Burnside and Mescheryakova (2003).

While M&O emphasize sudden stops as an important friction, there really isn't something in the model one could call a sudden stop. In Model 1 governments do hit borrowing limits, but there is no sense in which capital flow reversals play a role. Admittedly, Model 1 is too simple for it to be otherwise. It is only a model of the government budget constraint. Liability dollarization also does not play a role. Governments borrow in dollars but there is no role for the exchange rate or relative prices.

Model 2 improves on Model 1 by providing a more serious treatment of the government budget constraint that illustrates some of the effects of liability dollarization, ties revenue to the level of real activity in the economy, and introduces variation in government spending.

My comments on Model 2 require some understanding of the properties of the quantitative version of M&O's model, which is calibrated to the Mexican case. Importantly, most of the variation in the primary balance budget flows goes through the direct effects of the endowments,  $y_t^T$  and  $y_t^N$ , on revenue, and the indirect effects of changes in the relative price of nontraded goods,  $p_t^N$ , on both revenue and expenditure. The levels of government purchases,  $g_t^T$  and  $g_t^N$ , are virtually constant.<sup>17</sup> This is not necessarily a criticism of the model, since this does not imply that the real quantity of government purchases does not change. This can still occur due to changes in relative prices. Nonetheless, it is an important property of the model.

One problem for the model in terms of empirical realism is that—like many simple models—it generates unrealistic movements in the relative (or dollar) price of nontraded goods. In the model,  $p_t^N$  turns out to be highly correlated with  $y_t^T/y_t^N$ . One feature of sudden stops, and crises in emerging markets, is that the economy enters a recession, with the collapse in nontraded goods production being much more severe than in the traded goods sector. This implies that  $y_t^T/y_t^N$  rises and that  $p_t^N$  would rise. This is not surprising. The model is one in which the relative scarcity of the nontraded good drives its price. Unfortunately, this means that the model predicts a real exchange rate appreciation during a sudden stop or crisis. Similarly, during booms, emerging markets often experience a rise in nontraded output that outstrips the rise in traded output. In this case,  $y_t^T/y_t^N$  falls, which the model would translate into a decline of  $p_t^N$ . But the typical experience of an emerging market during such a period is that the real exchange rate appreciates. Additionally, the

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<sup>17</sup>M&O report that  $g^T$  and  $g^N$  are 3.6 and 8.6 percent of GDP, respectively, in the nonstochastic steady state of their model. Their standard deviations are 0.037 and 0.15 percent, respectively, which correspond to about 0.001 and 0.013 percent of GDP, respectively.

model unrealistically assumes that the endowments are mutually independent, which tends to overstate the model’s ability to generate a volatile real exchange rate.

Like Model 1, there are no sudden stops in Model 2. This is partly a problem of motivation given that M&O put a lot of emphasis on sudden stops. But it is also a problem of fact. Many of the significant changes in government indebtedness in emerging markets are the result of sudden stops or crisis episodes. The model is missing many of the important features of these events: (i) there are no banking crises, or other shocks, leading to a fiscal crisis, (ii) there is no role for nominal variables, so a collapse in the value of local currency does not have implications for the value of government debt, (iii) like Model 1, the model allows no role for variations in  $r_t$  in driving changes in debt, and (iv) unlike Model 1, the government never hits a borrowing limit, so there are no debt crises.<sup>18</sup>

Finally, there is a misleading policy message provided by M&O in their concluding remarks. They state that:

“Emerging market countries suffer of the syndrome of liability dollarization in public debt. That is, they are exposed to large fluctuations in domestic relative prices of nontradable goods to tradable goods while at the same time they leverage public debt denominated in units of tradable goods on the large fraction of revenues they collect from the nontradables sector of their economies.”

Though liability dollarization may be a problem, the reasoning provided by M&O is very misleading. The government has debt in dollars (traded goods), and, if the nontraded goods sector is larger than the traded goods sector, we have the implication that most revenue comes from taxing nontraded goods. If the relative price of these goods falls, this revenue declines. But once again, M&O’s emphasis on revenue is misplaced. What matters for fiscal sustainability is not revenue, but revenue net of primary expenditure, i.e. the primary balance. It is very likely that this will actually improve (in dollar terms) as the result of a crisis. To see why, consider that the primary balance (measured in dollars) in M&O’s model is

$$X_t^f = \tau(y_t^T + p_t^N y_t^N) - g_t^T - p_t^N(g_t^N + v_t). \quad (2.22)$$

Recall that they assume  $v_t = v$  for all  $t$ , and that to a first approximation,  $g_t^T$  and  $g_t^N$  are

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<sup>18</sup>This is because the government self-insures to guarantee repayment of its debt in all states of the world.

constant in their quantitative example, so that we can write

$$X_t^f \cong \tau(y_t^T + p_t^N y_t^N) - g^T - p_t^N(g^N + v). \quad (2.23)$$

Notice that we can rearrange this as

$$X_t^f \cong \tau y_t^T - g^T + p_t^N(\tau y_t^N - g^N - v). \quad (2.24)$$

Imagine a negative shock to the relative price of nontraded goods, i.e. a decline in  $p_t^N$ . The change in the primary balance is

$$\Delta X^f \cong (\tau y^N - g^N - v)\Delta p^N \quad (2.25)$$

where  $\Delta p^N$  is the change in the relative price, and  $y^N$  is the steady state level of nontraded goods output. How does this affect the primary balance? In M&O's model, which is realistically calibrated to the Mexican economy, nontraded goods represent 60.7 percent of production, i.e.  $y^N = 0.607$ . The tax rate is set at  $\tau = 0.239$  to replicate the ratio of revenue to GDP. Government purchases of goods and services are about 12 percent of GDP, but, importantly, more than two thirds of these purchases are on nontraded goods so that  $g^N = 0.086$ . Transfers represent about 9.5 percent of GDP so  $v = 0.095$ . Together these assumptions imply that  $\tau y^N - g^N - v = -0.036$ , so that  $\Delta X^f \cong -0.036\Delta p^N$ . A real exchange rate depreciation, by itself, helps the government.<sup>19</sup> Why is this the case? Even though the government gets most of its revenue (about 61%) from nontraded goods, an even greater proportion of its spending (about 83%) is on nontraded goods plus transfers. So the government benefits when the price of these goods declines. This finding contradicts the prevailing (and M&O's) view that liability dollarization means that a decline in  $p_t^N$ , by itself, "compromises the government's ability to service public debt and results in sharp declines in sustainable debt-output ratios."<sup>20</sup> It is likely to be a quite general result. Most emerging

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<sup>19</sup>This finding is consistent with Burnside, Eichenbaum and Rebelo's (2003) results in case study for Mexico.

<sup>20</sup>The reader may wonder why the discussion focuses on the primary balance and not the capital loss associated with a revaluation of the local currency against the dollar. Remember that because we work with the government budget constraint expressed in dollars, the dollar value of the government's debt does not change as the result of a depreciation. On the other hand, as the above analysis indicates, the dollar value of the primary balance which helps to service that debt rises. As Burnside, Eichenbaum and Rebelo (2004) show, doing the accounting in local currency units does not change the result. Just as the local currency value of dollar debt jumps up when the currency depreciates, the present value of the future primary surplus (which we have already argued will rise in dollar terms), will also jump up.

economies have large nontraded goods sectors, but have government purchases even more heavily skewed towards nontraded goods and transfers that are indexed to local prices.<sup>21</sup>

Nonetheless, dollarization *is* an issue for a number of reasons not touched on by M&O. Other shocks that hit the economy unambiguously hurt the government's budget. Clearly a recession will do this, other things held equal.<sup>22</sup> A banking crisis usually requires the government to take on substantial new debts as it bails out depositors and other creditors, or recapitalizes banks. Banking crises, thus, turn into fiscal crises. It would help the government, in such circumstances, if the dollar value of some of its debt declined. This would be the case, for example, if the government had local currency debt, and the banking crisis coincided with a currency crisis.<sup>23</sup>

Liability dollarization also matters because it is a problem for firms and banks who, like the government, have dollar debts, but who, unlike the government, generate positive net flows from their activities in the nontraded goods sector and negative net flows from their activities in traded goods.<sup>24</sup> These firms and banks will face insolvency if a major real exchange rate movement occurs. Their private balance sheet problem becomes a government problem if the government bails them out. This type of contingent liability is absent from M&O's model.

#### **2.4. The IMF's Approach to Fiscal Sustainability Analysis**

The IMF's approach to fiscal sustainability analysis is evolving, but conventional methods are described in two reports, IMF(2002) and IMF (2003). These reports indicate that fiscal sustainability analysis is treated by the IMF as both a forecasting tool, and as an instrument of policy dialogue with its member governments. Generally speaking, the IMF produces regular macroeconomic projections in its surveillance work, as well as for countries with active loans. These macroeconomic projections are fed into its projections of a government's debt dynamics, i.e. the forward simulation of the budget constraint, written as (2.13). The path of the debt stock under these projections, as well as possible statistical deviations from

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<sup>21</sup>See Burnside, Eichenbaum and Rebelo (2003) for a more general discussion and case studies of Mexico, Korea and Turkey.

<sup>22</sup>A recession may coincide with a real exchange rate depreciation, but the the latter, as we have seen will offset the effect of the recession on revenue.

<sup>23</sup>Of course, to issue such debt the government would have to pay a risk premium ex-ante.

<sup>24</sup>A profitable nontraded goods producer generates all his revenue from selling nontraded goods, but, presumably, uses both nontraded and traded goods as inputs. So he runs a surplus on nontraded goods and probably runs a deficit on traded goods.

this path, are presented. In this sense, the simulated debt dynamics represent a forecast combined with a confidence interval around that forecast. The next step in the analysis involves interpreting the implications of the debt forecast (and confidence band) for fiscal sustainability. This usually involves terminating the forecast at some future date at which standard long-run fiscal sustainability analysis, as described in Section 1, is applied.

**The Basic Framework** Consider (1.2), our generic description of the government's life-time budget constraint. In the IMF's basic template the public sector usually does not explicitly include the central bank, so we will exclude seigniorage revenue from (1.2) and write it as:

$$B_t - B_{t-1} = I_t - X_t, \quad (2.26)$$

where, again,  $B_t$  is the stock of public debt at the end of period  $t$ ,  $I_t$  is interest payments, and  $X_t$  is the primary balance, all measured in units of local currency. Conceptually, the IMF separates out grants and other non-debt creating international transfers, as well as privatization revenue from the rest of the primary balance. For simplicity, here we include them in  $X_t$ . The IMF's framework is most consistent with the following assumptions.

*Assumption Set 2.* Assume that (i) time is discrete, (ii) debt has a maturity of one period, (iii) debt is nominal (in other words, its face value is fixed in nominal terms) and (iv) debt issued at date  $t - 1$  pays a nominal interest rate,  $n_{t-1}$ .

With these assumptions (2.26) can be rewritten as

$$B_t = (1 + n_{t-1})B_{t-1} - X_t. \quad (2.27)$$

It is conventional to rewrite the stocks and flows in (2.27) as fractions of nominal GDP. Here we again define  $\bar{b}_t = B_t/(P_t y_t)$  and  $\bar{x}_t = X_t/(P_t y_t)$ , where  $P_t$  is the GDP deflator and  $y_t$  is real GDP. It is easy to show that with this notation, (2.27) can be rewritten as:

$$\bar{b}_t = \frac{(1 + n_{t-1})}{(1 + \pi_t)(1 + \gamma_t)} \bar{b}_{t-1} - \bar{x}_t, \quad (2.28)$$

where  $\pi_t = P_t/P_{t-1} - 1$  is the inflation rate and  $\gamma_t = y_t/y_{t-1}$  is the real growth rate. This implies that the change in the stock of debt is

$$\Delta \bar{b}_t = \frac{n_{t-1}}{(1 + \pi_t)(1 + \gamma_t)} \bar{b}_{t-1} - \frac{\gamma_t}{(1 + \pi_t)(1 + \gamma_t)} \bar{b}_{t-1} - \frac{\pi_t(1 + \gamma_t)}{(1 + \pi_t)(1 + \gamma_t)} \bar{b}_{t-1} - \bar{x}_t. \quad (2.29)$$

The first term on the right-hand side of (2.29) is interest payments on government debt as a fraction of GDP. The second term is a growth effect, which reflects the fact that in a growing economy, the debt-to-GDP ratio would fall even if the stock of debt were constant. The third term is an inflation effect that reflects the fact inflation erodes the face value of nominal debt, creating a capital loss for investors and a capital gain for the government.

The IMF's fiscal sustainability template is based on (2.29). Typically, completing the template would require decomposing 5 years of historical changes in the stock of debt into the 4 terms on the right-hand side of (2.29). It also would require simulating (2.28) forward for 5 to 10 years, using the IMF's projections for the interest rate  $n_t$ , the inflation rate,  $\pi_t$ , the real growth rate,  $\gamma_t$  and the primary balance,  $\bar{x}_t$ . Here we refer to this simulated path, computed at time 0, as  $\{\bar{b}_t^e\}_{t=1}^T$ , where  $T$  represent the maximal forecast horizon.

The sustainability template also suggests conducting the following exercises. First, when possible, compute the means and standard deviations of  $n_t$ ,  $\pi_t$ ,  $\gamma_t$  and  $\bar{x}_t$  in the previous 10 years. Then compute the following alternative simulations:

- Let all 4 of the variables equal their historical averages in the projections. Denote the path of debt according to this projection,  $\{\bar{b}_t^A\}_{t=1}^T$ .
- Let  $n_0$  and  $n_1$  be equal to their historical average plus two standard deviations, but let all others variables and  $n_2, n_3$ , etc., equal their values in the baseline projections. Denote the path of debt according to this projection,  $\{\bar{b}_t^n\}_{t=1}^T$ .
- Let  $\pi_1$  and  $\pi_2$  be equal to their historical average *less* two standard deviations, but let all others variables and  $\pi_3, \pi_4$ , etc., equal their values in the baseline projections. Denote the path of debt according to this projection,  $\{\bar{b}_t^\pi\}_{t=1}^T$ .
- Let  $\gamma_1$  and  $\gamma_2$  be equal to their historical average *less* two standard deviations, but let all others variables and  $\gamma_3, \gamma_4$ , etc., equal their values in the baseline projections. Denote the path of debt according to this projection,  $\{\bar{b}_t^\gamma\}_{t=1}^T$ .
- Let  $\bar{x}_1$  and  $\bar{x}_2$  be equal to their historical average *less* two standard deviations, but let all others variables and  $\bar{x}_3, \bar{x}_4$ , etc., equal their values in the baseline projections. Denote the path of debt according to this projection,  $\{\bar{b}_t^x\}_{t=1}^T$ .
- A combined one standard deviation shock of  $(n_{t-1}, \pi_t, \gamma_t, \bar{x}_t)_{t=1,2}$  relative to historical average, in the directions described above, with future realizations equal the baseline

projections. Denote the path of debt according to this projection,  $\{\bar{b}_t^C\}_{t=1}^T$ .

The alternative scenarios represent a type of sensitivity analysis.<sup>25</sup>

In IMF (2003) there is a further discussion of this type of sensitivity analysis, that suggests formalizing it by using a Monte Carlo simulation approach over the full  $T$ -period horizon.<sup>26</sup>

Let  $z_t$  be a vector containing  $n_t$ ,  $\pi_t$ ,  $\gamma_t$  and  $\bar{x}_t$ . Consider the vector autoregressive model

$$z_t = \beta_0 + \sum_{k=1}^K \beta_k z_{t-k} + \epsilon_t, \quad (2.30)$$

where  $\epsilon_t$  is an iid error term with 0 mean and covariance matrix  $\Sigma$ . This model could be estimated using historical data. A  $T$ -year Monte Carlo simulation exercise could be conducted, where  $\epsilon_t$  is drawn from a normal distribution, or a bootstrap procedure could be used, where  $\epsilon_t$  is drawn from the empirical distribution of the residuals. This approach represents a potential improvement to the sensitivity analysis, since it more naturally produces a meaningful confidence band for the projections. On the other hand, in practice it may be difficult to estimate a model such as (2.30) for many emerging market countries. Adequately long time series will often be hard to come by.

The IMF's template would usually conclude by taking the baseline simulation, and perhaps one of the others, and evaluating the debt stock at the end of period  $T$ ,  $\bar{b}_T^e$ . The steady-state analysis we discussed in Section 1 would then be applied. The primary balance needed to attain fiscal sustainability from date  $T$  forward would be

$$\bar{x} = \frac{n - \gamma - \pi(1 + \gamma)}{(1 + \pi)(1 + \gamma)} \bar{b}_T^e, \quad (2.31)$$

where  $n$ ,  $\pi$  and  $\gamma$  would represent the long-run projections for the interest rate, inflation and growth. This value of the primary balance would be compared to the long-run projection for the primary balance. The likelihood of the government being able to sustain  $\bar{x}$  would be framed around this comparison.

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<sup>25</sup>IMF (2002) also suggests three further scenarios. In the first of these, *standard* standard deviations are used in place of the historical standard deviations in computing  $\bar{b}_t^C$ . The term *standard* refers to the fact that the deviations are based on broad cross-country experience. In the second scenario, a one-time 30 percent exchange rate depreciation is imagined, while the other variables are left at the baseline values. To understand this scenario the model of the budget constraint must explicitly include external debt. The third scenario imagines a one-time increase in the debt-to-GDP ratio, which is meant to capture the possible effects of the realization of significant contingent liabilities, possibly in the banking sector.

<sup>26</sup>In IMF (2003) this approach is applied in an external debt sustainability exercise, but, in principle, there is no reason not to apply it to fiscal sustainability.

**Discussion** The IMF’s approach conforms most closely to traditional fiscal sustainability analysis, and, in fact, the simple analysis, discussed in Section 1, has often been extended in this direction by individual country studies in the past. It represents an improvement over the simple analysis because it recognizes the importance of short- and medium-term macroeconomic fluctuations in determining future debt burdens. Like the simple analysis it is based on forward looking projections of important macroeconomic variables, but these are allowed to vary over time, and uncertainty is explicitly used in the sensitivity analysis.

There are a number of ways in which the IMF approach could be improved or enhanced, and some aspects of the approach have serious shortcomings. Parts of the sensitivity analysis are atheoretical, and some of the simulated scenarios may be extremely unlikely. For example, in a country with volatile inflation, the nominal interest rate and the inflation rate are likely to be highly correlated. Scenarios in which one of these variables moves by two standard deviations and the other is held constant are highly unlikely. The combined scenario is also highly unlikely, given that the inflation rate and the interest rate move in opposite directions.<sup>27</sup> The sensitivity analysis does not take into account how the primary balance might respond to the stock of debt, nor to how the interest rate might depend on both the primary balance and the stock of debt. These shortcomings are more probably more important if the purpose of the analysis is to produce accurate forecasts. Adding structure to forecasting models could, presumably, reduce forecast error variance. On the other hand, these shortcomings might not be as critical if the purpose of the analysis is to engage in a policy discussion motivated by the projections. Presumably such discussions center on asking the government how its policies *would* respond to the state of the economy and its finances.<sup>28</sup>

The simulation approach, to some extent, deals with these issues, because it envisions a model in which the shocks to  $n_t$ ,  $\pi_t$ ,  $\gamma_t$  and  $\bar{x}_t$  are correlated. Nonetheless, since the model is purely statistical it may not capture some of the relationships among these variables that may be important from a theoretical standpoint. One aspect of this, is that the response of  $\bar{x}_t$  to  $\bar{b}_{t-1}$  is not modeled. This response has important implications for fiscal sustainability, and for forecast error confidence bands. To see why, consider a simplified version of (2.28),

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<sup>27</sup>Results in IMF (2003) using the stochastic simulation approach also suggest that the combined-shock scenario is highly unlikely.

<sup>28</sup>It seems less likely that a policy discussion involves the presentation to the government of a complete forecast by the IMF or World Bank; i.e. one that includes the presumed government policy response.

where  $n_t$ ,  $\pi_t$  and  $\gamma_t$  are treated as constants:

$$\bar{b}_t = (1 + \bar{r})\bar{b}_{t-1} - \bar{x}_t, \quad (2.32)$$

where  $\bar{r} = [n - g - \pi(1 + g)] / [(1 + \pi)(1 + \gamma)]$ . Imagine that a forecasting model of  $\bar{x}_t$  is built in which it is treated as a stationary 1st order autoregressive process that does not feed back on  $\bar{b}_{t-1}$ : i.e.  $\bar{x}_t = \bar{x} + \rho(\bar{x}_{t-1} - \bar{x}) + \epsilon_t$  with  $|\rho| < 1$ . We can then write

$$[1 - (1 + \bar{r})L]\bar{b}_t = -\bar{x} - (1 - \rho L)^{-1}\epsilon_t. \quad (2.33)$$

Premultiplying by  $(1 - \rho L)$  we have

$$(1 - \rho L)[1 - (1 + \bar{r})L]\bar{b}_t = -(1 - \rho)\bar{x} - \epsilon_t. \quad (2.34)$$

This equation has two important implications. First, it implies that unless  $\bar{x} = \bar{r}\bar{b}_0$ , the stock of debt will explode (or implode) in the absence of shocks. Second, it implies that regardless of the precise value of  $\rho$ , the stock of debt will be a nonstationary time series process. In this example, it will be a second order AR process, but one of its roots is explosive. This nonstationarity may not bother the practitioner, nor is it necessarily unrealistic in the leadup to a crisis, but it is worth pointing out. It also implies that the mean of the long-run forecast of  $\bar{b}_t$  will be divergent and that the confidence bands around forecast paths will widen with the forecast horizon.

As Bohn (1998) has shown, if the government takes deliberate corrective action as its debt stock rises, this nonstationarity is eliminated. To see this consider an alternative model in which  $\bar{x}_t = -\alpha + \rho\bar{b}_{t-1} + \epsilon_t$ , with  $0 < \rho < 1$ . The idea, here, is that there are random shocks to fiscal policy (the  $\epsilon_t$ s), but there is also a deliberate policy rule which increases the primary balance as the stock of debt rises. Under this law of motion, (2.32) becomes

$$[1 - (1 + \bar{r} - \rho)L]\bar{b}_t = \alpha - \epsilon_t. \quad (2.35)$$

In this case if  $\rho > \bar{r}$ , the law of motion is  $\bar{b}_t$  is stationary, and the  $\bar{b}_t$  fluctuates around an unconditional mean given by  $\bar{b} = \alpha / [1 - (1 + \bar{r} - \rho)]$ . The long-run forecast of  $\bar{b}_t$  is  $\bar{b}$  and the confidence regions associated with the forecasts will eventually be smaller (and vanish) as the forecast horizon lengthens.

Bohn (1998) also explores nonlinear reaction functions for  $\bar{x}_t$ . These can be specified in such a way that the government's rule for  $\bar{x}_t$  is not debt-stabilizing when  $\bar{b}_t$  is within some range, but becomes debt-stabilizing if the debt stock goes outside an upper or lower bound.

Another issue with the IMF approach is that neither IMF (2002) nor IMF (2003) puts much emphasis on how changes in the exchange rate affect debt dynamics, despite the fact that for many emerging markets movements in the exchange rate will explain many of the sharp changes in government debt. One of the scenarios mentioned in the IMF's template for fiscal sustainability analysis is a one-time 30 percent exchange rate depreciation. But there is little discussion of how exchange rate movements enter into the budget constraint. Suppose we reconsider (2.26) by making the following assumptions:

*Assumption Set 3.* Assume that (i) time is discrete, (ii) debt has a maturity of one period, (iii) all debt is nominal, but some,  $B_t^d$ , is issued in local currency, while other debt,  $B_t^f$ , is issued in dollars, and (iv) local currency debt issued at date  $t - 1$  pays a nominal interest rate,  $n_{t-1}$ , while foreign currency debt pays the dollar rate,  $n_{t-1}^f$ .

With these assumptions (2.26) can be rewritten as

$$B_t^d - B_{t-1}^d + S_t(B_t^f - B_{t-1}^f) = n_{t-1}B_{t-1}^d + S_t n_{t-1}^f B_{t-1}^f - X_t, \quad (2.36)$$

where  $S_t$  is the local currency per dollar exchange rate. Total debt is  $B_t = B_t^d + S_t B_t^f$ . The debt-to-GDP ratio is  $\bar{b}_t = \bar{b}_t^d + \bar{b}_t^f$ , where  $\bar{b}_t^d = B_t^d/(P_t y_t)$  and  $\bar{b}_t^f = S_t B_t^f/(P_t y_t)$ . With a considerable amount of algebra it can be shown that

$$\Delta \bar{b}_t = \bar{v}_t - \bar{x}_t - \frac{\gamma_t}{(1 + \pi_t)(1 + \gamma_t)} \bar{b}_{t-1} - \frac{\pi_t(1 + \gamma_t)}{(1 + \pi_t)(1 + \gamma_t)} \bar{b}_{t-1} + \frac{\delta_t}{(1 + \pi_t)(1 + \gamma_t)} \bar{b}_{t-1}^f \quad (2.37)$$

where  $\bar{v}_t$  is interest payment at time  $t$  as a fraction of GDP, and is given by

$$\bar{v}_t = \frac{n_{t-1} \bar{b}_{t-1}^d + (1 + \delta_t) n_{t-1}^f \bar{b}_{t-1}^f}{(1 + \pi_t)(1 + \gamma_t)}, \quad (2.38)$$

where  $\delta_t = S_t/S_{t-1} - 1$  is the rate of depreciation. Notice that relative to (2.29) we have the additional depreciation effect, which shows how a depreciation of the local currency raises the debt-to-GDP ratio if the government holds foreign debt. Sometimes (2.37) is rewritten by writing it as

$$\Delta \bar{b}_t = \bar{v}_t - \bar{x}_t - \frac{\gamma_t}{(1 + \pi_t)(1 + \gamma_t)} \bar{b}_{t-1} - \frac{\pi_t(1 + \gamma_t)}{(1 + \pi_t)(1 + \gamma_t)} \bar{b}_{t-1}^d + \frac{\delta_t - \pi_t(1 + \gamma_t)}{(1 + \pi_t)(1 + \gamma_t)} \bar{b}_{t-1}^f. \quad (2.39)$$

In this formulation the inflation effect only applies to domestic debt, and the effect stemming from external debt depends on a real exchange rate movement. Of course, the two formulations are equivalent. The important point is that the term relating to foreign debt

is quantitatively important, as are shocks to the exchange rate, in many emerging markets. They should not be treated as an afterthought.

Finally, as we saw above, focusing only on the effects of changes in the exchange rate on the debt stock can be highly misleading. Changes in the nominal exchange rate are often accompanied by persistent changes in the real exchange rate. As we saw in the discussion of Mendoza and Oviedo’s paper, these changes in the real exchange rate, in turn, have significant implications for the government’s primary balance. As Burnside, Eichenbaum and Rebelo’s (2003) case studies show, treating the primary balance as invariant to the real exchange rate would lead to highly misleading conclusions about fiscal policy.

### 3. Case Studies

We turn now to the three reports that include case studies: da Costa, Silva and Baghdassarian (2004), Mora (2004) and Mendoza and Oviedo (2004b). These papers implement variants of the four approaches mentioned so far, as well as methods of their own.

#### 3.1. Da Costa, Silva and Baghdassarian’s Case Study of Brazil

In their case study of Brazil, da Costa, Silva and Baghdassarian (DSB, 2004) consider three methods: (i) the VaR approach of Barnhill and Kopits (2003), (ii) the fair spreads model of Xu and Ghezzi (2003) and (iii) their own approach, which uses a small structural macroeconomic model built by the Brazilian Treasury.

**The VaR Approach** In the case of the VaR approach DSB point out right away that their approach is a little different than that of BK, because less detailed information on assets and liabilities is available for Brazil. On the asset side they do not have measures of tangible assets and the value of public sector enterprises. On the liability side they do not try to model specific contingent liabilities. They simply compare the value of the government’s outstanding debt securities to the present value of its residual (entire) primary balance:

$$W_t = E_t \left( \sum_{i=1}^{\infty} X_{t+i}/N_t^{t+i} \right) - B_t. \quad (3.1)$$

One of the main possible advantages of BK’s approach is that it provides a way of assessing the importance of large shocks—such as a permanent change in the world oil price in an oil producing country, or a banking crisis—relative to the government’s other obligations and potential revenue.

DSB find that their measures of  $E_t(\sum_{i=1}^{\infty} X_{t+i}/N_t^{t+i})$  vary substantially from one period to the next. Their measures of  $B_t$  do not. So the size of the VaR depends almost entirely on what happens to  $E_t(\sum_{i=1}^{\infty} X_{t+i}/N_t^{t+i})$  from one period to the next. The estimated VaR is also very large, with the standard deviation of net worth representing about 25 percent of its mean.

DSB do not provide enough details of their simulations to assess them. Nonetheless, one is quickly led to the conclusion that the VaR approach provides little to guide policy discussions. It focuses on a bottom line number and unless it is a very detailed analysis, it does not provide much information about where there the risks to fiscal sustainability lie. Neither does it indicate how risk changes over time.

**The Fair Spreads Approach** XG's fair spreads model is implemented straightforwardly by DSB. They model the change in treasury reserves in the same way as XG, and compute default probabilities at various horizons. To map these default probabilities to spreads they estimate the recovery rate by fitting the actual yield curve to the model's predicted yield curve over the period 1999–2001. The exogenous processes needed as inputs to the model are calibrated to high-frequency Brazilian data from the period 1999–2002.

In the end, DSB estimate default probabilities ranging from a low of 20 percent at a one year horizon to about 90 percent at a 7 year horizon. It is difficult to assess whether DSB's estimates are reasonable or not. One would either need a long sample for Brazil, or a short panel for a large number of countries, to verify whether defaults take place at the predicted frequency.

The main criticisms of these default probability estimates are the ones we saw above: (i) the interest rates faced by the government used to compute the default probabilities are exogenous, but should, in reality, reflect those probabilities, (ii) the debt-to-GDP ratio is held constant, so that all shocks are absorbed by reserves, (iii) the government makes no attempt to avoid default by borrowing and (iv) the default probabilities and spread calculations do not, in any obvious way, map into policy analysis.

**DSB's Structural Model Approach** For their third approach, DSB design their own model, which, in some ways, resembles the IMF's vector autoregression based model, but has more economic structure and pays more attention to the detailed structure of Brazilian debt. Apart from the budget constraint, the guts of the model consist of 4 equations. The

first equation is an aggregate demand equation (IS curve):

$$y_t = -\beta r_{t-1} + \delta e_{t-1} + \lambda y_{t-1} + \epsilon_t, \quad (3.2)$$

where  $y_t$  is the output gap—the difference between the log level of GDP and the trend implied by the Hodrick and Prescott (1997) filter— $r_t$  is the real interest rate and  $e_t$  is the logarithm of the real exchange rate. The second equation specifies the aggregate supply side (Phillips curve):

$$\Delta\pi_t = \alpha y_{t-1} + \gamma \Delta e_{t-1} + \eta_t, \quad (3.3)$$

where  $\pi_t$  is the inflation rate of the CPI. The third equation determines the value of the real exchange rate:

$$e_t = \chi m_t + \nu_t, \quad (3.4)$$

where  $m_t$  is the EMBI spread over US treasuries. The spread itself is modeled as:

$$m_t = \kappa m_{t-1} + \omega \bar{b}_{t-1} + u_t. \quad (3.5)$$

Finally, real interest rates are assumed to be determined by a Taylor rule:

$$r_t = \rho r_{t-1} + \psi(\pi_{t-1} - \pi^*) + \phi y_{t-1}, \quad (3.6)$$

where  $\pi^*$  is an inflation target. The variables  $\epsilon_t$ ,  $\eta_t$ ,  $\nu_t$  and  $u_t$  are iid error terms.

DSB combine this model with a model of the effective nominal interest cost on 4 types of Brazilian debt: (i) floating rate bonds, (ii) fixed rate bonds, (iii) exchange rate indexed bonds, and (iv) inflation indexed bonds. Since simulations of the model are only done one step ahead, marginal interest costs (the interest rates on newly issued bonds) for each instrument type are known. Marginal interest costs map into average interest costs differently for each type of bond. For floating rate bonds the average interest cost is just this year's short term interest rate. For the other types of bonds the average interest rate at date  $t$  is  $\sum_{s=0}^n \omega_{t-s} n_{t-s}$  where  $\omega_{t-s}$  represents the fraction of the debt issued at date  $t-s$  and  $n_{t-s}$  represents the marginal interest rate at that time. DSB assume that the maturity structure of multi-period debt is of the form  $\omega_{t-s} = (1 + n_{t-s})^{-s} / \sum_{s=0}^{\tau} (1 + n_{t-s})^{-s}$  where  $\tau$  is the maximal maturity. The structure of debt instruments across types is assumed to constant over time. It is not clear whether the model links  $r_t$  to the marginal short term interest rate that applies to floating rate debt.

DSB use the model to simulate debt dynamics recursively over time from the end of 2000 through the end of 2003. The primary balance is assumed to be constant as a fraction of GDP. At each step, the simulations take into account new information, so forecasts are only made one step ahead. Confidence bands around mean predictions are calculated using Monte Carlo methods. The model does quite well in 2001 and 2002, with the mean forecast of the debt-to-GDP ratio being within 3 percent of GDP of the actual outcome in 2001 (0.5 percent of GDP in 2002). In 2003, on the other hand, the forecast is too high by 8.5 percent of GDP and lies well outside the 95 percent confidence band.

DSB's model has a number of difficulties. First, without more serious modeling of asset price determination the model cannot be used to forecast forward more than one period. Second, the model is relatively undeveloped as a forecasting model—DSB argue that the model is too sensitive to initial conditions but they do not elaborate on what they mean by this. Third, the model, like many, assumes that the primary balance as a fraction of GDP is invariant to shocks. We have discussed, above, how this is unrealistic if there are significant shocks to the real exchange rate.

### **3.2. Mora's Case Studies of Colombia and Costa Rica**

In his case studies of Colombia and Costa Rica, Mora (2004) considers four methods: (i) the IMF's (2002) approach (ii) Mendoza and Oviedo's (2004a) model, (iii) his own approach, which uses a small model of the structural fiscal balance and (iv) an additional approach which attempts to estimate default probabilities.

**The IMF Approach** Mora applies the IMF's formal statistical approach by fitting VARs to historical Colombian and Costa Rican time series on the interest rate, the inflation rate, real growth rate, and primary balance. His results illustrate one of the main drawbacks of using the statistical approach. For both countries the sample is small, consisting of a couple of decades of annual data. For Colombia all 4 of the series appear to be integrated, and the estimated VAR provides little forecasting power. As a result Mora replaces most of the forecasting equations in the VAR with pure random walks before using the model to simulate data for the 2004–08 period. Since the simulation is forward looking Mora does not evaluate his projections.

Mora's quantitative results are puzzling. Colombia's initial debt-to-GDP ratio is 51 percent for the simulations. Costa Rica's is 40 percent. Despite the apparent greater degree of

volatility in Colombia’s economy, however, the confidence bands generated for the Colombian simulations are much narrower. Going out as far as 2008, the projected debt-to-GDP ratio has an average of 123 percent, and the upper boundary of the 95 percent confidence region is 205 percent. These numbers reflect Colombia’s explosive debt dynamics in recent years, and the lack of stationarity in Mora’s empirical model for the Colombian data. On the other hand, for Costa Rica, for 2008 the mean debt-to-GDP ratio is projected to be 73 percent, but the upper boundary of the 95 percent confidence region is 441 percent.<sup>29</sup> The projections for Costa Rica seem incredible, and illustrate the pitfalls of applying a statistical approach built from a template.

**Mendoza and Oviedo’s Model 1** Mora also applies Mendoza and Oviedo’s (2004a) first model to Colombia and Costa Rica. Recall that their model consists of a single equation

$$\bar{b}_t = \frac{1+r}{1+\gamma} \bar{b}_{t-1} - (\bar{t}_t - \bar{g}_t), \quad (3.7)$$

where  $\bar{b}_t$  is the debt-to-GDP ratio,  $r$  is a constant real interest rate,  $\gamma$  is a constant real growth rate,  $\bar{t}_t$  is the ratio of revenue to GDP and  $\bar{g}_t$  is the ratio of primary expenditure to GDP. The model implies a debt limit given by (2.15): i.e.  $\bar{b}_t \leq \bar{b}^* = (1+\gamma)(\underline{t} - \underline{g})/(r - \gamma)$ . Here  $\underline{t}$  is the worst possible realization of revenue, and  $\underline{g}$  is the lowest politically feasible level of government expenditure. To obtain the debt limit, the model requires specifying  $r$ ,  $\gamma$ ,  $\underline{t}$  and  $\underline{g}$ , while to use the model to simulate debt paths, one must also specify the law of motion of  $\bar{t}_t$  and the normal, constant, value of  $\bar{g}_t$ .

For Colombia, Mora uses data for 1990–2003 to calibrate the parameters. He chooses  $r = 0.065$ , and sets  $\gamma = 0.027$ , its sample average over the period. He sets  $\underline{t} = 0.103$ , which corresponds to its mean minus two standard deviations over the sample period. This implies  $\bar{b}^* = 2.8 - 27.25\underline{g}$ . The resulting debt limit is extremely sensitive to the choice of  $\underline{g}$ . If one sets  $\underline{g} \approx 0.084$ , then one obtains  $\bar{b}^* = 0.51$ , roughly Colombia’s current debt level. On the other hand, if one sets  $\underline{g}$  equal to its lowest observed value in the sample, 0.103, the debt limit is roughly 0.

For Costa Rica, Mora uses the same time period, and also sets  $r = 0.065$ . He sets  $\gamma = 0.047$ , its sample average over the period. He sets  $\underline{t} = 0.134$ , which corresponds to its mean minus two standard deviations over the sample period. This implies  $\bar{b}^* = 7.7498 - 57.834 \times \underline{g}$ .

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<sup>29</sup>The narrowness of the confidence region in the Colombian case probably reflects the arbitrary imposition of the random walk model for forecasting purposes.

For reasonable levels of  $\underline{g}$ , Costa Rica's debt limit is higher because it has faster growth, and a higher level of revenue in bad times. To obtain the same debt limit as Colombia, we would need to set  $\underline{g}$  to a much higher value, i.e.  $\underline{g} = 0.125$  implies  $\bar{b}^* = 0.51$  while, of course,  $\underline{g} = 0.134$  implies  $\bar{b}^* = 0$ . The lowest observed value of  $\bar{g}_t$  in Costa Rica in the sample period was 12.5 percent of GDP.

Mora then turns to a simulation study in which he estimates Markov chains for revenue, and calibrates  $\underline{g}$  so that both countries have a debt limit of  $\bar{b}^* = 0.5$ . The Markov chain is simple. In Colombia, revenue is assumed to lie in one of 7 intervals,  $(0, 11]$ ,  $(11, 12]$ ,  $(12, 13]$ ,  $\dots$ ,  $(16, 17]$ , while in Costa Rica it is assumed to lie in one of 6 intervals  $(0, 13]$ ,  $(13, 14]$ ,  $\dots$ ,  $(17, 18]$ . Mora constructs the Markov transition matrix by evaluating the empirical probability that revenue moved from one of these states to another. Clearly, with a small sample, he uses too many states, as many of these transitions never occurred, or occurred just once. Given that there are 42 free parameters to be identified (in the case of Colombia), the sample is insufficiently long. Within each interval the value of  $\bar{t}_t$  is assumed to equal the average of the values within that interval in the sample period.

The model can be used to simulate debt paths starting from arbitrary initial conditions. These simulations can be used to produce conditional forecasts of the future debt stock, or to ask what is the probability of hitting the debt limit within  $K$  periods. While these questions are interesting and the answers thought provoking it is not clear that there is sufficient empirical realism in the model to take these forecasts or probabilities seriously.

**Mora's Model of the Structural Balance** Mora's model begins by considering the law of motion for the debt-to-GDP ratio described by Mendoza and Oviedo (2004a), (2.13). Mora argues that cyclical fluctuations in output drive a lot of the variation in budget flows and interest rates, so that simple fiscal sustainability analysis based on current levels of these variables could be highly misleading. In other words, if one were to make projections of the debt-to-GDP ratio, under the assumption that the steady state values of  $r_t$ ,  $\gamma_t$  and  $\bar{x}_t$  were equal to their current values one would be easily misled.

Based on this argument, he suggests rewriting the government budget constraint to make explicit the distinction between cyclical (temporary) factors and structural (permanent) factors.<sup>30</sup> The first step involves decomposing output into its cyclical and trend components

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<sup>30</sup>The use of the terms permanent and temporary in reference to the Hodrick-Prescott filter's trend and cyclical components can be quite misleading. The Hodrick-Prescott filter's definition does not correspond to

using the Hodrick-Prescott filter.<sup>31</sup> The Hodrick-Prescott filter decomposes the logarithm of real GDP,  $\ln y_t$ , into a trend component,  $\ln y_t^*$ , and a mean-zero cyclical component,  $\ln y_t^c$ . So we have  $\ln y_t = \ln y_t^* + \ln y_t^c$ , implying that  $y_t = y_t^* y_t^c$ . The output gap is defined as  $GAP_t = (y_t - y_t^*)/y_t^* = y_t^c - 1$ .

The second step involves estimating elasticities of revenue and expenditure with respect to the cycle. For this purpose Mora suggests estimating the following equations:

$$g_t/y_t^* = c_g + \varepsilon_g GAP_t + \mu_t \quad (3.8)$$

$$t_t/y_t^* = c_\tau + \varepsilon_\tau GAP_t + v_t \quad (3.9)$$

where  $g_t$  and  $t_t$  are real revenue and real primary expenditure,  $c_g$  and  $c_\tau$  are constants,  $\varepsilon_g$  and  $\varepsilon_\tau$  represent elasticities with respect to the cyclical component of output and  $\mu_t$  and  $v_t$  are error terms.<sup>32</sup> Given the estimated elasticities, the structural components of primary expenditure and revenue are defined as

$$\bar{g}_t^* = \bar{g}_t - \hat{\varepsilon}_g GAP_t \quad (3.10)$$

$$\bar{t}_t^* = \bar{t}_t - \hat{\varepsilon}_\tau GAP_t. \quad (3.11)$$

The structural primary balance is defined as  $\bar{x}_t^* = \bar{t}_t^* - \bar{g}_t^* = \bar{t}_t - \bar{g}_t + (\hat{\varepsilon}_g - \hat{\varepsilon}_\tau)GAP_t$ .<sup>33</sup>

In the third step the real interest rate is broken into two components using the HP filter:  $r_t = r_t^* + r_t^c$ . We can then rewrite (2.13) as

$$\bar{b}_t = \frac{1 + r_t^*}{1 + \gamma_t} \bar{b}_{t-1} - (\bar{t}_t^* - \bar{g}_t^*) + \frac{r_t^c}{1 + \gamma_t} \bar{b}_{t-1} - (\bar{t}_t^c - \bar{g}_t^c). \quad (3.12)$$

Mora thinks of this as a decomposition of the debt stock into its permanent and temporary components,  $\bar{b}_t^*$  and  $\bar{b}_t^c$ , where

$$\bar{b}_t^* = \frac{1 + r_t^*}{1 + \gamma_t} \bar{b}_{t-1} - (\bar{t}_t^* - \bar{g}_t^*) \quad (3.13)$$

$$\bar{b}_t^c = \frac{r_t^c}{1 + \gamma_t} \bar{b}_{t-1} - (\bar{t}_t^c - \bar{g}_t^c) \quad (3.14)$$

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other standard definitions of the permanent and temporary components of a time series.

<sup>31</sup>Mora uses a smoothing parameter of  $\lambda = 100$  in his analysis of annual data. Burnside (2000) and Ravn and Uhlig (2001) suggest a value between 6 and 7 for annual data. The larger  $\lambda$  is the smoother the HP trend will be, so the latter papers would suggest that Mora's trend will be smooth relative to common practice.

<sup>32</sup>It is not clear from Mora's discussion whether real expenditures and revenue are expressed as a fraction of trend real GDP, or whether nominal expenditures and revenue are expressed as a fraction of trend nominal GDP.

<sup>33</sup>It should be pointed out that it is nonstandard to adjust all expenditure components for the effects of the business cycle when computing the "structural" primary balance. Standard practice usually only adjusts spending on transfer programs, which are likely to be countercyclical. For Mora's purposes, his approach is sensible, since he wishes to decompose budget flows into trend and cyclical components more for statistical rather than interpretational purposes.

When Mora estimates the model he finds  $\varepsilon_\tau$  statistically insignificantly different from 0 for both countries. This result seems surprising in light of Talvi and Végh's (2000) evidence for both countries, who report the correlation with output to be over 0.4. He estimates  $\varepsilon_g$  to be  $-0.21$  for Colombia and  $0.32$  for Costa Rica. These results are also surprising in light of Talvi and Végh's results, which suggest the correlation of government consumption with output is  $0.31$  in Colombia and  $0.80$  in Costa Rica. A possible explanation for this is that the trend growth rates of real revenue and real expenditure in the two countries do not correspond to the trend growth rate of real GDP. If this is the case then the variables  $g_t/y_t^*$  and  $t_t/y_t^*$  will have significant trends in them, while, by construction  $GAP_t$  will not. This will bias the coefficients  $\varepsilon_g$  and  $\varepsilon_\tau$  towards 0. Another possibility is that Mora's results are biased by his use of forward looking output forecasts (through 2010) to compute the decomposition of output into its trend and cyclical components.

Mora argues in favor of the following interpretation of his model: that debt should be regarded as unsustainable if  $\bar{b}_t^*$  exceeds its value in a long-run steady state, i.e. if

$$\bar{b}_t^* > \bar{b}^* = \frac{1 + \gamma^*}{r^* - \gamma^*}(\bar{t}^* - \bar{g}^*). \quad (3.15)$$

However, it is unclear that such an interpretation makes sense, for the simple reason that the structural component of debt, as defined by (3.13) has the potential to be highly variable, and to violate (3.15) even when fiscal policy is designed to satisfy the lifetime budget constraint. The easiest way to understand this argument is to consider the case where there are no business cycles, the growth rate,  $\gamma_t$ , is a constant  $\gamma$ , that also equals the constant  $\gamma_t^*$ , and the real interest rate,  $r_t$ , is a constant  $r$ , that also equals the constant  $r_t^*$ . Notice that, in this case,  $\bar{t}_t^* = \bar{t}_t$  and  $\bar{g}_t^* = \bar{g}_t$  as long as the practitioner has enough data to ascertain that the “true” values of  $\varepsilon_g$  and  $\varepsilon_\tau$  are 0. Using these facts we would find that

$$\bar{b}_t^* = \bar{b}_t = \frac{1 + r}{1 + \gamma} \bar{b}_{t-1} - (\bar{t}_t - \bar{g}_t). \quad (3.16)$$

If Mora's criterion were used, debt would be deemed to be unsustainable whenever it rose above the long-run steady state level. Clearly the criterion is too strict.

**Estimating Default Probabilities** Mora extends his analysis by arguing that, in reality, default is possible, since sovereign borrowers cannot be compelled to repay debt in all states of the world, though they may find it optimal to repay in most circumstances. He imagines a world population by risk-neutral investors who lend to the government as long as their

expected return corresponds to the risk free rate. If default is complete, in the sense that recovery rates are zero upon default, then the borrowing rate should obey

$$(1 - \theta_t)(1 + r_t) = 1 + r_t^f, \quad (3.17)$$

where  $r_t^f$  is the risk free rate in world markets,  $r_t$  is the rate at which the government borrows and  $\theta_t$  is the probability of default.

Mora also argues that it is natural to think of the default probability as

$$\theta_t = \Pr(\bar{b}_t^* > \bar{b}^*) \quad (3.18)$$

where  $\bar{b}^*$  is now given by

$$\bar{b}^* = \frac{1 + \gamma^*}{r^f - \gamma^*}(\bar{t}^* - \bar{g}^*), \quad (3.19)$$

where  $r^f$  is the steady state risk free rate. The argument he gives in favor of using the risk free rate to define  $\bar{b}^*$  is that as long as the debt stock stays below  $\bar{b}^*$  the government ought to be able to borrow at the risk free rate, and it will also not need to default.

Mora then uses a recursive approach to estimating  $\theta_t$ . Given a particular  $\theta_t$  to obtain nominal interest rates using (3.17), he then uses the model of the previous section to simulate debt dynamics for  $\bar{b}_t^*$ . These debt dynamics, themselves, produce estimates of  $\theta_t$  from (3.18). If the starting estimate is within a tolerance of the final estimate the procedure stops. Otherwise it is run recursively by taking the final estimate as the next starting value. With this procedure, Mora finds very high default probabilities for both countries as we move into the latter part of this decade. However, as I argued in the previous section, Mora's insolvency criterion is probably too strict. And, the assumption of zero recovery is, also, probably too extreme.

A further conceptual problem with Mora's approach is that the probability of a government exceeding its natural debt limit is probably not the same as its probability of defaulting. Consider, again, Mendoza and Oviedo's model, in which debt evolves according to (3.7). The debt limit that arises in that model only has meaningful content if we take seriously the notion that the government will avoid default in all states of the world, or that investors are simply unwilling to lend unless default is ruled out. Once we allow for default the whole meaning of the debt limit breaks down.

But imagine that somehow, out of equilibrium, the government exceeds its debt limit in Mendoza and Oviedo's model. Will it default? Chances are it will not, at least not on

the basis of ability to pay. Recall that Mendoza and Oviedo’s debt limit is the amount of debt the government can repay even if a string of exceptionally bad revenue outcomes happens in perpetuity. Since such an event is highly unlikely, so is default if the government only marginally exceeds Mendoza and Oviedo’s debt limit. This highlights the distinction between *inevitable* default and an inability to commit to repay, which indicates a (possibly very small) probability of default.

The interpretation of Mora’s debt limit is different. As I argued above, in the case where there are no business cycles, his debt limit would be exceeded 50 percent of the time if the distribution of the debt stock were stationary and symmetric around its steady state level. It is clear that interpreting this as a default probability is inappropriate.

### 3.3. Mendoza and Oviedo’s Case Studies

Mendoza and Oviedo (M&O, 2004b) conduct four case studies of countries in Latin America: Brazil, Colombia, Costa Rica and Mexico. They apply their first model to data on the four economies and draw the following conclusions: Brazil and Colombia are near their natural debt limits, Mexico was near its limit in 1998, and Costa Rica is well inside its natural debt limit.

Recall that the natural debt limit in M&O’s model is  $\bar{b}^* = (1 + \gamma)(\underline{t} - \underline{g})/(r - \gamma)$ , where  $\gamma$  is the real growth rate,  $r$  is the real interest rate,  $\underline{t}$  is the lowest possible realization of the ratio of revenue to GDP and  $\underline{g}$  is the lowest level of primary expenditure that is politically feasible. To determine natural debt limits for each country, M&O set  $\gamma$  equal to the sample average of the growth rate of real GDP in the period 1961–2000, in each of their benchmark scenarios. They set  $r = 0.05$  for all countries. They calibrate  $\underline{t}$  and  $\underline{g}$  to fiscal data over the period 1990–2002. They set  $\underline{t}$  equal to the sample average of  $\bar{t}_t$  minus two standard deviations. They set  $\underline{g}$  so that  $\bar{b}^*$  corresponds to the maximal level of  $\bar{b}_t$  observed within the sample. The findings are summarized in Table 2.

In their benchmark exercise M&O do not use their model to calculate the natural debt limit. Rather they interpret the highest observed debt level as the debt limit and infer from this what the lowest politically acceptable level of primary government expenditure is. Thus, the model is not really used to answer the question: How much money can the government borrow? Rather, it is used to answer the question: How can we explain the apparent differing abilities of governments to borrow? The answer M&O offer is that this depends on the willingness of the different governments to cut expenditure in a crisis.

TABLE 2

MENDOZA AND OVIEDO'S BENCHMARK EXERCISE

	Brazil	Colombia	Costa Rica	Mexico
$r$	0.05	0.05	0.05	0.05
$\gamma$	0.0255	0.0186	0.0183	0.022
$\underline{t}$	0.138	0.104	0.181	0.193
$\bar{b}^*$	0.561	0.505	0.533	0.549
Implied value of $\underline{g}$	0.125	0.088	0.164	0.177

Source: Mendoza and Oviedo (2004b).

Since, in M&O's sample the 4 governments have apparently similar abilities to borrow, the model really answers the question: Why can they borrow up to about the same amount despite having different distributions of tax revenue? Abstracting from differences in the real growth rates (which are also important) the answer is that the governments with more volatile revenue, as measured by  $\underline{t}$ , which are Brazil and Colombia, also have the lowest politically feasible levels of  $\bar{g}_t$ . Brazil is also helped by the fact that its growth is the fastest in the group.

M&O then hold  $\underline{t}$  and  $\underline{g}$  fixed at the values indicated in Table 2 and explore the sensitivity of the natural debt limit,  $\bar{b}^*$ , to changes in either  $r$  or  $\gamma$ . These changes are motivated, in the case of  $r$ , as a general increase in the cost of international borrowing and, in the case of  $\gamma$ , a growth slowdown. Not surprisingly, as their analysis concludes, the debt limits in these alternative scenarios are much lower.

Finally, M&O compare their results to what a practitioner of standard fiscal sustainability analysis might have concluded about each country. To do this, they imagine that this practitioner might compute the sustainable debt level as  $\bar{b} = (1 + \gamma)(\bar{t} - \bar{g})/(r - \gamma)$ , where  $\bar{t}$  and  $\bar{g}$  are the average values within their sample. As Table 3 indicates, standard analysis concludes that Brazil could only sustain a very small debt level, because its primary balance has averaged close to zero, while Colombia would actually have to hold a net asset position because its primary balance has been negative on average. The reason that M&O's method concludes that much higher levels of debt are sustainable in these two countries is that it presumes a much higher primary balance is *feasible*, in the state of the world where revenues are low. One might question whether this is reasonable. Take, for example, the Brazilian case. M&O imagine a scenario in which, on average,  $\bar{t}_t$  and  $\bar{g}_t$  are around 19 percent of GDP.

TABLE 3

STANDARD FISCAL SUSTAINABILITY ANALYSIS VERSUS  
MENDOZA AND OVIEDO'S APPROACH

	Brazil	Colombia	Costa Rica	Mexico
$r$	0.05	0.05	0.05	0.05
$\gamma$	0.0255	0.0186	0.0183	0.022
$\bar{t}$	0.193	0.126	0.203	0.230
$\bar{g}$	0.192	0.128	0.185	0.193
Implied value of $\bar{b}$	0.035	-0.053	0.548	1.315

Source: Mendoza and Oviedo (2004b).

But, if a sequence of bad shocks is realized, and debt rises to its limit, the government will be able to slash primary spending by 6.5 percent of GDP, to just 12.5 percent of GDP.

In the case of Mexico, standard fiscal sustainability analysis would conclude that much more debt was sustainable because, on average, the primary balance has been quite large. M&O argue that the true debt limit is much lower because the Mexican government could not credibly cut primary expenditure more than 1.6 percent of GDP in a crisis, to 17.7 percent of GDP. While this interpretation is necessary to rationalize the model, is it reasonable? Is the political economy of Mexico so different from that of Brazil? This remains as an open question.

#### 4. Conclusions

In this paper we have reviewed recent developments in the literature on fiscal sustainability analysis. We surveyed four new methodological approaches to fiscal sustainability analysis that attempt to model uncertainty: Barnhill and Kopits (2003), IMF (2002, 2003), Xu and Ghezzi (2003) and Mendoza and Oviedo (2004a). In summary I would argue that each of these papers provides important insights into fiscal sustainability analysis, but only IMF (2002) provides a thorough enough discussion to be of general use to practitioners.

Barnhill and Kopits' (2003) VaR approach would probably best be applied in the assessment of specific contingent assets or liabilities. But their paper provides little guidance about how their approach should be used in generic fiscal sustainability analysis, where the focus is on the government's overall primary balance. It also is thin in terms of providing a

mapping from assessments of VaR to policy conclusions. This comes out in da Costa, Silva and Baghdassarian's (2004) case study of Brazil. Apart from providing an overall measure of net worth, the authors do not draw many conclusions from their use of the VaR approach.

Xu and Ghezzi's (2003) approach might be useful to investment banks in assessing default probabilities, but it is not very closely related to fiscal sustainability. They argue that when a country's reserves run out it defaults. They provide a statistical model of the reserve level in order to estimate the default probability. Apart from a number of technical problems with their model, there are important conceptual issues. Most importantly, their method does not allow for any fiscal policy adjustment by a country with a falling reserve level or a rising debt level. Apart from this bringing the applicability of their model into doubt as a forecasting tool, it also renders it devoid of policy content. This makes it less than useful to practitioners attempting to engage a government on policy issues. Again, this comes out in da Costa, Silva and Baghdassarian's (2004) case study of Brazil. They provide estimates of default probabilities for Brazil, but they do not use their results to reach a set of policy conclusions.

Mendoza and Oviedo's (2004a) approach is most useful for its insight that a government's ability to borrow may depend more on its willingness (ability) to cut expenditure, relative to revenue, during bad times, than it does on the government's average primary balance. Nonetheless, their paper oversimplifies the procyclical fiscal policy problem by putting all its emphasis on revenue volatility. It also sends somewhat misleading messages about dollarization. As Mora's (2004) and Mendoza and Oviedo's (2004b) case studies reveal, it is less than clear that we should take the quantitative predictions of these models very seriously.

Despite its shortcomings, many of them acknowledged in IMF (2002, 2003), and discussed in this paper, the IMF's framework for fiscal sustainability analysis is quite useful to the practicing economist. It gives some idea of the degree of uncertainty associated with future debt paths, and is easily adapted to policy discussions with government officials. Nonetheless, it has serious shortcomings as a forecasting tool, has difficulty to interpret confidence bands, and does not put enough emphasis on the role of the real exchange rate in fiscal dynamics.

The literature of fiscal sustainability analysis will continue to evolve. In the meantime, practitioners would do well to raise their level of analysis at least to the level of IMF (2002, 2003), bearing in mind the insights in Mendoza and Oviedo (2004a), as well as those to be found in other parts of the theoretical literature, for example, Burnside, Eichenbaum and

Rebelo (2003). The literature has not advanced sufficiently at this stage to argue that there is a good approach to estimating default probabilities from a fully specified model of the budget constraint. Nor has the literature found an adequate way of mapping backward from bond spreads to default probabilities. Nonetheless, it is not obvious that these advances stand in the way of high quality fiscal analysis that can be used in a sensible policy dialogue with country authorities.

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