Behavioral Seminar in Accounting

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November 10, 2011

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1 Preface

I made these notes to aid my preparation for the behavioral comprehensive exam at Duke University.

2 Kahneman and Tversky [1979]

2.1 Background

Let $A = \{a_i\}_{i=1}^n$ be the set of outcomes. Then $\mathcal{G}_\mathcal{S}$, the set of simple gambles (on A), is given by

$$\mathcal{G}_\mathcal{S} \equiv \left\{ (p_i \circ a_i)_{i=1}^n \mid p_i \geq 0, \sum_{i=1}^n p_i = 1 \right\}$$

Recall the **axioms of choice under uncertainty** [Jehle and Reny, 2001]:

1. Completeness—For any two gambles, g, $g' \in \mathcal{G}$, either $g \succsim g'$ or $g' \succsim g$.

2. Transitivity—For any three gambles, g, g', $g'' \in \mathcal{G}$, if $g \succsim g'$ and $g' \succsim g''$, then $g \succsim g''$.

   Note: Axioms 1 and 2 imply that the finitely many elements of A are ordered by $\succsim$.

3. Continuity—For any $g \in \mathcal{G}$, there is some probability, $\alpha \in [0,1]$, such that $g \sim (\alpha \circ a_1, (1-\alpha) \circ a_n)$.

4. Monotonicity—For all $\alpha, \beta \in [0,1]$,

   $$(\alpha \circ a_1, (1-\alpha) \circ a_n) \succsim (\beta \circ a_1, (1-\beta) \circ a_n)$$

   iff $\alpha \geq \beta$.

   Note: Axiom 4 rules out the case in which the decision maker is indifferent among all the outcomes in A.

5. Substitution—If $g = (p_1 \circ g^1, \ldots, p_k \circ g^k)$, and $h = (p_1 \circ h^1, \ldots, p_k \circ h^k)$ are in $\mathcal{G}$, and if $h^i \sim g^i \; \forall \; i$, then $h \sim g$.

   Note: Axioms 1 and 5 imply that when the agent is indifferent between two gambles, he must be indifferent between all convex combinations of them.

6. Reduction to Simple Gambles—For any $g \in \mathcal{G}$, if $(p_1 \circ a_1, \ldots, p_n \circ a_n)$ is the simple gamble induced by g, then $(p_1 \circ a_1, \ldots, p_n \circ a_n) \sim g$.

7. Independence—Axioms 5 and 6 imply that if $(p_1 \circ a_1, \ldots, p_n \circ a_n) \sim (q_1 \circ a_1, \ldots, q_n \circ a_n)$, then $\forall \; \alpha \in [0, 1]$, and every simple gamble $(r_1 \circ a_1, \ldots, r_n \circ a_n)$,

$$((\alpha p_1 + (1-\alpha)r_1) \circ a_1, \ldots, (\alpha p_n + (1-\alpha)r_n) \circ a_n),$$
$$\sim$$
$$((\alpha q_1 + (1-\alpha)r_1) \circ a_1, \ldots, (\alpha q_n + (1-\alpha)r_n) \circ a_n).$$

   That is, when we combine each of two gambles with a third in the same way, the individual's ranking of the two new gambles is *independent* of the third gamble.

2.2 Critique

The application of expected utility theory to choices between prospects, or gambles, is based on the following:

1. Expectation—$U(\{x_i, p_i\}_{i=1}^n) = \sum_{i=1}^n p_i u(x_i)$.

2. Asset Integration—$(\{x_i, p_i\}_{i=1}^n)$ is acceptable at asset position w iff $U(\{w + x_i, p_i\}_{i=1}^n) > u(w)$

3. Risk Aversion—u is concave ($u'' < 0$).

2.2.1 Limitations

The reliance on hypothetical choices raises obvious questions about the validity of the method and the generalizability of the results. Lab experiments have been designed to obtain precise measures of utility and probability from actual choices, but they typically involve artificial gambles for small stakes and a large repetition of very similar problems.

The use of the method relies on the assumption that people often know how they would behave in actual situations of choice, and on the further assumption that the subjects have no special reason to disguise their true preferences. If people are reasonably accurate in predicting their choices, the presence of common and systematic violations of EU theory in hypothetical problems provides presumptive evidence against that theory.

2.2.2 Certainty, Probability, and Possibility

People overweight outcomes that are considered certain, relative to outcomes which are "merely" probable—the *certainty effect*.

Consider a variation of the *Allais paradox*:

Problem 1: Choose between

A: 2,500 with probability .33,	B: 2,400 with probability 1
2,400 with probability .66,	
0 with probability .01;	

Problem 2: Choose between

C: 2,500 with probability .33,	D: 2,400 with probability .34,
0 with probability .67;	0 with probability .66

Most subjects chose B and C, which is a violation of EU theory. With $u(0) = 0$, $B \succ A$ implies

$$u(2,400) > .33u(2,500) + .66u(2,400)$$
$$.34u(2,400) > .33u(2,500)$$

while $C \succ D$ implies the reverse inequality!

$$.33u(2,500) + .67u(0) > .34u(2,400) + .66u(0)$$
$$.33u(2,500) > .34u(2,400)$$

The results suggest the following empirical generalization concerning the matter in which the independence axiom is violated: if $(y, pq) \sim (x, p)$, then $(y, pqr) \succ (x, pr)$ for $0 < p, q, r < 1$.

2.2.3 The Reflection Effect

What if gains are replaced by losses? Reflection of prospects around 0 reverses the preference order—the *reflection effect*.

1. The reflection effect implies that risk aversion in the positive domain is accompanied by risk seeking in the negative domain.

2. The preferences between the negative prospects also violate the expectation principle.

3. The reflection effect eliminates aversion for uncertainty or variability as an explanation of the certainty effect.

Certainty appears to increase the aversiveness of losses as well as the desirability of gains.

2.2.4 Probabilistic Insurance

In *probabilistic insurance*, you pay half of the regular premium. When shit hits the fan, there is a 50% chance that you pay the other half of the premium and the insurance company covers all the losses, and a 50% chance that you get back your insurance payment and suffer all the losses. Lab tests showed that reducing the probability of a loss from p to $p/2$ is less valuable than reducing the probability of that loss from $p/2$ to 0.

Yet, EU theory with a concave u implies that probabilistic insurance is superior to regular insurance. That is, if at asset position w, one is just willing to pay a premium y to insure against a probability p of losing x, then one should definitely be willing to pay a smaller premium ry, to reduce the probability of losing x from p to $(1-r)p, 0 < r < 1$.

Formally, if $(w-x, p; w, 1-p) \sim (w-y)$, then $(w-x, (1-r)p; w-y, rp; w-ry, 1-p) \succ (w-y)$. We must show that

$$pu(w-x) + (1-p)u(w) = u(w-y)$$
$$\Rightarrow$$
$$(1-r)p \cdot u(w-x) + rp \cdot u(w-y) + (1-p)u(w-ry) > u(w-y)$$

Proof. WLOG, we set $u(w-x) = 0$ and $u(w) = 1$. Hence,

$$pu(w-x) + (1-p)u(w) = u(w-y)$$
$$p \cdot 0 + (1-p) \cdot 1 = u(w-y)$$
$$1-p = u(w-y)$$

We wish to show that

$$(1-r)p \cdot 0 + rp \cdot (1-p) + (1-p)u(w-ry) > 1-p$$
$$rp + u(w-ry) > 1$$
$$u(w-ry) > 1-rp$$

which holds iff $u'' < 0$. Consider a graph:

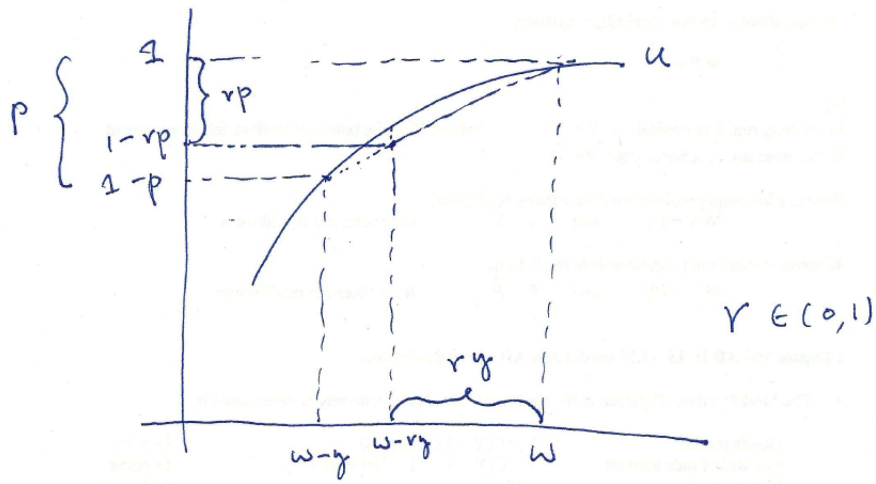

Figure 1: A graphical argument.

□

2.2.5 The Isolation Effect

When different decompositions of a pair of prospects lead to different preferences, we have the *isolation effect*. The reversal of preferences due to the dependency among events is particularly significant because it violates the basic supposition of a decision-theoretical analysis, namely that choices between prospects are determined solely by the probabilities of final states.

2.3 Theory

Prospect theory distinguishes between two phases in the choice process:

1. editing, where a preliminary analysis of the offered prospects often yields a simpler representation of these prospects; and

2. evaluating, where the prospect of highest value is chosen.

The function of the editing phase is to organize and reformulate the options so as to simplify subsequent evaluation and choice:

1. Gains and losses are defined relative to some neutral reference point, which usually corresponds to the current asset position. However, the location of the reference point, and the consequent *coding* of outcomes as gains or losses, can be affected by the formulation of the offered prospects, and by the expectations of the decision maker.

2. Prospects can sometimes be simplified by *combining* the probabilities associated with identitical outcomes.

3. Some prospects contain a riskless component that is *segregated* from the risky component in the editing phase.

4. The essence of the isolation effect is the discarding, or *cancellation*, of components that are shared by the offered prospects.

Following the editing phase, the decision maker is assumed to evaluate each of the edited prospects, and to choose the prospect of highest value. The overall value, denoted V, is expressed in terms of two scales, π and v:

1. π associates with each probability p a decision weight $\pi(p)$, which reflects the impact of p on the overall value of the prospect. But π is not a probability measure!

2. v assigns to each outcome x a number, $v(x)$, which reflects the subjective value of that outcome. It measures the value of deviations from the reference point.

The basic equation of the theory describes the manner in which π and v are combined to determine the overall value of regular prospects (i.e. neither strictly positive nor strictly negative). If $(x, p; y, q)$ is a regular prospect, then

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y) \tag{1}$$

where $v(0) = 0, \pi(0) = 0$, and $\pi(1) = 1$. As in utility theory, V is defined on prospects, while v is defined on outcomes.

The evaluation of strictly positive and strictly negative prospects follows a different rule. In teh editing phase, such prospects are segregated intwo two components:

(i) the riskless component, and

(ii) the risky component.

If $p + q = 1$ and either $x > y > 0$ or $x < y < 0$, then

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)] \tag{2}$$

That is, the value of a strictly positive or strictly negative prospect equals the value of the riskless component plus the value-difference between the outcomes, multiplied by the weight associated with the more extreme outcome. The essential feature of Eq. (4) is that a decision weight is applied to the value-difference $v(x) - v(y)$, which represents the risky component of the prospect, but not to $v(y)$, which represents the riskless prospect.

Note that

$$\begin{aligned} v(y) + \pi(p)[v(x) - v(y)] &= v(y) + \pi(p)v(x) - \pi(p)v(y) \\ &= \pi(p)v(x) + [1 - \pi(p)]v(y) \\ &= \pi(p)v(x) + \pi(q)v(y) \end{aligned}$$

If $\pi(p) + \pi(q) = 1$, then we have Eq. (3).

The equations of prospect theory retain the general bilinear form that underlines EU theory. However, Kahneman and Tversky [1979] are compelled to assume that values are attached to changes rather than to final states, and that decision weights do not coincide with stated probabilities. These departures from EU theory must lead to normatively unacceptable consequences, including

1. inconsistencies,

2. intransitivities, and

3. violations of dominance.

When the decision maker does not have the opportunity to discover that his preferences could violate decision rules that he wishes to obey, anomalies implied by prospect theory are expected to occur.

2.3.1 The Value Function

An essential feature of prospect theory is that the carriers of value are changes in wealth or welfare, rather than final states. The emphasis on changes as the carriers of value should not be taken to imply that the value of a particular change is independent of initial position. Value should be treated as a function in two arguments: the asset position that serves as reference point, and the magnitude of the change from that reference point.

Kahneman and Tversky [1979] hypothesize that the value function for changes of wealth is normally concave above the reference point ($v''(x) < 0 \; \forall \; x > 0$) and often convex below it ($v''(x) > 0 \; \forall \; x < 0$). That is, the marginal value of both gains and losses generally decreases with their magnitude.

A salient characteristic of attitudes to changes in welfare is that losses loom larger than gains. The aggravation that one experiences in losing a sum of money appears to be greater than the pleasure associated with gaining the same amount. Moreover, the aversiveness of symmetric fair bets generally increases with the size of the stake. That is, if $x > y \geq 0$, then $(y, .50; -y, .50) \succ (x, .50; -x, .50)$. According to Eq. (3),

$$v(y) + v(-y) > v(x) + v(-x)$$

$$\Rightarrow$$

$$v(-y) - v(-x) > v(x) - v(y)$$

Setting $y = 0$ yields $v(x) < -v(-x)$, and letting $y \to x$ yields $v'(x) < v'(-x)$, provided that v' exists.

Proof. Without loss of generality, let $x > y \geq 0$.

$$\lim_{y \to x} \frac{v(-y) - v(-x)}{x - y} > \lim_{y \to x} \frac{v(x) - v(y)}{x - y}$$

$$\lim_{y \to x} \frac{v'(-x)}{1} > \lim_{y \to x} \frac{v'(x)}{1}$$

$$v'(-x) > v'(x)$$

□

Thus, the value function for losses is steeper than the value function for gains!

To summarize, the value function is

(i) defined on deviations from the reference point,

(ii) generally concave for gains and commonly convex for losses, and

(iii) steeper for losses than for gains.

2.3.2 The Weighting Function

In prospect theory, the value of each outcome is multiplied by a decision weight. Decision weights are not probabilities; they do not obey the probability axioms and should not be interpreted as measures of degree or belief. Decision weights measure the impact of events on the desirability of prospects, and not merely the perceived likelihood of these events. The two scales coincide (i.e. $\pi(p) = p$) if the expectation principle holds, but not otherwise.

π is an increasing function of p, with $\pi(0) = 0$ and $\pi(1) = 1$. That is, outcomes contingent on an impossible event are ignored, and the scale is normalized so that $\pi(p)$ is the ratio of the weight associated with the probability p to the weight associated with the certain event.

For small values of p, π is a subadditive function of p (i.e. $\pi(rp) > r\pi(p)$ for $0 < r < 1$. Kahneman and Tversky [1979] propose that very low probabilities are generally overweighted, that is, $\pi(p) > p$ for small p.

It is important to distinguish overweighting, which refers to a property of decision weights, from the overestimation that is commonly found in the assessment of the probability of rare events. *Subcertainty* refers to the property that for all $0 < p < 1, \pi(p) + \pi(1 - p) < 1$.

The slope of π in the interval $(0, 1)$ can be viewed as a measure of the sensitivity of preferences to changes in probability. Subcertainty entails that π is regressive with respect to p, that is, preferences are generally less sensitive to variations of probability than the expectation principle would dictate. Thus, subcertainty captures that the sum of the weights associated with complementary events is typically less than the weight associated with the certain event.

Recall that violations of the independence axiom conform to the following rule: if $(y, pq) \sim$

(x,p), then $(y,pqr) \succ (x,pr)$ for $0 < p,q,r < 1$. By Eq. (3),

$$\pi(p)v(x) = \pi(pq)v(y)$$

$$\iff$$

$$\frac{v(x)}{v(y)} = \frac{\pi(pq)}{\pi(p)}$$

$$\Rightarrow$$

$$\pi(pr)v(x) \leq \pi(pqr)v(y)$$

$$\iff$$

$$\frac{v(x)}{v(y)} \leq \frac{\pi(pqr)}{\pi(pr)}$$

$$\Rightarrow$$

$$\frac{\pi(pq)}{\pi(p)} \leq \frac{\pi(pqr)}{\pi(pr)}$$

Thus, for a fixed ratio of probabilities, the ratio of the corresponding decision weights is closer to unity when the probabilities are low than when they are high, the property of *subproportionality*. It imposes considerable constraints on the shape of π: it holds iff $\log \pi$ is a convex function of $\log p$.

An obvious objection to the assumption that $\pi(p) \neq p$ involves comparisons between prospects of the form $(x,p;x,q)$ and $(x,p';x,q')$, where $p+q = p'+q' < 1$. Since any individual will surely be indifferent between the two prospects—they are the same gamble—it could be argued that $\pi(p)+\pi(q) = \pi(p')+\pi(q') \Rightarrow \pi$ is the identity function. But this argument is invalid in the present theory, which assumes that the probabilities of identical outcomes are combined in the editing of prospects.

A more serious objection to the nonlinearity of π involves potential violations of dominance. Suppose $x > y > 0, p > p'$, and $p+q = p'+q' < 1$; hence $(x,p;y,q) \succ (x,p';y,q')$. If preference obeys dominance, then

$$\pi(p)v(x) + \pi(q)v(y) > \pi(p')v(x) + \pi(q')v(y)$$

$$\Longleftrightarrow$$

$$\frac{\pi(p) - \pi(p')}{\pi(q') - \pi(q)} > \frac{v(y)}{v(x)}$$

Hence, as $y \to x, \pi(p) - \pi(p') \to \pi(q') - \pi(q)$. Since $p - p' = q' - q, \pi$ must be essentially linear, or else dominance must be violated.

Direct violations of dominance are prevented in the present theory by the assumption that dominated alternatives are detected and eliminated prior to the evaluation of prospects. The theory does permit indirect violations of dominance (e.g. $A \succ B \succ C \succ A$).

Finally, the present theory concerns the simplest decision task in which a person chooses between two available prospects.

3 Tversky and Kahneman [1992]

Any adequate descriptive theory of choice must meet a minimal challenge of five major phenomena of choice:

1. *Framing effects*—The rational theory of choice assumes description invariance, but a wealth of experimental evidence suggests that framing yields systematically different preferences.

2. *Nonlinear preferences*—Maurice Allais famously showed that the difference between probabilities of 0.99 and 1.0 has more impact on preferences than the difference between 0.10 and 0.11.

3. *Source dependence*—Daniel Ellsberg demonstrated that people's unwillingness to bet on an uncertain event depends on the source of uncertainty; people may be averse to ambiguity.

4. *Risk seeking*—You may recall from one of Hüseyin Yıldırım's lectures that explanations to economic phenomena become boring when we assume that people are risk loving. But risk-seeking choices are indeed consistently observed in two classes of decision problems:

   (i) People often prefer a small probability of winning a large prize over the expected value of that prospect.

   (ii) Risk seeking is prevalent when people must choose between a sure loss and a substantial probability of a larger loss.

5. *Loss aversion*—Losses loom larger than gains!

3.1 Cumulative prospect theory

In the classical expected utility (EU) theory, the utility of an uncertain gamble is the sum of the utilities of the outcomes, each weighted by its probability:

$$U(g) = \sum_i p_i u(a_i)$$

Kahneman and Tversky [1979] proposed two major modifications:

1. the carriers of value are gains and losses, not final assets or states of wealth; and

2. the value of each outcome is multiplied by a decision weight, not by an additive probability:

$$U(g) = \sum_i \pi_i v(a_i)$$

In Kahneman and Tversky [1979], π is a monotonic transformation of p. But two problems arise.

1. It does not always satisfy stochastic dominance.

2. It is not readily extended to gambles with large numbers of outcomes.

To solve these problems, instead of transforming each p separately, Tversky and Kahneman [1992] transform the entire cumulative distribution function separately to gains and to losses. Prospect theory is thus extended to uncertain as well as risky prospects with any number of outcomes while preserving most of Kahneman and Tversky [1979]'s essential features.

Let S be a finite set of states of nature; $A \subseteq S$ is an event. Exactly one state occurs, which is unknown to the decision maker. X denotes a set of consequences, or outcomes. X is assumed to have a neutral outcome, 0, and all other elements of X are gains or losses.

An uncertain prospect $f : S \to X$ assigns to each state $s \in S$ a consequence $f(s) = x \in X$. To define the cumulative functional, we arrange the outcomes of each prospect in increasing order. f is then represented as a sequence of pairs (x_i, A_i), which yields x_i if A_i occurs, where $x_i > x_j$ iff $i > j$, and (A_i) is a partition of S.

We assign to each f a number $V(f)$ such that f is preferred to or indifferent to g iff $V(f) \geq V(g)$. A *capacity* is a nonadditive set function that generalizes the standard notion of probability; it assigns to each $A \subset S$ a number $W(A)$ satisfying

$$W(\phi) = 0$$

$$W(S) = 1$$

$$W(A) \geq W(B)$$

whenever $A \supset B$.

Cumulative prospect theory asserts that there exists a strictly increasing value function $v : X \to \mathbb{R}$, satisfying $v(x_0) = v(0) = 0$, and capacities W^+ and W^- such that for $f = (x_i, A_i), -m \leq i \leq n$,

$$\begin{aligned} V(f) &= V(f^+) + V(f^-) \\ V(f^+) &= \sum_{i=0}^{n} \pi_i^+ v(x_i) \\ V(f^-) &= \sum_{i=-m}^{0} \pi_i^- v(x_i) \end{aligned} \tag{1}$$

where the decision weights $\pi^+(f^+) = \{(\pi_i^+)\}_{i=1}^n$ and $\pi^-(f^-) = \{(\pi_j^-)\}_{j=-m}^0$ are defined by

$$\begin{aligned} \pi_n^+ &= W^+(A_n) \\ \pi_{-m}^- &= W^-(A_{-m}) \\ \pi_i^+ &= W^+(\cup_{j=1}^n A_i) - W^+(\cup_{j=i+1}^n A_j) \\ \pi_i^- &= W^-(\cup_{j=-m}^i A_i) - W^-(\cup_{j=-m}^{i-1} A_j) \end{aligned}$$

Letting $\pi_i = \pi_i^+$ if $i \geq 0$ and $\pi_i = \pi_i^-$ if $i < 0$, (1) reduces to

$$V(f) = \sum_{i=-m}^{n} \pi_i v(x_i) \tag{2}$$

π_i^+ associated with a positive outcome is the difference between the capacities of the events

1. "the outcome is at least as good as x_i," and

2. "the outcome is strictly better than x_i."

π_i^- associated with a negative outcome is the difference between the capacities of the events

1. "the outcome is at least as bad as x_i," and

2. "the outcome is strictly worse than x_i."

Thus, π_i can be interpreted as the marginal contribution of the respective event, defined in terms of W^+ and W^-. If each W is additive, and hence a probability measure, then $\pi_i = p(A_i)$. It follows that for both positive and negative prospects,

$$\sum_{i=1}^{n} \pi_i = 1$$

For mixed prospects, the sum can be either smaller or greater than one.

If $f = (x_i, A_i)$ is given by a probability distribution $p(A_i \equiv p_i$, it can be viewed as a probabilistic or risky prospect (x_i, p_i). Decision weights are then defined by

$$\begin{aligned}
\pi_n^+ &= w^+(p_n) \\
\pi_{-m}^- &= w^-(p_{-m}) \\
\pi_i^+ &= w^+(\sum_{j=1}^{n} p_j) - w^+(\sum_{j=i+1}^{n} p_j) \\
\pi_i^- &= w^-(\sum_{j=-m}^{i} p_j) - w^-(\sum_{j=-m}^{i-1} p_j)
\end{aligned}$$

w^+ and w^- satisfy $w' > 0, w : [0,1] \to [0,1], w^+(0) = w^-(0) = 0$, and $w^+(1) = w^-(1) = 1$.

4 Narrow Framing and Myopic Loss Aversion [Thaler, 1999]

Consider the following gamble / lottery / prospect: heads, you get \$200; tails, you pay \$100 [Samuelson, 1963]. Samuelson's Colleague (SC) famously replied, "I won't bet because I would feel the \$100 loss more than the \$200 gain. But I'll take you on if you promise to let me make 100 such bets."[1] Is it rational to decline if the gamble is one-shot but accept if it's repeated 100 times?

4.1 The Original [Samuelson, 1963]

Samuelson said it wasn't: "if you would *always* refuse to take favorable odds on a single toss, you must rationally refuse to participate in any (finite) sequence of such tosses."

Proof. If you will not accept one toss, you cannot accept two, since the latter consists of the (presumably unwise) decision to accept one *plus* the open decision to accept a second. Even if you were forced to accept one toss, you would cut your further (utility) losses and refuse the second. By mathematical induction, it follows that we rule out any sequence at all. □

Samuelson warned, however, that his theorem "does not say one must always refuse a sequence if one refuses a single venture: if, at higher income levels the single tosses become acceptable, and at lower levels the penalty of losses does not become infinite, there might well be a long sequence that is optional."

4.2 Two Decades Later [Tversky and Bar-Hillel, 1983]

Consider a set of gambles played by tossing n coins and having the player receive \$200 for each heads and pay \$100 for each tails. Let X_k be the gamble on the kth coin, $1 \leq k \leq n$, and let $S_k = \sum_{i=1}^{k} X_i$ denote the multiple gamble that consists of tossing coins 1 through k—in particular, $S_1 = X_1$ and $S_{k+1} = S_k + X_{k+1}$.

SC rejects a single toss, therefore

$$w \succ w + X_1 \tag{3}$$

[1] SC thus foreshadowed Kahneman and Tversky [1979]'s proclamation that "losses loom larger than gains."

Furthermore, the single toss is unacceptable at any level of wealth that can be achieved by 100 tosses, hence we stipulate, with Samuelson,

$$w + y \succ w + y + X_1 \tag{4}$$

for $-10,000 \leq y \leq 20,000$. We also assume that if X and Y are independent, and if for every outcome $y \in Y$,

$$w + y \succ w + y + X$$

then

$$w + Y \succ w + Y + X \tag{5}$$

That is, if X is unacceptable at any level of wealth $w + y$ that might result from playing Y, then X is also unacceptable at $w + Y$ where one does not know for sure which of these levels of wealth one will obtain.

Assumption (5) is weaker than expected utility theory. In conjunction with Assumption (4), however, it implies that for every $1 \leq k \leq 100$, $S_k \succ S_{k+1}$.

Proof. For any $s \in S_k$,

$$w + s \succ w + s + X_{k+1}$$

by Assumption (4) and X_1 and X_{k+1} being identically distributed. Hence, by Assumption (5),

$$\begin{aligned} w + S_k &\succ w + (S_k + X_{k+1}) \\ &= w + S_{k+1} \end{aligned}$$

and by transitivity,

$$w \succ w + S_1 \succ \cdots \succ w + S_{100}$$

Given Assumptions (4) and (5), therefore, the rejection of S_1 implies the rejection of S_{100}.

$\square$

Thus, Tversky and Bar-Hillel rigorously bolster Samuelson's argument.

4.3 Mental Accounting [Thaler, 1999]

Suppose SC's preferences are a piecewise linear version of the prospect theory value function with a loss aversion factor of 2.5:

$$u(x) = \begin{cases} x & \text{for } x \geq 0 \\ 2.5x & \text{for } x < 0 \end{cases}$$

A single gamble is unattractive:

$$\begin{aligned} \frac{1}{2}u(200) + \frac{1}{2}u(-250) &= 100 - 125 \\ &= -25 \\ &< 0 \\ &= u(0) \end{aligned}$$

But what about two plays? If each play is treated as a separate event, then two plays are twice as bad as one play. If they are combined, however, then the two-bet gamble yields positive expected utility and becomes even more attractive as the number of plays increases:

$$\begin{aligned} \frac{1}{4}u(400) + \frac{1}{2}u(100) + \frac{1}{4}u(-200) &= 100 + 50 - 125 \\ &= 25 \\ &> 0 \\ &= u(0) \end{aligned}$$

Loss-averse people are more willing to take risks if they combine many bets together than if they consider them one at a time. So why was SC unwilling to play the gamble once? We need a combination of loss aversion **and** one-bet-at-a-time mental accounting. Evaluating projects one at a time, rather than as part of an overall portfolio, can lead to an extreme unwillingness to take risks.

5 Between versus Within-subjects Designs

Any categorical explanatory variable (i.e. exogenous) for which

1. each subject experiences all of the levels is a **within-subjects factor**.

2. each subject experiences only one of the levels is a **between-subjects factor**.

Any experiment that has at least one within-subjects factor uses a **within-subjects design**, while an experiment that uses only between-subjects factors is a **between-subjects design** [Seltman, 2011].

As suggested by Kahneman and Tversky, the between-subjects design provides a clean test of the subject's natural reasoning process. The within-subjects design draws attention to the independent variable of interest and thus gives the subject a chance to detect and correct errors and inconsistencies in their responses; it thus tells how subjects address any conflict between what they do and what they know [Tan et al., 2002].

6 Heuristics versus Biases

A *heuristic* is a mental shortcut [Bonner, 2008]. A *bias* is a departure from a normative rational theory [Gilovich and Griffin, 2002]. Heuristics can lead to biases, but only if the decision prescribed by the heuristic departs from a normative rational theory.

In the *representativeness heuristic*, probabilities are evaluated by the degree to which A is representative of B; that is, by the degree to which A resembles B. It can lead to biases because similarity, or representativeness, is not influenced by several factors that should affect judgments of probability. For example, when Tversky and Kahneman showed subjects brief personality sketches and prior outcome probabilities and asked them to estimate the probability that an individual was in a certain career, subjects evaluated based on the degree to which the description was representative of the career's stereotype and ignored prior probabilities.

Subjects used prior probabilities correctly when they had no personality sketches. But prior probabilities were again ignored if the personality sketch was modified to be totally uninformative! [Tversky and Kahneman, 1974]

7 Limitations of Archival Research

"Archival research cannot resolve the disputes over the purported problems or benefits associated with the current piecemeal or potential full-fair-value-income measurement because archival designs cannot vary how banks measure and report income to test whether (or how) capital-markets participants would use full-fair-value income to assess risk and share values" [Hirst et al., 2004].

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