

Problem Set 4

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Economics 302

1 Problem 1

1.1 A

For $n = 1$:

$$\begin{aligned}K_0 &= J + 2 \\ \text{Player 1 chooses } j_0 &\in \{1, \dots, J\} \\ K_1 &= J - j_0 + 2 \in \{2, \dots, J + 1\} \\ \text{Player 2 can play } j_1 &= (K_1 - 1) \in \{1, \dots, J\} \\ K_3 &= 1 \quad \therefore \text{Player 1 loses}\end{aligned}$$

Assume the induction hypothesis (property is true for n). For $n + 1$:

$$\begin{aligned}K_0 &= (n + 1)(J + 1) + 1 \\ \text{Player 1 chooses } j_0 &\in \{1, \dots, J\} \\ K_1 &= J - j_0 + 1 + n(J + 1) + 1 \quad \text{where } J - j_0 + 1 \in \{1, \dots, J\} \\ \therefore \text{Player 2 can play } j_1 &= J - j_0 + 1\end{aligned}$$

Now, Player 1 has to start with $n(J + 1) + 1$ sticks. By the induction hypothesis, Player 1 will lose.

1.2 B

If \exists a non-negative integer n satisfying $K = n(J + 1) + 1$, then $K = n(J + 1) + 1 + m$ where $m \in \{1, \dots, J\}$. Player 1 can play $m = (K - 1) \bmod (J + 1)$, forcing Player 2 to start play with $n(J + 1) + 1$ sticks on the table. By Part A, Player 2 loses.

1.3 C

$$j^*(k) = \begin{cases} m & \text{if } m = (k - 1) \bmod (J + 1) \neq 0 \\ \text{any valid choice} & \text{otherwise} \end{cases}$$

2 Problem 2

2.1 A

First, note that the same player wins in every SPNE of this game.

Suppose that Player 2 wins in every SPNE. This implies that no matter what Player 1 plays, Player 2 has a winning strategy. Therefore there is a winning strategy for Player 2 if Player 1 chomps $(1, 1)$. However, no matter what (j, k) Player 2 chomps, Player 1 could have preempted it by playing (j, k) instead of $(1, 1)$ in the first stage. Thus there exists a winning strategy for Player 1 that begins with playing (j, k) in the first stage. By this contradiction, Player 1 wins in every SPNE.

2.2 B

Player 1 can chomp $(K - 1, K - 1)$ and then mirror Player 2's moves (i.e., if Player 2 chomps (x, y) , chomp (y, x)). This rule is not a strategy because it is not a complete contingent plan (i.e., it does not specify actions off the equilibrium path).

2.3 C

Game tree for Chomp(2,2):

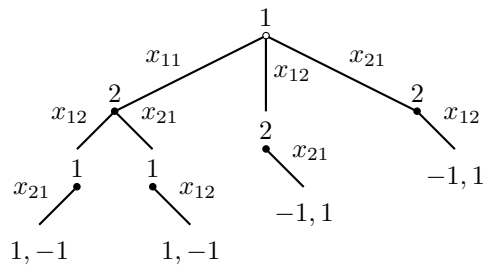


Figure 1: Chomp(2,2)

2.4 D

Excluding cases where x_{22} is the only choice left:

$$a_{11}(\emptyset) = x_{11}$$

$$a_{21}(x_{11}) = x_{12} \vee x_{21}, a_{21}(x_{12}) = x_{21}, a_{21}(x_{21}) = x_{12}$$

$$a_{12}(\{x_{11}, x_{12}\}) = x_{21}, a_{12}(\{x_{11}, x_{21}\}) = x_{12}$$

3 Problem 3

3.1 A

Symmetric NE of the stage 2 game:

$$F(b; m) = \begin{cases} (b/v)^{\frac{1}{m-1}} & \text{if } m \geq 2 \\ 1 & \text{if } m = 1 \end{cases}$$

For the stage 1 game, let $p_i = \sigma_i(\text{exit})$ and solve for the symmetric NE:

$$\begin{aligned} p^{n-1}(v - \epsilon) + (1 - p^{n-1})(-\epsilon) &= 0 \\ p^* &= \sigma_i^*(\text{exit}) = (\epsilon/v)^{\frac{1}{n-1}} \end{aligned}$$

The SPNE: $\{(\sigma_i^*(\text{exit}); F(b; m)) : i \in N\}$

3.2 B

The expected equilibrium payoff to each potential participant is 0

3.3 C

$$\begin{aligned} R &= m\epsilon + 1(m \geq 2)v \\ E[R] &= E[m]\epsilon + E[1(m \geq 2)]v = E[m]\epsilon + Pr[m \geq 2]v \\ E[R] &= E[m]\epsilon + (1 - Pr[m = 0] - Pr[m = 1])v \\ m &\sim \text{Bin}(1 - p^*, n) \\ E[R] &= (1 - p)n\epsilon + (1 - p^n - np^{n-1}(1 - p))v \\ &= n\epsilon - pn\epsilon + v - vp^n - vnp^{n-1} + vnp^n \\ &= n\epsilon - pn\epsilon + v - vp^n - vn(\epsilon/v) + vnp^n \\ &= -pn\epsilon + v - vp^n + vnp^n \\ &= v - vp^n = v - v(\epsilon/v)^{\frac{n}{n-1}} \end{aligned}$$

Expected revenue is decreasing in $\epsilon > 0$. If the seller could choose ϵ , she would set it to equal zero.

4 Problem 4

4.1 A

Pure strategy SPNE:

$$\{(L; L, R; L, L), (R; R, R; R, L)\}$$

4.2 B

From the SPNE above: LLL, RRL

From non-credible threats: LR (threat of RRR), RL (threat of LR or LLR)

5 Problem 5

5.1 A

NE of the third stage game: $NE_3 = \{(\sigma_1(f), \sigma_3(F)) : (1, 0), (0, 1), (0.5, 0.5)\}$

Corresponding expected payoffs: $\{(7, 10, 7), (7, 10, 7), (3.5, 5, 3.5)\}$

5.2 B

$NE = \{(R; l; NE_3)\}$

5.3 C

If Player 1 expects $(3.5, 5, 3.5)$ returns in the third stage game, but believes that Player 2 expects $(7, 10, 7)$, i.e., coordination, then Player 1 would play L to prevent Player 2 from playing l .