What's in a Poll?
Incentives for Truthful Reporting in Pre-Election Opinion Surveys*

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Abstract

We examine the ability of pre-election polls to aggregate information about voter preferences. We show that if the electorate is small and voting costs are negligible, then an equilibrium exists in which citizens report their true political preferences. If the electorate is large or voting costs are significant, however, then no such equilibrium exists because poll respondents possess incentives to influence the voting behavior of others by misreporting their true preferences. We find that when a truthful equilibrium does exist, a poll can raise expected welfare by discouraging turnout among members of the minority.

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1 Introduction

Pre-election polls are ubiquitous in modern democracies. Although many organizations and media outlets conduct polls, their predictions, nevertheless, often turn out to be incorrect. In 2002, most polls predicted that the Democrats would retain control of the US Senate, but the Republicans took control handily. In Australia's 1993 general elections, nearly every pre-election poll predicted a Liberal party victory, yet the Labour party won. Perhaps most famously, in the 1948 US presidential election, the Chicago Tribune erroneously ran the headline "Dewey Defeats Truman" after the most influential pollsters reported a large majority preferred Dewey[1]. Such inaccuracies may, of course, arise simply due to sampling error, but there may also be a less innocuous explanation. A common problem for pollsters is that a significant number of citizens often display a "last minute shift in candidate preference"[2]. Could these poll respondents be acting strategically?

Since voting is costly in time and effort, many citizens may choose to abstain if the election is in little doubt. If it is highly likely that the candidate a citizen supports will win, then he has an incentive to avoid incurring the cost of voting by free-riding off the efforts of like-minded voters. If members of the electorate believe that polls accurately reflect political preferences, then poll participants often possess incentives to report strategically. If, for example, an individual reports his preference for candidate B when he actually prefers candidate A and his report is regarded as genuine, then the other citizens will believe that A is less likely to win the election. This both reduces the incentive for A's supporters and increases the incentives for B's supporters to free ride, i.e. to abstain from voting.

In this paper we examine whether or not an equilibrium exists in which citizens truthfully report their preferences to a pollster. We find that the result depends critically on the cost of voting and the size of the electorate. When the cost of voting is negligible, there is little benefit to free-riding and truthful reporting can be supported in a perfect Bayesian equilibrium. However, when the cost of voting is significant or the population is large, no truthful equilibrium exists. In these settings, free-riding is attractive and poll respondents have an incentive to lie in order to try and influence the probability with which other citizens vote. This result is important since it indicates that efforts

[1] http://www.financialexpress.com/columnists/full_column.php?content_id=58541.

[2] "Possible Consequences of Non-Response for Pre-Election Surveys", The Pew Research Center.

to ascertain the underlying preferences of the electorate may often be in vain.

Our theory is built upon a model consisting of a round of cheap talk followed by an election with private valuations, costly voting and two alternatives. In the first stage, agents simultaneously make a report to a pollster. The pollster then reveals the profile of reports and agents simultaneously decide whether or not to vote. Within this setting we analyze a small population (3 citizens) then a population of arbitrary size (n citizens). We demonstrate that truthful reporting can occur in equilibrium if and only if the cost of voting is small and the population is not too large. We find that when a truthful equilibrium exists, a poll can raise expected welfare by discouraging turnout among the minority.

Our research builds upon the seminal work of Palfrey and Rosenthal (1983). These authors explore equilibrium expected turnout using a costly voting model with perfect information. If citizens in our model all report their true preferences in the polling stage, then the resulting continuation game coincides exactly with the setting studied by Palfrey and Rosenthal.

In addition to this study, there are several other recent papers combining communication and strategic voting. Gerardi and Yariv (2003) examines the effect of communication on different voting rules in a private information setting. They find that communication leaves a wide range of voting rules equivalent with respect to the sequential equilibrium outcomes that obtain. Coughlan (2000) allows agents to reveal their private information (a noisy signal about the state of the world) prior to making a final vote in a two alternative setting. He finds that agents will truthfully reveal their signal if and only if their preferences are sufficiently similar. Both of these papers differ from ours by introducing a common value component in agents' payoffs (e.g., a jury wants to convict if and only if the defendant is guilty). Additionally, both papers assume voting is costless. Austen-Smith and Feddersen (2002) analyzes a three voter, two alternative model with costless voting. In this framework agents' preferences are private in two dimensions. Each agent knows his bias and receives a noisy signal about the state of the world. When players simultaneously send messages prior to the voting stage, they find that a majority rule induces more information transmission and fewer decision-making errors than a unanimity rule. Austen-Smith and Feddersen (2006) examines a similar setup and shows that in a committee of arbitrary size, uncertainty of preferences generally permits full information revelation only under nonunanimous voting rules. Meirowitz (2005a) analyzes a two alternative, majority rule, costless voting model in which agents privately observe both their bias and a noisy signal. He finds that all agents will truthfully report their signals if and only if they are sufficiently optimistic that a majority of the population shares their bias and that truth telling can be more easily supported in small populations. Fey (1997) finds that pre-election polls can provide information about the viability of candidates and allow voters to coordinate on a Duvergarian outcome in a three candidate setting. In his model, however, voters do not consider how their poll responses will affect other agents' behavior.

Meirowitz (2005b) provides an alternative reason to suspect the veracity of polls in a two-candidate, majority-rule election when candidates use polling data to inform their policy selections. In his three-stage model: voters first report their ideal points to a polling service; candidates then use poll responses to select policy platforms; and voters finally cast a ballot for one of the candidates. Within this setting, Meirowitz finds that truthful reporting is generally not an equilibrium because agents have an incentive to misrepresent their ideal points in order to influence the candidates' platform selections. In our model, voters are not able to affect the policy alternatives, yet still they possess incentives to misreport their preferences.

Finally, Goeree and Grosser (2006) and Taylor and Yildirim (2005) also study the impact of information on equilibrium electoral outcomes. In both of these papers, however, the informational regimes are taken as exogenous. The focus of our investigation is to model explicitly the informational content of polls. Interestingly, Goeree and Grosser (2006) and Taylor and Yildirim (2005) find that more information reduces expected welfare in the respective settings they study. This contrasts with our finding that polls can raise expected welfare when a truthful equilibrium exists. The reason for this difference is that in the earlier studies, citizens are modelled as being more or less informed about a parameter governing the distribution of voter preferences. In this paper, however, citizens learn about the actual realization of voter preferences. Hence, whether welfare increases or decreases in response to more information appears to be sensitive to the type of information that is revealed.

In the following section we discuss the primitives of our model. Sections 3 and 4 contain the analysis of the three voter and n voter electorates respectively. A welfare analysis is presented in Section 5. Section 6 contains a discussion of our results when poll respondents are permitted to report that they are undecided. We conclude with some brief remarks in Section 7. Proofs not appearing in the text have been relegated to the Appendix.

2 The Model

Consider a population comprised of $n \geq 3$ risk-neutral citizens who can cast a vote in an election between two alternatives, 0 and 1. An agent may be one of two types, θ_0 or θ_1. A type θ_i citizen prefers alternative i. Each citizen privately knows his own type, but believes that the other agents' preferences are determined by realizations of i.i.d. random variables, where the probability a citizen is type θ_0 is $p \in (0,1)$. After observing his type, each citizen in the population makes a report, r $\in \{\theta_0, \theta_1\}$, to a pollster.[3] The pollster, who is not strategic, then aggregates the information and publicly reveals it to the citizens. Each citizen observes the outcome of the poll and then decides either to abstain, or vote for his preferred alternative.[4] Agents decide whether or not to vote simultaneously. If a citizen's preferred alternative wins the election, then he receives gross payoff normalized to 1, otherwise he receives 0. Casting a vote costs $c \in \left(0, \frac{1}{2}\right]$ for each agent, the election is determined by majority rule and ties are broken by the toss of a fair coin.

The equilibrium concept is a refinement of perfect Bayesian equilibrium outlined fully below. It should be noted here, however, that it is not necessary to explicitly specify a belief system because we investigate the existence of truthful equilibria in which beliefs are perforce degenerate (i.e., the reported profile of preferences is believed to be correct with probability one). Moreover, since every constellation of preferences occurs with positive probability, no profile of reports is off the path of play.

[3]To preserve symmetry and tractability, it is assumed that every citizen in the electorate participates in the poll. It seems quite likely, however, that our results would continue to hold qualitatively if only a subset of citizens were actually surveyed. Such a model would, however, be very technical and unwieldy.

[4]Abstaining strictly dominates voting for one's less preferred alternative.

3 The Three Citizen Population

Suppose $n = 3$. To determine the conditions under which an equilibrium exists in which all citizens truthfully report their preferences to the pollster, we first derive voting probabilities and expected payoffs assuming truth telling and then derive the conditions under which an agent would prefer to deviate from honest disclosure. With three citizens there are two possible situations to consider, either all three citizens are the same type or there are two of one type and one of the other. Hence, assume initially that agents report truthfully in the polling stage, and let α_x, $x \in \{2, 1, 0\}$, be the probability a citizen votes in the election stage when there are x other citizens of his type.

3.1 The Three-Citizen Majority

When all three citizens have the same preferences, there exists a unique symmetric Nash equilibrium of the complete information voting game where α_2 is given by the indifference condition

$$\frac{1}{2}(1 - \alpha_2)^2 = c.$$

The left side of this expression is 1/2 times the probability that a given citizen's vote is pivotal and the right side is his cost of voting. In general, a citizen's vote is pivotal if it either breaks or creates a tie. The factor 1/2 derives from the fact that a tie-breaking vote raises the probability that the citizen's preferred alternative is implemented from 1/2 to 1, and a tie-creating vote raises the probability from 0 to 1/2. When all citizens are the same type, the only possible tie occurs if no-one votes. Hence, the probability that a given citizen's vote is pivotal in this situation is the probability that the other two agents abstain. Solving the above expression yields

$$\alpha_2 = 1 - \sqrt{2c}.$$

Each citizen receives an expected equilibrium payoff of

$$\pi_2 \equiv 1 - c$$

in this situation. To see this, note that each individual is indifferent between voting and abstaining.

Moreover, a citizen who votes in this scenario receives $1 - c$ with certainty because no one ever votes for the other alternative.

3.2 The Two-Citizen Majority

When there are two citizens of one type and one of the other, then there is again a unique symmetric equilibrium of the voting game. The probability a member of the majority votes, α_1, once more equates 1/2 times the probability of being pivotal to the cost of voting:

$$\frac{1}{2}((1 - \alpha_1)(1 - \alpha_0) + \alpha_1\alpha_0 + (1 - \alpha_1)\alpha_0) = c.$$

A member of the majority breaks a tie if either both or neither of the other two agents vote, and he creates a tie if the minority agent votes and the other member of the majority abstains.

If he votes, the minority citizen breaks or creates a tie in every event except when both of the majority citizens vote. Thus, the minority citizen is indifferent between voting and abstaining if

$$\frac{1}{2}(1 - \alpha_1^2) = c,$$

or

$$\alpha_1 = \sqrt{1 - 2c}.$$

Substituting into the equation above and reducing we get

$$\alpha_0 = 1 - \sqrt{1 - 2c}.$$

Using these probabilities and some simple algebra reveals that a majority citizen receives expected equilibrium payoff

$$\pi_1 \equiv \sqrt{1 - 2c},$$

and the minority citizen receives

$$\pi_0 \equiv 1 - \sqrt{1 - 2c} - c.$$

3.3 Incentives to Misreport

In order to derive the conditions under which a truth-telling equilibrium exists, we must ensure that no single agent can gain from manipulating the beliefs of the other agents by misreporting his preferences. In particular, since the probability a citizen votes depends upon the *perceived* realization of types, an agent who is believed to be honest could potentially benefit by misreporting his preferences in the polling stage. Hence, we now derive the expected payoff in the election stage to an agent who lies but is perceived to be honest by the other citizens.

Suppose all agents turn out to be the same type. If one of the agents has lied to the pollster in this case, then the other two citizens vote with probability α_1. From abstaining the lying agent receives expected payoff

$$1 - \frac{1}{2}(1-\alpha_1)^2 = \sqrt{1-2c} + c.$$

and from voting he obtains $1 - c$. Since

$$\sqrt{1-2c} + c > 1 - c,$$

the liar abstains and receives

$$\hat{\pi}_2 \equiv \sqrt{1-2c} + c$$

in this case.

Next, suppose there are two citizens in the majority and one in the minority. If an agent in the majority has lied, then the other two citizens will have incorrect beliefs about which alternative has majority support. Hence, the agent who is really in the minority will vote as if he were in the majority and vice versa. Thus, if the liar abstains he receives expected payoff

$$\alpha_0((1-\alpha_1) + \frac{1}{2}\alpha_1) + \frac{1}{2}(1-\alpha_0)(1-\alpha_1) = 1 - \sqrt{1-2c}.$$

On the other hand, if the liar votes he receives

$$\alpha_0 + (1-\alpha_0)((1-\alpha_1) + \frac{1}{2}\alpha_1) - c = \frac{1}{2}.$$

Thus if

$$\frac{1}{2} > 1 - \sqrt{1-2c} \;\Rightarrow$$

$$c < \frac{3}{8},$$

then the liar votes and receives

$$\hat{\pi}_1 \equiv \frac{1}{2}.$$

(Throughout this section we suppose $c < \frac{3}{8}$. Proposition 2 in the next section reveals that if $c \geq \frac{3}{8}$, then deviating from truthful reporting yields a higher expected payoff to an agent for every possible realized profile of voter preferences; i.e., no truth-telling equilibrium exists.)

Finally, if the citizen in the minority has lied, then the two members of the majority believe everyone is of the same type. Hence, they vote with probability α_2. If the liar abstains he expects

$$\frac{1}{2}(1-\alpha_2)^2 = c,$$

and if he votes he expects

$$(1-\alpha_2)^2 + \frac{1}{2}2\alpha_2(1-\alpha_2) - c = \sqrt{2c} - c.$$

Since

$$\sqrt{2c} - c > c,$$

if the agent in the minority has lied, then he votes and expects

$$\hat{\pi}_0 \equiv \sqrt{2c} - c.$$

3.4 Truth Telling

We are now in a position to examine the conditions under which truthful reporting can be supported in a perfect Bayesian equilibrium. Consider an arbitrary agent. The probability that the other two citizens are type θ_0 is p^2; the probability that one of them is type θ_0 and the other is type θ_1 is $2p(1-p)$; and the probability they are both type θ_1 is $(1-p)^2$.

Assuming all other agents report truthfully, we can use the expected payoffs obtained above to derive the condition for a type θ_0 citizen to report truthfully. Specifically, the difference in the expected utilities from truth telling and lying must be non-negative,

$$p^2(\pi_2 - \hat{\pi}_2) + 2p(1-p)(\pi_1 - \hat{\pi}_1) + (1-p)^2(\pi_0 - \hat{\pi}_0) \geq 0.$$

The condition for truth telling by a type θ_1 citizen is analogous, with p replaced by $1-p$. Noting this, suppose that a type θ_0 citizen prefers to report truthfully. Then a sufficient condition for a type θ_1 citizen also to prefer truthful reporting is,

$$\begin{aligned}&(1-p)^2(\pi_2 - \hat{\pi}_2) + 2p(1-p)(\pi_1 - \hat{\pi}_1) + p^2(\pi_0 - \hat{\pi}_0)\\ \geq\ &p^2(\pi_2 - \hat{\pi}_2) + 2p(1-p)(\pi_1 - \hat{\pi}_1) + (1-p)^2(\pi_0 - \hat{\pi}_0)\end{aligned}$$

which (after tedious algebra) reduces to $p \leq \frac{1}{2}$.

Thus when $p < \frac{1}{2}$, only the truth-telling condition for type θ_0 is relevant, and analogously, when $p > \frac{1}{2}$, only the truth-telling condition for type θ_1 is relevant.

Proposition 1. *A truthful equilibrium is most easily supported when* $p = \frac{1}{2}$*. Even in this case, however, a truthful equilibrium exists if and only if* $c \leq \frac{3-\sqrt{5}}{4}$*.*

Proof. Without loss of generality, suppose that $p \leq \frac{1}{2}$ so that only the truth-telling condition for type θ_0 is relevant. Expanding this condition yields

$$p^2(3 - 2c - \sqrt{2c} - 4\sqrt{1-2c}) - p(3 - 2\sqrt{2c} - 4\sqrt{1-2c}) + 1 - \sqrt{2c} - \sqrt{1-2c} \geq 0.$$

Differentiating with respect to p leaves:

$$2p(3 - 2c - \sqrt{2c} - 4\sqrt{1-2c}) - (3 - 2\sqrt{2c} - 4\sqrt{1-2c})$$

When $c < \frac{3}{8}$, the second term in this expression is negative. Additionally, the first term differs from the second term only in that it has a $-2c$ instead of a $-\sqrt{2c}$. Therefore, for $p < \frac{1}{2}$ the expression is positive; i.e., the truth-telling condition is monotone increasing with respect to p. (The truth-telling condition for type θ_1 is monotone decreasing for $p > \frac{1}{2}$.) Hence, as p approaches $\frac{1}{2}$, honest reporting becomes more attractive. Setting $p = \frac{1}{2}$ in the truth-telling condition for type θ_0 and performing simple algebra yields the condition

$$c \leq \frac{3 - \sqrt{5}}{4}.$$

□

Notice that if all citizens are the same type, an agent does strictly better if he has misreported. By representing himself as a member of the opposition in this situation, an agent reduces free-riding within his own party. By the same token, if an agent is actually in the minority, misreporting his type increases free-riding among the opposition. The only situation in which a citizen prefers to have reported truthfully occurs when he winds up in a 2-versus-1 majority. In this case, the alternative preferred by the majority is implemented with higher probability in equilibrium. Hence, a member of the majority does not want to lie (and switch the perceived majority) in this case.

This reveals the intuition for why reporting the truth is most attractive for $p = \frac{1}{2}$. Under highly asymmetric distributions, it is likely that all three agents are of the same type; so that lying is relatively attractive. When $p = \frac{1}{2}$, however, the most likely scenario is a 2-versus-1 majority, making truth-telling relatively attractive.

The main lessons to be derived from the three-citizen version of the model are that truthful reporting in the polling stage can be supported in equilibrium only if the distribution of types is fairly symmetric and the cost of voting is relatively low. In the following section we demonstrate that the forces underlying Proposition 1 continue to hold in larger electorates as well.

4 The n Citizen Population

In this section we analyze the general case when there are an arbitrary number of citizens in the population.[5]

4.1 Equilibrium Selection

When there are more than three citizens in the population who all report truthfully, Palfrey and Rosenthal (1983) showed that there exist multiple equilibria of the ensuing voting game. In their terminology, both "Mixed-Pure" and "Totally Quasi-Symmetric" equilibria exist. In a "Mixed-Pure" equilibrium all members of one side vote randomly with the same probability, while members of the other side act asymmetrically, some voting with probability one and the rest abstaining. We regard such equilibria as implausible since they implicitly require a great degree of coordination among the agents who behave asymmetrically – precisely which agents are to vote and which are to abstain? We, therefore, restrict attention to a "Totally Quasi-Symmetric" equilibrium, in which all agents of the same type vote with the same probability; i.e., what is now commonly called a "type-symmetric" equilibrium. It is standard in symmetric Bayesian games (e.g. auctions) to focus on such equilibria. Note also that the unique equilibrium in the three-voter case is type-symmetric.

Let ϕ_i be the probability that a type θ_i agent votes. If there are $\frac{n}{2}$ agents of each type, then we suppose the agents play the type-symmetric equilibrium in which $\phi_0 = \phi_1 = 1$.[6] Otherwise, the equilibrium voting probabilities are given in the following lemma.

Lemma 1. *Without loss of generality, suppose there are $n-k$ type θ_0 citizens and k type θ_1 citizens, where $k \leq \frac{n-1}{2}$ (i.e., alternative 0 has strict majority support). Then there exists an equilibrium of the voting game in which $\phi_1 = 1 - \phi_0$ and*

$$\frac{1}{2}P(\phi_0; k, n) = c, \tag{1}$$

[5] In large electorates it may at first seem that there is little reason for polling. While it is true the sample average converges to p, the variance of the binomial distribution does not converge to zero. Since we are interested in the exact population distribution, polls can potentially convey valuable information even in arbitrarily large populations.

[6] In this case, this is the only equilibrium satisfying $\phi_0 = \phi_1$.

where

$$P(\phi_0;k,n) \equiv \binom{n-1}{k}\phi_0^k(1-\phi_0)^{n-k-1} + \binom{n-1}{k-1}\phi_0^{k-1}(1-\phi_0)^{n-k}.$$

Proof. The probability a type θ_0 is pivotal is

$$\sum_{j=0}^{k}\binom{n-k-1}{j}\binom{k}{j}\phi_0^j(1-\phi_0)^{n-k-1-j}\phi_1^j(1-\phi_1)^{k-j}$$

$$+\sum_{j=0}^{k-1}\binom{n-k-1}{j}\binom{k}{j+1}\phi_0^j(1-\phi_0)^{n-k-1-j}\phi_1^{j+1}(1-\phi_1)^{k-j-1},$$

where the first term is the probability his vote breaks a tie and the second term is the probability his vote creates a tie. When the above sum equals $2c$ a type θ_0 is indifferent between voting and not voting. Similarly, in order for type θ_1 to mix we must have

$$\sum_{j=0}^{k-1}\binom{n-k}{j}\binom{k-1}{j}\phi_0^j(1-\phi_0)^{n-k-j}\phi_1^j(1-\phi_1)^{k-1-j}$$

$$+\sum_{j=0}^{k-1}\binom{n-k}{j+1}\binom{k-1}{j}\phi_0^{j+1}(1-\phi_0)^{n-k-j-1}\phi_1^j(1-\phi_1)^{k-1-j} = 2c.$$

Substituting for $2c$ and setting $\phi_0 = 1-\phi_1$ leads to

$$\phi_0^k(1-\phi_0)^{n-k-1}\sum_{j=0}^{k}\binom{n-k-1}{j}\binom{k}{j} + \phi_0^{k-1}(1-\phi_0)^{n-k}\sum_{j=0}^{k-1}\binom{n-k-1}{j}\binom{k}{j+1}$$

$$= \phi_0^{k-1}(1-\phi_0)^{n-k}\sum_{j=0}^{k-1}\binom{n-k}{j}\binom{k-1}{j} + \phi_0^k(1-\phi_0)^{n-k-1}\sum_{j=0}^{k-1}\binom{n-k}{j+1}\binom{k-1}{j}.$$

Using the combinatoric identity

$$\sum_{j=0}^{k}\binom{m}{j}\binom{n}{k-j} = \binom{m+n}{k},$$

we see that

$$\sum_{j=0}^{k-1}\binom{n-k-1}{j}\binom{k}{j+1} = \sum_{j=0}^{k-1}\binom{n-k-1}{j}\binom{k}{k-j-1} = \binom{n-1}{k-1},$$

$$\sum_{j=0}^{k}\binom{n-k-1}{j}\binom{k}{j}=\sum_{j=0}^{k}\binom{n-k-1}{j}\binom{k}{k-j}=\binom{n-1}{k},$$

$$\sum_{j=0}^{k-1}\binom{n-k}{j}\binom{k-1}{j}=\sum_{j=0}^{k-1}\binom{n-k}{j}\binom{k-1}{k-1-j}=\binom{n-1}{k-1},$$

$$\sum_{j=0}^{k-1}\binom{n-k}{j+1}\binom{k-1}{j}=\sum_{j=0}^{k-1}\binom{n-k}{n-k-j-1}\binom{k-1}{j}=\binom{n-1}{n-k-1}=\binom{n-1}{k}.$$

Thus, the two sums are equal. □

The function $P(\phi_0; k, n)$ is the probability that a member of the majority is pivotal when all other majority citizens vote with probability ϕ_0 and all minority citizens vote with probability $1-\phi_0$. In general there are two solutions to (1), giving rise to two type-symmetric equilibria of the voting game. To see this, note that

$$\frac{\partial P}{\partial \phi_0}=\binom{n-1}{k-1}\left(\frac{1}{k}\right)\phi_0^{k-2}(1-\phi_0)^{n-k-2}[k(k-1)(1-\phi_0)^2-(n-k-1)(n-k)\phi_0^2].$$

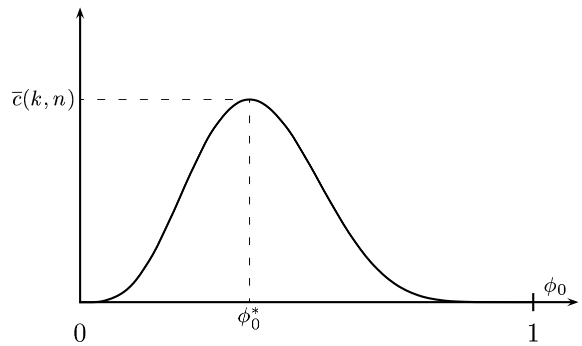
It is straightforward to verify that this expression is positive when $\phi_0 < \phi_0^*$, zero when $\phi_0 = \phi_0^*$, and negative when $\phi_0 > \phi_0^*$, where

$$\phi_0^* \equiv \frac{\sqrt{k(k-1)}}{\sqrt{(n-k-1)(n-k)}+\sqrt{k(k-1)}}.$$

Hence $P(\phi_0; k, n)$ is single-peaked and (as Palfrey and Rosenthal (1983) showed) there exist two solutions to (1) for all $c < \bar{c}(k,n)$, where

$$\bar{c}(k,n) \equiv \frac{1}{2}P(\phi_0^*; k, n) = \frac{(n-1)!(\sqrt{k(n-k-1)}+\sqrt{(n-k)(k-1)})}{k!(n-k)!\sqrt{(k-1)(n-k-1)}}\,\frac{[(n-k)(n-k-1)]^{\frac{n-k}{2}}[k(k-1)]^{\frac{k}{2}}}{2[\sqrt{(n-k)(n-k-1)}+\sqrt{k(k-1)}]^{n-1}}$$

Remark 1. As the population grows large, $\bar{c}(k,n)$ approaches zero. In order to ensure existence of a type-symmetric equilibrium, it is, therefore, necessary to assume n is finite and $c \leq \bar{c}(k,n)$.



Below we study the type-symmetric equilibrium in which the probability that a member of the majority votes is given by the larger of the two values for ϕ_0. There is a straightforward justification for this restriction. Specifically, it can be shown that in the equilibrium associated with the smaller value for ϕ_0, no truth-telling equilibrium ever exists; i.e. some citizens always lie to the pollster in equilibrium.

4.2 Existence of a Truthful Equilibrium

In the three-citizen setting, it was possible to express explicitly the difference in expected utilities from truth-telling and lying. This allowed us to characterize fully the conditions for the existence of a truthful equilibrium. With more than three citizens, however, explicit calculation of payoffs is not possible. Rather than characterizing the conditions under which a truthful equilibrium exists, we derive a strong sufficient condition implying that an agent does better when deviating from truth telling under every possible realized profile of voter preferences.

Definition 1. Suppose that there are k citizens in the minority. The profile of voter preferences is *critical* if $k = \frac{n-1}{2}$ when n is odd or $k = \frac{n}{2}$ when n is even.

A critical profile of voter preferences corresponds to the thinnest possible majority when n is odd and a tie when n is even. The following lemma reveals that a single agent who deviates from truthful reporting in the polling stage receives a higher expected payoff under every non-critical profile of voter preferences.

Lemma 2. *Consider an arbitrary agent, i. If all citizens other than i report truthfully and they all believe i's report, then i's expected payoff is higher under every non-critical realization of voter preferences if he lies in the polling stage.*

Proof. An individual in the majority does better from lying in the polling stage if it increases the probability the rest of the majority votes (Since $\phi_0 = 1 - \phi_1$ it also decreases the probability that a member of the minority votes). If all citizens tell the truth, then the probability a member of the majority votes is determined by

$$\frac{1}{2}P(\phi_0; k, n) = c.$$

Now, if one member of the majority lies, then the rest of the population mistakenly believes there is an extra person in the minority. Therefore, everyone else votes with a probability implicitly determined by

$$\frac{1}{2}P(\phi_0; k+1, n) = c.$$

(Note that we need $k < n - k - 1$ for this equation to be valid. This implies that the citizen in question cannot switch the perceived majority regardless of his report.)

We know $P(\phi_0; k, n)$ and $P(\phi_0; k+1, n)$ are both single-peaked and attain their respective maxima when

$$\phi_0^*(k) = \frac{\sqrt{k(k-1)}}{\sqrt{(n-k-1)(n-k)} + \sqrt{k(k-1)}}$$

and

$$\phi_0^*(k+1) = \frac{\sqrt{k(k+1)}}{\sqrt{(n-k-2)(n-k-1)} + \sqrt{k(k+1)}}.$$

Additionally, $P(\phi_0; k, n)$ and $P(\phi_0; k+1, n)$ intersect only when

$$\binom{n-1}{k-1}\phi_0^{k-1}(1-\phi_0)^{n-k} = \binom{n-1}{k+1}\phi_0^{k+1}(1-\phi_0)^{n-k-2},$$

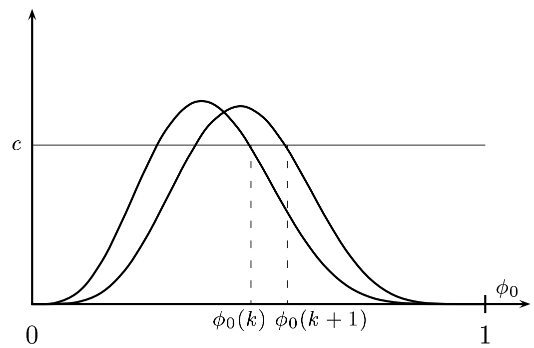
or

$$(1-\phi_0)^2 = \left(\frac{n-1-k}{k+1}\right)\left(\frac{n-k}{k}\right)\phi_0^2,$$

or

$$\tilde{\phi_0} = \frac{\sqrt{k(k+1)}}{\sqrt{k(k+1)} + \sqrt{(n-k-1)(n-k)}}.$$

Now, $\phi_0^*(k) < \tilde{\phi_0} < \phi_0^*(k+1)$, so the two functions intersect between their maxima. Since both functions equal 0 when $\phi_0 = 0$ or 1, for any given c, $\phi_0(k+1) > \phi_0(k)$.



Thus, if $k < n-k-1$, a member of the majority wants to lie if everyone else reports truthfully. Note that since $\phi_1 = 1 - \phi_0$, this also implies that a member of the minority does better if he has lied under the same circumstances.

□

The intuition for this result is similar to that for the three-agent case. The bigger the difference between the majority and the minority groups, the lower the probability that a member of the majority votes and the higher the probability a member of the minority votes, owing to the free-rider effect. Thus, when a member of the majority lies he decreases the perceived gap between the groups, reducing free riding in his own group and increasing it in the minority. Similarly, when a member of the minority lies, he increases the perceived gap between the groups, reducing free riding in his own group and increasing it in the majority.

In fact, the only situation in which a citizen might benefit from truth telling is when the realized profile of voter preferences is critical and he is in the (possibly weak) majority. When the profile of voter preferences is critical and c is relatively small, the probability that a majority citizen votes is significantly higher than the probability that a minority citizen votes. An agent who lies in this case will switch the perceived majority, causing his group to lose the election with high probability. As c increases, however, the difference between ϕ_0 and ϕ_1 diminishes, and for c sufficiently high, the probability that a minority agent votes can even exceed the probability that a majority agent votes. In particular, define $\underline{c}$ implicitly by

$$\underline{c} \equiv \begin{cases} \frac{1}{2}P(\frac{1}{2}; \frac{n-1}{2}, n), & \text{if n is odd} \\ \frac{1}{2}P(\frac{1}{2}; \frac{n-2}{2}, n), & \text{if n is even.} \end{cases}$$

If n is odd, $c = \underline{c}$, and a critical distribution of voter preferences obtains, then $\phi_0 = \phi_1 = \frac{1}{2}$ whether a majority citizen reports truthfully or lies. Hence, the benefit to truth telling in this situation is zero while the benefit from lying is strictly positive for all other realizations of voter preferences. If n is even, $c = \underline{c}$, and there are $\frac{n}{2}$ citizens of each type, an agent actually does strictly better if he has lied about his preferences. The following proposition formalizes this discussion.

Proposition 2. *Define*

$$\tilde{c} \equiv \begin{cases} \overline{c}(\frac{n-1}{2}, n), & \textit{if } n \textit{ is odd} \\ \overline{c}(\frac{n-2}{2}, n), & \textit{if } n \textit{ is even.} \end{cases}$$

(i) $\underline{c} \leq \tilde{c}$.

(ii) *If* $c \in [\underline{c}, \tilde{c}]$ *and all citizens report truthfully in the polling stage, then for every realization of voter preferences there exists a type-symmetric equilibrium of the voting game,* (ϕ_0, ϕ_1).

(iii) *However, if* $c \in [\underline{c}, \tilde{c}]$*, then there does not exist an equilibrium in which all citizens report truthfully in the polling stage and then play according to* (ϕ_0, ϕ_1).

Proof. We prove the claim for odd n. The proof for even n is similar, though slightly more involved.

By Lemma 2 we already know that if $k < n - k - 1$, an agent does better by deviating from truthful reporting. In Lemma 3 in the Appendix, we show that $\overline{c}(k, n)$ is minimized when $k = \frac{n-1}{2}$. Thus if $c \leq \tilde{c}$, type-symmetric equilibria of the voting game exist for all realizations of voter preferences.

Next, observe that

$$c = 0 \ \Rightarrow \ \phi_0 = 1.$$

Moreover,

$$\frac{\partial \phi_0}{\partial c} = \frac{1}{\frac{\partial P}{\partial \phi_0}} < 0.$$

Suppose $k = \frac{n-1}{2}$. By construction

$$c = \underline{c} \ \Rightarrow \ \phi_0 = \frac{1}{2}.$$

Hence, if

$$c = \tilde{c} \ \Rightarrow \ \phi_0 \leq \frac{1}{2},$$

then it follows that $\underline{c} \leq \tilde{c}$. If $c = \tilde{c}$, then a majority citizen votes with probability

$$\phi_0^* = \frac{\sqrt{\frac{n-1}{2}\frac{n-3}{2}}}{\sqrt{\frac{n-1}{2}\frac{n+1}{2}} + \sqrt{\frac{n-1}{2}\frac{n-3}{2}}} < \frac{1}{2}.$$

Hence, $\underline{c} < \tilde{c}$. Moreover, if $c \in (\underline{c}, \tilde{c}]$ then $\phi_0 < \frac{1}{2}$. A majority citizen who lies in this case raises the probability that other majority citizens vote to $1 - \phi_0$ and correspondingly lowers the probability that minority citizens vote to ϕ_0. This establishes the claim. □

It should be noted that $c \in [\underline{c}, \tilde{c}]$ is a strong sufficient condition for the non-existence of a truthful equilibrium. Specifically, this condition implies that an agent who deviates from truthful reporting receives a higher payoff under *every* possible realization of voter preferences. As noted above, if $c < \underline{c}$, lying turns out to be costly in the event that an agent is a member of the thinnest possible majority. In all other cases, however, the benefits from lying are strictly positive. Thus, no truth-telling equilibrium typically exists even for voting costs significantly lower than $\underline{c}$. In the three-citizen setting, for example, $\underline{c} = \frac{3}{8}$, while Proposition 1 indicates that no truthful equilibrium exists if $c \geq \frac{1}{5}$. The following result shows that even if c is arbitrarily small, no truthful equilibrium exists if the population of citizens is sufficiently large.

Proposition 3. *For any $c > 0$ there exists $\underline{n}$ such that*

$$n \geq \underline{n} \ \Rightarrow \ c > \underline{c}.$$

That is, if $n \geq \underline{n}$, then no truthful equilibrium exists.

Proof. We prove the claim for odd n. The proof for even n is analogous. Define

$$c_1(n) \equiv \binom{n-1}{\frac{n-1}{2}} \left(\frac{1}{2}\right)^n$$

and

$$c_2(n) \equiv \binom{n-1}{\frac{n-3}{2}} \left(\frac{1}{2}\right)^n.$$

Then

$$c_1(n) + c_2(n) = \underline{c}.$$

To establish the claim, we show that $c_i(n)$ is decreasing and $\lim_{n\to\infty} c_i(n) = 0$ for $i = 1, 2$.

First, note that

$$\frac{c_1(n+2)}{c_1(n)} = \frac{n}{n+1}.$$

Since $c_1(3) = \frac{1}{4}$, this reveals

$$c_1(n) = \frac{1}{2} \prod_{j=0}^{\frac{n-1}{2}-1} \frac{2j+1}{2j+2}. \tag{2}$$

Since $\frac{2j+1}{2j+2} < 1$, $c_1(n)$ is decreasing. Moreover, $c_1(n) \leq a(n)$ for the sequence

$$a(n) \equiv \prod_{j=0}^{n} \sqrt{\frac{j+1}{j+2}}.$$

Simple algebra reveals

$$a(n) = \frac{1}{\sqrt{n+2}},$$

so $\lim\limits_{n\to\infty} a(n) = 0$. Hence, $\lim\limits_{n\to\infty} c_1(n) = 0$.

Next, observe that

$$
\begin{aligned}
& c_2(n+2) < c_2(n) \\
\Leftrightarrow & \left(\frac{n(n+1)}{\left(\frac{n-1}{2}\right)\left(\frac{n+3}{2}\right)} \right) \left(\frac{1}{4} \right) < 1 \\
\Leftrightarrow & \; 3 < n.
\end{aligned}
$$

Hence, $c_2(n)$ is decreasing. Moreover

$$c_2(n) < c_1(n) \;\; \Leftrightarrow \;\; \frac{1}{\frac{n+1}{2}} < \frac{1}{\frac{n-1}{2}}.$$

Since $\lim_{n\to\infty} c_1(n) = 0$, this implies $\lim_{n\to\infty} c_2(n) = 0$ as well. □

As noted above, the only possible benefit from truthful reporting occurs when a citizen turns out to be in the thinnest possible majority. This benefit vanishes, however, as the population increases in size while the benefit from lying in all other situations remains strictly positive.

Proposition 1 indicated that if the population consists of only three citizens, then truthful reporting in the polling stage can be supported in equilibrium if and only if the cost of voting is sufficiently low. We have seen that for a population of arbitrary size, no truthful equilibrium exists when c is high. The next proposition reveals that a truthful equilibrium does exist if c is low enough.

Proposition 4. *If the voting cost, c, is sufficiently small, then an equilibrium in which all citizens truthfully report their preferences in the polling stage exists.*

Proof. From (1) we see

$$\lim_{c\to 0} \phi_0 = 1,$$

and correspondingly

$$\lim_{c\to 0} \phi_1 = 0.$$

This implies that the benefit from lying under non-critical profiles of voter preferences vanishes in the limit. If, however, the profile of voter preferences turns out to be critical, then the cost of lying is very high when c is small. Specifically, suppose n is odd and $k = \frac{n-1}{2}$. If all citizens report truthfully, then the majority group wins with probability one in the limit. If, on the other hand, a majority citizen lies, then the minority group wins with probability one. Similarly, suppose n is even and $k = \frac{n}{2}$. If all citizens report truthfully, then the election is decided by a coin toss in the limit. If, on the other hand, one citizen lies, then his group loses with certainty. Since ϕ_0 is continuous in c and $P(\phi_0, k, n)$ is continuous in ϕ_0, A truthful equilibrium exists for sufficiently small $c > 0$ as well. □

As c approaches zero, the probability that a majority citizen votes converges to one and the probability that a minority citizen votes converges to zero. This has two important implications. First, the free-rider effect – and hence the incentive to lie – vanishes. Second, the cost of lying under a critical realization of voter preferences becomes very large. Specifically, a single lie alters the perceived majority and changes the outcome of the election with probability one if n is odd and probability $1/2$ if n is even. Hence, an equilibrium in which citizens report their true preferences exists provided c is sufficiently small and the electorate is not too large.

5 Welfare

In the previous section it was shown that polls can convey information about voter preferences only if the cost of voting is small and the population is not too large. An important question is whether or not polls enhance expected voter welfare when they do convey information.

To answer this question, first consider a setting without a pre-election poll. If $p \in (0, 1)$, then the probability that a given citizen's vote is pivotal is strictly positive regardless of how the rest of the population votes. Hence, for c sufficiently small, all citizens vote and the majority wins the election with probability one in equilibrium. With a truthful pre-election poll, on the other hand, we have seen that citizens vote probabilistically. This has two important implications relative to the setting with no poll. First, there is a positive probability that the alternative favored by the majority will lose the election. Second, the probability that all citizens vote is less than one. Hence, with a truthful poll, both the expected benefits and the expected costs of an election are lower. The question is which effect is larger. This is answered for the case of small c as follows.

Proposition 5. *When c is small, expected welfare is higher with a pre-election poll than without one.*

Proof. First, consider the setting with a poll and suppose a truthful equilibrium obtains. The expected utility to an agent derived from the population with k people in the minority is given by

$$\left(\frac{n-k}{n}\right)\Bigg[\phi_0\Bigg(1-\sum_{j=2}^{k}\binom{k}{j}\phi_1^j(1-\phi_1)^{k-j}\sum_{r=0}^{j-2}\binom{n-k-1}{r}\phi_0^r(1-\phi_0)^{n-k-r-1}$$

$$-\frac{1}{2}\sum_{j=1}^{k}\binom{k}{j}\phi_1^j(1-\phi_1)^{k-j}\binom{n-k-1}{j-1}\phi_0^{j-1}(1-\phi_0)^{n-k-j}-c\Bigg)$$

$$+(1-\phi_0)\Bigg(1-\sum_{j=1}^{k}\binom{k}{j}\phi_1^j(1-\phi_1)^{k-j}\sum_{r=0}^{j-1}\binom{n-k-1}{r}\phi_0^r(1-\phi_0)^{n-k-r-1}$$

$$-\frac{1}{2}\sum_{j=0}^{k}\binom{k}{j}\phi_1^j(1-\phi_1)^{k-j}\binom{n-k-1}{j}\phi_0^j(1-\phi_0)^{n-j-k-1}\Bigg)\Bigg]$$

$$+\left(\frac{k}{n}\right)\Bigg[\phi_1\Bigg(1-\sum_{j=2}^{n-k}\binom{n-k}{j}\phi_0^j(1-\phi_0)^{n-k-j}\sum_{r=0}^{j-2}\binom{k-1}{r}\phi_1^r(1-\phi_1)^{k-r-1}$$

$$-\frac{1}{2}\sum_{j=1}^{n-k}\binom{n-k}{j}\phi_0^j(1-\phi_0)^{n-k-j}\binom{k-1}{j-1}\phi_1^{j-1}(1-\phi_1)^{k-j}-c\Bigg)$$

$$+(1-\phi_1)\Bigg(1-\sum_{j=1}^{n-k}\binom{n-k}{j}\phi_0^j(1-\phi_0)^{n-k-j}\sum_{r=0}^{j-1}\binom{k-1}{r}\phi_1^r(1-\phi_1)^{k-r-1}$$

$$-\frac{1}{2}\sum_{j=0}^{n-k}\binom{n-k}{j}\phi_0^j(1-\phi_0)^{n-k-j}\binom{k-1}{j}\phi_1^j(1-\phi_1)^{k-j-1}\Bigg)\Bigg].$$

The first term above is the probability a citizen is in the majority given k people in the minority. This is multiplied by the majority's expected utility. If he votes, which occurs with probability ϕ_0, the majority wins unless the minority out votes the remaining majority by at least two. If there is a tie, the majority wins with probability $\frac{1}{2}$. The next two terms represent the probability a member of the majority abstains multiplied by the corresponding expected utility. The remaining terms represent the expected utility of a member of the minority.

Suppose $c = 0$. If there is no poll, then all citizens vote with probability one. If there is a poll, then all members of the majority vote and all members of the minority abstain with probability one. In either case, the majority wins with certainty. Since voting is costless, welfare is identical in the two settings. When the cost of voting is increased infinitesimally, all citizens continue to vote with probability one if there is no poll. If there is a poll, however, a slight increase in cost alters voting behavior. To determine how this affects welfare, we differentiate the expected utility above with respect to c and evaluate at $c = 0$. This yields:

$$\frac{n-k}{n}\left(\frac{\partial \phi_0}{\partial c} - \phi_0 - \frac{\partial \phi_0}{\partial c}\right) + \frac{k}{n}\left(\frac{\partial \phi_1}{\partial c} - \phi_1 - \frac{\partial \phi_1}{\partial c}\right) = -\frac{n-k}{n}.$$

When $c = 0$, $\phi_0 = 1$ and $\phi_1 = 0$. Hence, all terms containing ϕ_1 or $(1 - \phi_0)$ drop out. Notice that for all values of j, every term in each summation above contains either ϕ_1 or $(1 - \phi_0)$ raised to a power greater than or equal to two. This eliminates all these expressions.

When the population distribution has k members in the minority, the increase in cost decreases expected utility by a factor of $\frac{n-k}{n}$. Therefore, total expected welfare is decreased by a factor of

$$w = \begin{cases} \sum_{j=0}^{\frac{n-1}{2}} \left(\binom{n}{j} p^j (1-p)^{n-j} + \binom{n}{n-j} p^{n-j} (1-p)^j\right)\left(\frac{n-j}{n}\right), & \text{if n is odd} \\ \sum_{j=0}^{\frac{n-2}{2}} \left(\binom{n}{j} p^j (1-p)^{n-j} + \binom{n}{n-j} p^{n-j} (1-p)^j\right)\left(\frac{n-j}{n}\right) + \binom{n}{\frac{n}{2}} p^{\frac{n}{2}} (1-p)^{\frac{n}{2}}, & \text{if n is even} \end{cases}$$

Without polls, all citizens incur the cost of voting and total expected welfare is decreased by a factor of one. With polls, expected welfare decreases by a factor strictly less than one. The minority abstains with high probability when the population is polled, and the majority votes with probability close to one. Some welfare is lost since the minority wins with positive probability, however this is dominated by the savings in cost from lower turnout. □

When the cost of voting is small and there is no pre-election poll, all citizens vote and the majority wins with probability one. When agents are allowed to make reports, however, polls act as a coordination device. The majority votes with probability close to one, while the minority votes with probability close to zero. In both situations the majority wins with very high probability, but with a poll, expected turnout– and hence aggregate expected voting cost – is significantly lower. The savings in expected aggregate voting costs outweigh the loss in expected benefits, and a pre-election poll, therefore, results in higher expected welfare over all.

6 Undecided Voters

Many polls give respondents the option to report that they are "undecided." It is straightforward to extend the analysis above to allow for this possibility. Specifically, suppose that there is a third type of agent, θ_U, who is indifferent between the two alternatives. Note that such a citizen never votes in equilibrium and has no reason to misreport his type to the pollster. What about the incentives facing the other types of agents?

Consider the possibility of an equilibrium in which all agents report their true preferences. As before, an agent who is not undecided fairs better by misreporting his type whenever the realized profile of voter preferences is non-critical. Moreover, the opportunity for a citizen to misreport his type as θ_U may undermine the existence of a truthful equilibrium. To see this, suppose that the number of voters who are not undecided is odd and consider an agent who is in the thinnest possible majority. When the only possible reports to the pollster are θ_0 or θ_1, it was shown above that such an agent would prefer to report truthfully if c was sufficiently low. The reason for this is that lying changes the perceived majority and agents in the majority vote with much higher probability than those in the minority when c is small. If, however, a member of the thinnest possible majority reports his preferences as θ_U, then he changes the perceived profile of preferences to a tie. It can be shown that misreporting his type as undecided and voting results in a higher expected payoff than truthful revelation in this setting for all values of $c > 0$. Hence, enriching the model to allow for undecided voters may make it even harder to support truthful disclosure in equilibrium.

7 Conclusion

Given the prevalence of pre-election polls and the considerable attention they receive, it is important to understand their impact on voting behavior and electoral outcomes. In this paper we have attempted a first pass at these issues in the context of a strategic voting model where citizens have intrinsic tastes over two political alternatives and voting is costly. Our results suggest that polls are most apt to convey real information about voter preferences only when the cost of voting is quite small and the electorate is not too large.

The incentive for an agent to misreport his true preferences to a pollster derives from a desire to influence the voting behavior of other citizens. In particular, the equilibrium probability that an individual votes is decreasing in the number of other citizens who share his preferences. Hence, by lying, an agent can increase turnout among citizens who prefer the same alternative as him and decrease turnout among citizens who prefer the opposing alternative. The only situation in which an agent prefers truthful reporting is when he is in the thinnest possible majority. Even in this case, however, the return to truthful reporting vanishes when the cost of voting is sufficiently high.

When a truthful equilibrium does exist, polls have two effects. They raise the probability that the alternative favored by the minority wins the election, and they reduce expected turnout. Hence, polls reduce both the expected benefits and the expected costs of an election. The savings in aggregate expected voting costs, however, outweigh the loss in expected benefits, resulting in higher welfare over all.

Our analysis raises several related issues. First, if no truthful equilibrium exists, then it would be edifying to know whether or not any partially revealing equilibria can be supported. Some cursory analysis of this question suggests that if there does not exist an equilibrium in which all citizens report truthfully, then there does not exist an equilibrium in which a poll conveys any useful information at all. Second, to preserve symmetry and tractability, we have assumed that every citizen in the electorate participates in the pre-election poll. It seems very likely, however, that our results would continue to hold qualitatively if only a subset of citizens were actually surveyed. The driving force behind our findings is a free-rider effect under which the probability an agent votes is decreasing in the number of other citizens who share his preferences. There is no reason to suppose that this effect would not be present in settings where only a subset of citizens participated in a poll. Moreover, to the extent that the free-rider effect is present in such a setting, poll respondents will continue to possess incentives to misreport their true preferences.

The main message of this paper is that there are often reasons to suspect the veracity of pre-election polls. By definition, if a poll conveys useful information, then it influences voting behavior. This implies that poll participants often possess incentives to respond strategically. Even in a setting where responses are likely to be genuine, the incentive of the individual or organization conducting the poll to fully and honestly disclose survey results may be in question. Unbiased polls in which participants respond sincerely and pollsters fully disclose survey data can enhance overall welfare - - our findings suggest, however, that such polls may be quite rare.

Appendix

Lemma 3. *$\bar{c}(k,n)$ attains its minimum when $k = \frac{n-1}{2}$.*

Proof. If we show that $\bar{c}(k+1,n) < \bar{c}(k,n)$ for any $k < \frac{n-1}{2}$, we will have the desired result.

$$2\bar{c}(k,n) = \binom{n-1}{k}(\phi_0^*(k))^k(1-\phi_0^*(k))^{n-1-k} + \binom{n-1}{k-1}(\phi_0^*(k))^{k-1}(1-\phi_0^*(k))^{n-k}$$

Substituting in for $\phi_0^*(k)$ yields

$$\bar{c}(k,n) = \frac{(n-1)!(\sqrt{k(n-k-1)}+\sqrt{(n-k)(k-1)})}{k!(n-k)!\sqrt{(k-1)(n-k-1)}}\frac{[(n-k)(n-k-1)]^{\frac{n-k}{2}}[k(k-1)]^{\frac{k}{2}}}{2[\sqrt{(n-k)(n-k-1)}+\sqrt{k(k-1)}]^{n-1}}.$$

Now, $\bar{c}(k+1,n) < \bar{c}(k,n)$ if

$$\frac{(k+1)(\sqrt{k(n-k-1)}+\sqrt{(n-k)(k-1)})}{\sqrt{(k-1)(n-k-1)}}\frac{[(n-k)(n-k-1)]^{\frac{n-k}{2}}[k(k-1)]^{\frac{k}{2}}}{[\sqrt{(n-k)(n-k-1)}+\sqrt{k(k-1)}]^{n-1}}$$

$$> \frac{(n-k)(\sqrt{(k+1)(n-k-2)}+\sqrt{k(n-k-1)})}{\sqrt{k(n-k-2)}}\frac{[(n-k-1)(n-k-2)]^{\frac{n-k-1}{2}}[k(k+1)]^{\frac{k+1}{2}}}{[\sqrt{(n-k-1)(n-k-2)}+\sqrt{k(k+1)}]^{n-1}}$$

We will take the proof in three parts. First,

$$\sqrt{(n-k-1)(n-k-2)}+\sqrt{k(k+1)} > \sqrt{(n-k)(n-k-1)}+\sqrt{k(k-1)}.$$

Notice that the left-hand side is the same as the right-hand side with $k+1$ replacing k. Next we show that $\sqrt{(n-k)(n-k-1)}+\sqrt{k(k-1)}$ is increasing in k.

$$\frac{d\left((\sqrt{(n-k)(n-k-1)}+\sqrt{k(k-1)})\right)}{dk}$$

$$= \frac{1}{2}[k(k-1)]^{-\frac{1}{2}}(2k-1) + \frac{1}{2}[(n-k)(n-k-1)]^{-\frac{1}{2}}(2k-2n+1).$$

This is positive if

$$[(n-k)(n-k-1)]^{\frac{1}{2}}(2k-1) > (2n-2k-1)[k(k-1)]^{\frac{1}{2}}.$$

Which, after some algebra, reduces to

$$k < \frac{n}{2}.$$

Secondly,

$$\frac{\sqrt{k(n-k-1)} + \sqrt{(n-k)(k-1)}}{\sqrt{(k-1)(n-k-1)}} > \frac{\sqrt{(k+1)(n-k-2)} + \sqrt{k(n-k-1)}}{\sqrt{k(n-k-2)}}.$$

Rewriting and dividing by $\sqrt{k(n-k-1)}$ yields

$$\sqrt{k(n-k-2)}\left(1 + \sqrt{\frac{(n-k)(k-1)}{k(n-k-1)}}\right) > \sqrt{(k-1)(n-k-1)}\left(1 + \sqrt{\frac{(k+1)(n-k-2)}{k(n-k-1)}}\right).$$

Rearranging leads to

$$\frac{\sqrt{n-k-2}(k - \sqrt{(k+1)(k-1)})}{\sqrt{k}} > \frac{\sqrt{k-1}(n-k-1-\sqrt{(n-k)(n-k-2)})}{\sqrt{n-k-1}}.$$

Notice $\sqrt{n-k-2} > \sqrt{k-1}$ and $\sqrt{n-k-1} > \sqrt{k}$ when $k < \frac{n-1}{2}$. Additionally,

$$k - \sqrt{k^2-1} > n-k-1-\sqrt{(n-k-1)^2-1},$$

since, $a - \sqrt{a^2-1}$ is decreasing in a.

Lastly,

$$(k+1)[(n-k)(n-k-1)]^{\frac{n-k}{2}}[k(k-1)]^{\frac{k}{2}} \geq (n-k)[(n-k-1)(n-k-2)]^{\frac{n-k-1}{2}}[k(k+1)]^{\frac{k+1}{2}}.$$

Collapsing,

$$\left(\frac{n-k}{n-k-2}\right)^{n-k-2}\left(\frac{n-k-1}{n-k-2}\right) \geq \left(\frac{k}{k-1}\right)\left(\frac{k+1}{k-1}\right)^{k-1}.$$

Let $a = n - k$ and $b = k + 1$. This yields

$$\left(\frac{a}{a-2}\right)^{a-2}\left(\frac{a-1}{a-2}\right) > \left(\frac{b}{b-2}\right)^{b-2}\left(\frac{b-1}{b-2}\right).$$

(Note $b \geq 3$ since we need $k \geq 2$ for the expression for $\bar{c}(k,n)$ to be valid.) When $a = 4$ and $b = 3$, the above inequality binds. We show that

$$\left(\frac{a}{a-2}\right)^{a-2}\left(\frac{a-1}{a-2}\right)$$

is increasing in a for $a \geq 4$. Differentiating the above expression yields

$$(a-2)a^{a-3}(a-2)^{1-a}(a-1) + (\ln a)a^{a-2}(a-2)^{1-a}(a-1) - (a-1)(a-2)^{-a}a^{a-2}(a-1)$$

$$-\ln(a-2)(a-2)^{1-a}a^{a-2}(a-1) + a^{a-2}(a-2)^{1-a},$$

which reduces to

$$a^{a-3}(a-2)^{-a}[(a-2)^2(a-1) + a(a-2) - a(a-1)^2 + (a-2)(a-1)a(\ln a - \ln(a-2))].$$

We need to show

$$(a-2)^2(a-1) + a(a-2) - a(a-1)^2 + (a-2)(a-1)a(\ln a - \ln(a-2)) \geq 0,$$

or equivalently,

$$(a-2)(a-1)a[\ln a - \ln(a-2)] \geq 2a^2 - 5a + 4.$$

Note that

$$\ln a - \ln(a-2) > \frac{2}{a-1}.$$

Thus,

$$(a-2)(a-1)a[\ln a - \ln(a-2)] > 2a(a-2) \geq 2a^2 - 5a + 4$$

if $a \geq 4$.

$\square$

References

[1] Austen-Smith, D. and Feddersen, T., 2005, "Deliberation and Voting Rules," *In Social Choice and Strategic Decisions: Essays in Honor of Jeffrey S. Banks,* eds. David Austen-Smith and John Duggan. Heidelberg:Springer

[2] Austen-Smith, D. and Feddersen, T., 2006, "Deliberation, Preference Uncertianty, and Voting Rules," *American Political Science Review,* 100, 209-217

[3] Coughlan, P., 2000, "In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Strategic Voting," *American Political Science Review,* 94, 375-93

[4] Feddersen, T. and W. Pesendorfer, 1997, "Voting Behavior and Information Aggregation in Elections with Private Information," *Econometrica*, 65, 1029-58

[5] Fey, M., 1997, "Stability and Coordination in Duverger's Law: A Formal Model of Preelection Polls and Strategic Voting," *American Political Science Review*, 91, 135-47

[6] Gerardi, D. and L. Yariv, 2003, "Putting Your Ballot Where Your Mouth Is - An Analysis of Collective Choice with Communication," Working Paper, Department of Economics, Yale University

[7] Goeree, J. and J. Grosser, 2006, "False Consensus Voting and Welfare Reducing Polls,". *Economic Theory,* forthcoming.

[8] Meirowitz, A., 2005a "In defense of exclusionary deliberation: communication and voting with private beliefs and values," Working Paper, Department of Politics, Princeton University

[9] Meirowitz, A., 2005b, "Polling games and information revelation in the Downsian framework," *Games and Economic Behavior*, 51, 464-489

[10] Palfrey, T. and H. Rosenthal, 1983, "A Strategic Calculus of Voting," *Public Choice*, 41, 7-53

[11] Taylor, C. and H. Yildirim, 2005, "Public Information and Electoral Bias," Working Paper, Department of Economics, Duke University