# Privacy and Information Acquisition in Competitive Markets

**Abstract**

Personal privacy is studied in the context of a competitive product (or labor) market. Firms initially post prices (or wages) they promise to charge (or pay) individuals whose applications are ultimately approved. Contracts are incomplete because the amount of information firms acquire about applicants cannot be observed. When information acquisition corresponds to searching for bad news, firms search too hard in equilibrium. Consumers can ameliorate this by demanding inefficiently small levels of output. If economic characteristics differ across groups of applicants and price discrimination is prohibited, then members of the high-risk group are subjected to more scrutiny and suffer disproportionately high rejection rates. When information acquisition corresponds to searching for good news, firms acquire too little information about their applicants in equilibrium. Finally, if rejected applicants remain in the market and continue to apply to different firms, then the resulting adverse selection may be so severe that all parties would be better off if no information was collected at all.

JEL Classifications: C73, D61, D83, L14.
Key Words: Privacy, Information Acquisition, Quantity Discrimination, Incomplete Contracts.

# 1 Introduction

Recent years have witnessed dramatic advances in virtually all aspects of information technology including: storage, processing, and transmission. Between 1988 and 2000, the cost of a gigabyte of hard disk storage fell from about $11,500 to around a dollar (Acquisti and Varian 2002 p. 1). Between 1989 and 2000, the number of transistors on a standard PC mother board increased by a factor of 42 (www.intel.com/research/silicon/mooreslaw.htm). Between 1990 and 1998, the number of nodes on the Internet increased from 313,000 to 29,670,000 (Coffman and Odlyzko 1998, table 7).

Such innovations in information technology have revolutionized many industries. They have reshaped modern economies and profoundly impacted society. They have given rise to new markets for new goods and services while at the same time spawning new concerns regarding questions of public policy. The question at the heart of this paper concerns the impact of the revolution in information technology on personal privacy.

Privacy is, of course, not a new concern, but the extraordinary advances in information technology occurring over the past 15 years have brought it to the forefront of public debate.[1] The focus of the debate involves rights over the collection, storage, and sale of personal data such as an individual's credit history, medical records, or criminal convictions. Under the federal Fair Credit Reporting Act, for example, an insurance under writer, creditor, landlord or any other business with a "permissible purpose" may procure a credit report on an applicant without his knowledge or permission.[2] Besides financial records and payment histories, "credit" reports often contain data on current and past: employment, place of residence, criminal and/or civil judgments. Firms may also procure an "investigative report" which is a credit report that is based on interviews with

[1]To see the recent consumer privacy initiatives launched by the Federal Trade Commission, visit www.ftc.gov/privacy/index.html.

[2]An exception is prospective employers who must obtain an applicants permission before procuring a credit report.

employers, co-workers, neighbors or others (see www.pirg.org/consumer/credit/#finpri). In addition to the three national credit repositories, Experian, Equifax, and Trans Union, there are many specialized consumer reporting agencies who collect and sell personal data including the Medical Information Bureau, and a host of employee and tenant screening companies. What is more, virtually all of these agencies do business over the Internet, permitting firms to obtain data on their prospective customers or employees at low cost and with great speed.

These dramatic advances in information-acquisition technology have generated both social benefits and costs. The benefits derive from a more efficient allocation of goods and services and job assignments. Insurance policies, debt contracts, and apartment leases can be tailored to better fit the characteristics of prospective customers. Likewise, employment and college-enrollment offers can be better matched with the skills and aptitudes of applicants. There are, however, two costs associated with the increase in information acquisition. First, there is obviously a direct resource cost involving the collection, storage, and processing of data. Second, there is a cost in terms of personal privacy. Specifically, while consumers and/or prospective employees are typically willing to divulge personal information in order to secure more favorable contract terms, firms are unlikely to collect the efficient amount of information when screening applicants.

In this paper, a relatively simple model that conforms with the preceding discussion is presented and analyzed. In the first stage of the game, firms that sell homogeneous goods or services post prices they promise to charge applicants who are ultimately approved. In the second stage, each consumer applies to purchase the good or service from one of the firms.[3] Next, the firms acquire information about each of their applicants.[4] The outcome of this information acquisition is a bivariate signal indicating that each consumer is either *qualified*, in which case he is permitted to

[3]Although the analysis is presented in terms of a product market, it can easily be recast in terms of a labor market where prospective employees apply to firms for jobs.

[4]See Bramoulle and Kranton (2002) for an interesting analysis involving incentives for information acquisition in a different context.

buy the good or service at the posted price, or *unqualified*, in which case his application is rejected. Two informational environments are considered, one in which firms search for bad news about their applicants and one in which they search for good news. The key assumption underlying the model is that contracts are incomplete because the amount of information firms acquire cannot be observed. Since firms cannot commit to information-acquisition levels, they are left to compete only in prices.[5]

In the case of searching for bad news, firms post the lowest price consistent with zero economic profit in equilibrium. Unfortunately, this low price gives them incentives to acquire excessive amounts of information about their applicants.[6] In other words, all consumers would be better off *ex ante* if the firms posted higher prices and acquired less information. It is shown that when the marginal cost of information acquisition is relatively high, welfare is larger under a regulatory regime that vests privacy rights with consumers. Also, in an effort to preserve their privacy, consumers typically demand inefficiently low levels of output. Economic discrimination is investigated in a setting with two groups of consumers, a high-risk group and a low-risk group. In the absence of regulation, members of the high-risk group face a higher equilibrium price than members of the low-risk group. Banning price discrimination accentuates quantity discrimination by leading to a situation in which high-risk applicants are subjected to more scrutiny and suffer disproportionately high rejection rates, although their overall welfare rises.

In the case of searching for good news, by contrast, firms post prices that result in positive profit in equilibrium. They, nevertheless, acquire too little information and reject too many applicants. Finally, it is shown that when rejected consumers can continue to apply for the good at different firms, the resulting adverse selection seriously undermines the market and can generate a situation

[5] Bernheim and Whinston (1999) also consider contracting in an environment where only a subset of variables are verifiable.

[6] In the context of an agency model, Khalil (1997) finds that the probability that the principal will audit the agent is higher when the principal cannot commit.

in which all parties would be better off if no information was collected at all.

Until recently, little economic research on privacy had been written since the pioneering works by Hirshleifer (1980), Stigler (1980), and Posner (1981). Three new contributions to the privacy literature are: Acquisti and Varian (2002), Calzolari and Pavan (2002), and Taylor (2002). Inspired by observations of price discrimination on the Internet, these papers investigate monopolistic settings in which the purchasing history of consumers can be used to formulate personalized offers. Rather than *price* discrimination by monopolists, however, the current paper investigates *quantity* discrimination in a competitive market. The demand for customer information by a monopolist often does generate undesirable social outcomes, but one hardly expects monopolists to act in the interest of social efficiency. The distortions identified in this paper, on the other hand, arise in a competitive setting where one might plausibly expect efficient information acquisition to obtain.

The findings here are reminiscent of – though distinct from – those presented in Hirshleifer (1971). In his celebrated paper, Hirshleifer showed that, given equilibrium prices, the private benefit of information acquisition typically outweighs the social benefit. Indeed, in a pure exchange setting, information may have no social value at all, because it results only in a redistribution of wealth from ignorant agents to informed ones. In the current paper, by contrast, some information acquisition is typically desirable from a social perspective, but agents possess private incentives to collect either too much or too little, depending on the information-acquisition technology they possess. Also, equilibrium price determination is a key ingredient in the analysis presented here.

Perhaps it is most appropriate to view this paper as adding an important caveat to the original privacy articles by Hirshleifer (1980), Stigler (1980), and Posner (1981). These authors argued that privacy should not be a concern in a competitive setting where market forces ensure that the marginal benefit of information acquisition equals the marginal cost. The central theme of this paper is that if information acquisition is not observable, then competitive pressure will lead to a divergence between the marginal private benefit of information acquisition and the marginal social benefit. In such a setting, firms will possess incentives to systematically collect the wrong amount of information about prospective customers and/or employees resulting in too little trade in equilibrium.

## 2 The Model

Consider the market for a good in which there is uncertainty about cost-relevant consumer characteristics. Examples include: a life or health insurance market in which there is uncertainty about genetic factors, a credit market in which there is uncertainty about default risk, a rental housing market in which there is uncertainty about the risk of property damage, and a college market in which there is uncertainty about scholastic aptitude.[7] The supply side of the market is composed of at least two identical risk-neutral expected profit maximizing firms. The demand side of the market consists of a continuum of *ex ante* identical consumers with unit measure. Each consumer is a risk-neutral expected utility maximizer who receives incremental utility of $v > 0$ from consuming one unit of the good and zero from consuming additional units.[8] A consumer is one of two possible types. In particular, the cost of supplying the good to him either turns out to be low, $c_L \geq 0$, or high, $c_H > c_L$. The realization of a consumer's type is not contractually verifiable. Also, in order for the model to be interesting, it is assumed that $c_H > v > c_L$. In other words, it is efficient to serve only low-cost consumers.

The proportion of high-cost consumers in the population is $\lambda > 0$. Information is initially incomplete and symmetric. In particular, consumers do not know their own types. Hence, it is appropriate to think of a single representative consumer whose probability of being a high-cost type

[7] In a labor-market setting, employers would be uncertain about the characteristics of potential workers such as their productivity or the health status of their children.

[8] Risk aversion would strengthen the findings presented below. Also, the unit-demand assumption is relaxed in Section 8.

is $\lambda$.

At the beginning of the game, each firm $j$ announces a price $p_j \in \mathbb{R}_+$ at which it commits to sell a unit of the good to a consumer whose application is ultimately approved. These price announcements are made publicly and simultaneously. The consumer then either *applies* to purchase the good from one of the firms or chooses not to apply to any firm. If he does not apply for the good, then the game ends and all parties receive their reservation payoffs of zero.

If a consumer applies to purchase the good, then the firm he selects may acquire information about him. Specifically, the firm chooses a 'sample size' or search intensity $n \geq 0$.[9] The search intensity, $n$, is unobservable and unverifiable. The cost to the firm of acquiring information about an applicant is $kn$, where $k > 0$. A firm that chooses search intensity $n$ receives $n$ conditionally independent Bernoulli' signals $X_1, \ldots, X_n$, where

$$\Pr\{X_i = 1|c\} = \begin{cases} 1, & \text{if } c = c_L, \\ 1-\alpha, & \text{if } c = c_H \end{cases}$$

The parameter $\alpha \in (0,1)$ is intrinsic signal strength. If $\alpha = 1$, then a single signal is fully informative, and if $\alpha = 0$, then the signals contain no information at all. This process is interpreted as follows. Each firm chooses a file containing $n$ records, $X_1, \ldots, X_n$, (e.g., a payment or job history) for each of its applicants. Each record in the file is either positive ($X_i = 1$) or negative ($X_i = 0$). Since the probability of a false negative is zero in this setting, it is appropriate to regard the firm as searching records for 'bad news' about its applicants.[10]

Note that it is possible to summarize all the information contained in an applicant's file with

[9] The sample size, $n$, is treated as a continuous choice variable for convenience. Also, it is assumed that the technology involves simultaneous rather than sequential sampling. All results remain qualitatively unchanged under sequential sampling.

[10] The analogous setting in which firms search for 'good news' is considered in Section 9.

the sufficient statistic

$$S_n \equiv \min\{X_1, \ldots, X_n\}.$$

Specifically, if $S_n = 0$, then at least one of the records was negative and the applicant is certainly type $c_H$, and if $S_n = 1$, then all the records were positive and the applicant is type $c_L$ with probability

$$\frac{(1-\lambda)}{\lambda(1-\alpha)^n + (1-\lambda)} > (1-\lambda).$$

If $S_n = 0$, then the applicant is regarded as *unqualified*, and if $S_n = 1$, then he is regarded as *qualified*.

After acquiring information, a firm must decide whether to approve the consumer's application (i.e., sell him the good at the price it posted). Approval results in a payoff of $v - p_j$ for the consumer and an expected payoff of $p_j - E[c|S_n] - kn$ for the firm. Rejection results in a payoff of zero for the consumer and $-kn$ for the firm.

It is notationally convenient to define the positive constant

$$m \equiv -\frac{k}{\ln(1-\alpha)}.$$

This is a measure of the efficacy of the information-acquisition technology. Lower values of $m$ correspond to better technologies involving low sampling costs and/or high intrinsic signal strength.

## 3 The First-Best Solution

In this section the socially efficient information acquisition and allocation policy is characterized. To this end, suppose that a planner interested in maximizing the expected utility of consumers operated the firms subject to a zero-profit constraint. In particular, define the function

$$AC(n) \equiv \frac{\lambda(1-\alpha)^n c_H + (1-\lambda)c_L + kn}{\lambda(1-\alpha)^n + (1-\lambda)}. \tag{1}$$

This is the expected cost of gathering information about a consumer and supplying him the good conditional on observing $S_n = 1$. In other words, it is the cost to a firm per accepted application, or its average cost of operation. A firm that makes zero expected profit must charge a price $p$ to qualified applicants and select a search intensity $n$ such that $p = AC(n)$.

Given the information-acquisition technology, the planner should clearly pursue one of the following three possible strategies:

**Policy 1.** Acquire information $n > 0$ about the consumers and sell to the qualified ones at a price of $AC(n)$.

**Policy 2.** Acquire no information and sell to all consumers at a price of $\lambda c_H + (1-\lambda)c_L$.

**Policy 3.** Acquire no information and sell to no-one.

Policy 3 corresponds to abandoning the market and obviously yields welfare of zero. If the planner elects not to abandon the market, then she must solve the following problem in order to choose optimally between Policies 1 and 2:

$$\max_{(p,n)} U(p,n) \equiv \left(\lambda(1-\alpha)^n + (1-\lambda)\right)(v-p) \quad \text{s.t. } p = AC(n).$$

A consumer's expected utility, $U(p,n)$, is the product of two terms, the probability of being permitted to buy the good and the surplus from buying it. Note that the probability of being allowed to buy the good is decreasing in the amount of information acquisition, $n$. Hence, a consumer's taste for privacy stems from the fact that the more the firm knows about him, the less likely it is to sell him the good. This, however, does not imply that it is necessarily optimal to set $n = 0$. Specifically, there is generally a trade-off between higher values of $n$ and lower values of $p$ deriving from the zero-profit constraint. To see this, define welfare by $W(n) \equiv U(AC(n), n)$. The planner's problem can then be written:

$$\max_{n \geq 0} W(n) = \lambda(1-\alpha)^n(v - c_H) + (1-\lambda)(v - c_L) - kn. \tag{2}$$

There are two 'costs' and one 'benefit' associated with $W(n)$. The first term is negative and represents the social cost of allocating the good to the high-cost consumers who are mistakenly regarded as qualified; the second term is positive and represents the social benefit of allocating the good to the low-cost consumers; and the third term is the cost of information acquisition. Policy 2 dominates Policy 1 if and only if a corner solution to (2) obtains at $n = 0$. At such a solution, the marginal cost of mistakenly allocating the good to a consumer is less than the marginal cost of acquiring information about him, while these costs are equalized at an interior solution.

Differentiating $W(n)$ yields

$$W'(n) = \ln(1-\alpha)\lambda(1-\alpha)^n(v - c_H) - k.$$

Observe that $W'(n)$ decreases with $n$ and is negative for sufficiently large $n$. An interior solution to (2) obtains, therefore, if and only if $W'(0) > 0$, or

$$m < \lambda(c_H - v). \tag{3}$$

If (3) holds, then the solution, $n^*$, is defined implicitly by the condition

$$\lambda(1-\alpha)^{n^*}(c_H - v) = m, \tag{4}$$

and if (3) does not hold, then the solution to (2) is $n^* = 0$. In other words, consumers are willing to give up some privacy for a lower price if and only if (3) holds. This makes sense. Consumers prefer Policy 1 to Policy 2 when the information-acquisition technology is relatively good (i.e., $m$ is relatively small) or the social cost of misallocation is relatively high (i.e., $\lambda(c_H - v)$ is large) because these are the situations in which the zero-profit price, $AC(n)$, declines rapidly.

While condition (3) is necessary and sufficient for Policy 1 to dominate Policy 2, it remains to determine the conditions under which Policy 3 (abandoning the market) is optimal. As a first step in answering this question, consider the following definition.

**Definition 1 (Viability).** The market is said to be *ex ante* viable if

$$v > \lambda c_H + (1-\lambda)c_L.$$

The market is *ex ante* viable if each consumer's valuation for the good exceeds the unconditional expected cost of supplying it to him. Observe that Policy 3 cannot be optimal in this case because Policy 2 (acquiring no information and selling the good to everyone) delivers positive welfare. Even if the market is not *ex ante* viable, however, Policy 1 may be preferable to abandoning the market.

**Lemma 1 (The Abandonment Boundary).** *If the market is not ex ante viable, then there exists a unique number $m^\dagger \in (0, \lambda(c_H - v)$ such that Policy 1 delivers positive welfare* iff $m < m^\dagger$*.*

**Proposition 1 (The First-Best Solution).** *The socially efficient plan is characterized as follows.*

**(i)** *If the market is* ex ante *viable and* $m < \lambda(c_H - v)$*, or if the market is not* ex ante *viable and* $m < m^\dagger$*, then Policy 1 is optimal; i.e., the planner should acquire information in accordance with (4) and sell the good to qualified consumers for* $p = AC(n^*)$*.*

**(ii)** *If the market is* ex ante *viable and* $m \geq \lambda(c_H - v)$*, then Policy 2 is optimal; i.e., the planner should acquire no information and sell the good to everyone for* $p = \lambda c_H + (1-\lambda)c_L$*.*

**(iii)** *If the market is not* ex ante *viable and* $m \geq m^\dagger$*, then Policy 3 is optimal; i.e., the planner should acquire no information and sell the good to no-one.*

This result is intuitive. It says that the market should be abandoned if and only if it is not *ex ante* viable and information is too costly. If this is not the case, then it is efficient either to acquire information about consumers and sell the qualified ones the good for $AC(n^*)$, or to acquire no information and sell the good to all consumers for $\lambda c_H + (1-\lambda)c_L$. As noted above, the most interesting aspect of this finding is that when Policy 1 is optimal, consumers are willing to sacrifice some privacy in an effort to secure the good at a lower price.

# 4 Market Equilibrium

The market game has four stages: price announcements by firms, application by consumers, information acquisition and allocation of the good by firms. As usual, derivation of a pure-strategy subgame perfect Nash equilibrium (referred to as just an equilibrium below) requires analyzing these stages in reverse order.

When deciding on its information acquisition and allocation plan, a firm should pursue one of the same three alternatives identified in Section 3: acquire information about its applicants and sell to the qualified ones; acquire no information and sell to all applicants; or acquire no information and sell to no-one.

Unless it is optimal to abandon the market, a firm posting price $p$ will choose $n$ to maximize

$$\Pi(p,n) \equiv \lambda(1-\alpha)^n(p-c_H)+(1-\lambda)(p-c_L)-kn.$$

Define the critical price

$$p^\dagger \equiv c_H - \frac{m}{\lambda}. \tag{5}$$

For $p < p^\dagger$, the optimal search intensity for the firm is defined implicitly by the following first-order condition

$$\lambda(1-\alpha)^{\overline{n}(p)}(c_H - p) = m, \tag{6}$$

and for $p \geq p^\dagger$, the optimal search intensity $\overline{n}(p) \equiv 0$.

Observe that for $p < p^\dagger$, lower prices induce firms to acquire more information about each applicant, resulting in a lower probability of sale. The question is which prices consumers find attractive.

**Definition 2 (Relevant Prices).** A price $p \in \mathbb{R}_+$ is said to be relevant if $p < v$ and $\Pi(p, \overline{n}(p)) \geq 0$.

Since a firm will reject all of its applicants without acquiring information if $\Pi(p, \overline{n}(p)) < 0$, only relevant prices yield an applicant positive expected utility in the continuation equilibrium. In particular, a consumer's expected utility from applying to purchase the good at relevant price $p$ is

$$U(p, \overline{n}(p)) = \left(\lambda(1-\alpha)^{\overline{n}(p)} + (1-\lambda)\right)(v-p). \tag{7}$$

The first term in this expression is the probability of having his application approved, which is increasing in $p$ (the privacy effect), and the second term is the surplus from acceptance, which is decreasing in $p$. The following lemma indicates that even though lower prices involve less privacy, consumers will apply to one of the firms posting the lowest relevant price in the market.

**Lemma 2 (Demand).** *A consumer's expected continuation payoff, $U(p, \overline{n}(p))$, is strictly decreasing in $p$.*

In light of Lemma 2, if a firm posts the lowest relevant price $p$, then in the continuation equilibrium it earns expected profit per application of

$$\Pi(p, \overline{n}(p)) = \lambda(1-\alpha)^{\overline{n}(p)}(p - c_H) + (1-\lambda)(p - c_L) - k\overline{n}(p) \tag{8}$$

Next, observe that in equilibrium $\Pi(p, \overline{n}(p))$ must equal zero. It cannot be negative, or a firm posting $p$ could profitably deviate by offering a non-relevant price. On the other hand, if $\Pi(p, \overline{n}(p)) > 0$, then one of the least profitable firms in the market would benefit by deviating to a price slightly less than $p$ and attracting all the applicants.

**Lemma 3 (The Competitive Price).** *If the market is* ex ante *viable or if $m < m^\dagger$, then there exists a unique relevant price $\overline{p}$ such that $\Pi(\overline{p}, \overline{n}(\overline{p})) = 0$. If the market is not* ex ante *viable and $m \geq m^\dagger$, then no relevant price exists.*

This lemma says that there is a unique relevant price $\overline{p}$ satisfying $\overline{p} = AC(\overline{n}(\overline{p}))$ if and only if there is positive surplus available in the market (i.e., first-best welfare is not zero). It is now possible to characterize the equilibrium outcome of the game.

**Proposition 2 (Market Equilibrium).** *The unique equilibrium outcome is characterized as follows.*

**(i)** *If the market is* ex ante *viable and $m < \lambda(1-\lambda)(c_H - c_L)$, or if the market is not* ex ante *viable and $m < m^\dagger$, then: at least two firms post the price $\overline{p} = AC(\overline{n}(\overline{p}))$; no firm posts a lower price; consumers apply to the low-price firms; the firms acquire information $\overline{n}(\overline{p}) > 0$ about their applicants and sell the good to the qualified ones.*

**(ii)** *If the market is* ex ante *viable and* $m \geq \lambda(1-\lambda)(c_H - c_L)$*, then: at least two firms post the price* $\overline{p} = \lambda c_H + (1-\lambda)c_L$*; no firm posts a lower price; consumers apply to the low-price firms; the firms acquire no information and sell the good to all applicants.*

**(iii)** *If the market is not* ex ante *viable and* $m \geq m^{\dagger}$*, then equilibrium payoffs to all parties are zero because all firms post non-relevant prices.*

This result parallels Proposition 1 in several respects. In particular, one of three possible types of market equilibrium prevails depending on parameter values. There may be a *Type 1 equilibrium* in which firms price competitively, acquire information about their applicants, and sell to the qualified ones; or there may be a *Type 2 equilibrium* in which firms price competitively, acquire no information, and sell to everyone; or there may be a *Type 3 equilibrium* in which the market is inactive.

These three types of equilibria correspond closely to the three potentially optimal policies identified in Proposition 1. Indeed, the parameter values giving rise to a Type 3 equilibrium in which the market is inactive are the same as those under which Policy 3 (abandoning the market) is efficient. On the other hand, while Type 1 and Type 2 equilibria are similar in spirit to implementation of Policy 1 and Policy 2 respectively, there is a key difference concerning the incentives for information acquisition across the two settings that is explored in the next section.

## 5 The Equilibrium Level of Privacy

The following lemma characterizes the function $AC(n)$ for parameter values under which the market equilibrium involves positive information acquisition.

**Lemma 4 (Minimum Average Cost).** *If the market is* ex ante *viable and* $m < \lambda(1-\lambda)(c_H - c_L)$

*or if the market is not* ex ante *viable and* $m < m^{\dagger}$*, then* $AC(n)$ *is U-shaped and*

$$\overline{n}(\overline{p}) = \underset{n \geq 0}{\operatorname{argmin}}\, AC(n).$$

With this lemma in hand, it is possible to prove the following key result.

**Proposition 3 (Excessive Information Acquisition).** *If the market is* ex ante *viable and* $m < \lambda(1-\lambda)(c_H - c_L)$*, or if the market is not* ex ante *viable and* $m < m^{\dagger}$*, then* $\overline{n}(\overline{p}) > n^*$ *and* $AC(\overline{n}(\overline{p})) < AC(n^*)$*. That is, firms collect too much information about their applicants and price too low in any Type 1 equilibrium.*

Proposition 3 is easily understood. It arises from a divergence between the social and private cost of misallocation. Specifically, the social cost of awarding the good to a high-cost consumer is $c_H - v$ while the private cost to a firm from misallocation is $c_H - \overline{p}$. In a Type 1 equilibrium, competition ensures that $\overline{p} < v$, and hence, firms have higher incentives to acquire information about their applicants than is socially efficient. In particular, the firms do not account for the positive consumer surplus $v - \overline{p}$ derived from selling the good to a high-cost consumer.[11]

Note from Lemma 4 that the equilibrium price $\overline{p}$ is the lowest price that firms can post and still break even. Indeed, $\overline{p}$ is 'too low' in the sense that consumers would be happier to face a higher price and preserve more privacy. In particular, a search intensity of $n^*$ and price of $p = AC(n^*)$ would generate an *ex ante* Pareto improvement since all consumers would be better off and firms would still make zero expected profit. The problem here derives from the unobservability of the

[11] It is possible to achieve efficiency in a Type 1 equilibrium by considering a richer but less realistic contract space. In particular, suppose each firm $j$ promises to give each of its applicants an up-front payment of $r_j$ and to charge the qualified ones $p_j$ for the good. The equilibrium of this game will involve $\overline{r}$ equaling first-best welfare, $\overline{p} = v$, and $\overline{n}(\overline{p}) = n^*$. In other words, in equilibrium the consumers 'sell the expected surplus' to the firms, which generates the correct incentives for information acquisition. Contracts that involve positive payments from the firms to unqualified applicants, however, seem both unrealistic and unrobust. For instance, if (as is considered in Section 10 below) a consumer whose application is rejected at one firm can apply to buy the good from a different one, then paying unqualified applicants will certainly not give rise to an efficient equilibrium outcome.

search intensity, $n$. Because $n$ is not contractible, firms cannot commit to investigate applicants efficiently. Specifically, given that the firms will acquire information according to $\overline{n}(p)$, consumers will apply to purchase the good for the lowest relevant price that is offered. Hence, the combination of competition and the non-contractibility of $n$ result in a price that is too low and too little privacy relative to the social optimum.

# 6 Privacy Rights

Since firms possess incentives to acquire too much information about their applicants in a Type 1 equilibrium, it is interesting to investigate a setting in which consumers have the right to complete privacy; i.e., a setting in which firms are required to supply the good to all applicants. Clearly, if the market is not *ex ante* viable, then this assignment of rights will cause the market to shut down. On the other hand, if the market is *ex ante* viable, then requiring the firms to serve all consumers may generate and *ex ante* Pareto improvement relative to a Type 1 equilibrium.

**Proposition 4 (Efficient Assignment of Rights).** *Suppose that the market is* ex ante *viable.*

**(i)** *If $m \in [\lambda(c_H - v), \lambda(1-\lambda)(c_H - c_L))$, then assigning privacy rights to consumers is socially optimal and generates strictly higher welfare than a Type 1 equilibrium.*

**(ii)** *There exists $\epsilon \in (0, \lambda(c_H - v))$ such that $m < \epsilon$ implies that a Type 1 equilibrium in which firms have the right to investigate applicants generates higher welfare than assigning privacy rights to consumers.*

This result says that it is better to allocate privacy rights to consumers when the market is *ex ante* viable and the information-acquisition technology is not very good or the social cost of misallocation $c_H - v$ is small. This makes sense. These are precisely the cases in which the socially-efficient search intensity, $n^*$, is small. Hence, the inefficiency deriving from allowing no information acquisition ($n = 0$) is less than that deriving from the excessive information acquisition ($n = \overline{n}(\overline{p})$) that would occur in a Type 1 equilibrium. On the other hand, as the information-acquisition technology becomes perfect, the social cost from excessive information acquisition vanishes, and it is better to permit firms to investigate their applicants.

## 7 Discrimination

In order to explore the interplay between privacy and economic discrimination, consider a variant of the model in which the population is composed of two identifiable groups of consumers (e.g., males and females, minorities and non-minorities, or young and old). Suppose that one group has a larger proportion of high-cost individuals than the other group, $\lambda_H > \lambda_L$. Denote the fraction of the population in the high-risk group by $\theta \in (0, 1)$.

Left unregulated, firms will naturally discriminate economically between the two groups both with respect to price and information acquisition. Suppose in this case that a Type 1 equilibrium obtains in both market segments and denote the prices posted to the high and low-risk groups respectively by $\overline{p}_H$ and $\overline{p}_L$.[12]

**Lemma 5 (Economic Discrimination).** *In any Type 1 equilibrium, high-risk applicants face a higher price and receive lower expected utility than low-risk applicants; i.e.,* $\overline{p}_H > \overline{p}_L$ *and* $U_H(\overline{p}_H, \overline{n}_H(\overline{p}_H)) < U_L(\overline{p}_L, \overline{n}_L(\overline{p}_L))$.

The fact that consumers in the high-risk group have lower expected equilibrium utility than those in the low-risk group is not surprising. It is, however, somewhat striking that high-risk applicants are not necessarily investigated more intensively in equilibrium. While the direct affect on equilibrium search intensity from a rise in $\lambda$ is positive, there is a countervailing indirect affect

[12] It is straightforward to verify that a type 1 equilibrium will obtain in both market segments if and only if either $v > \lambda_H c_H + (1 - \lambda_H)c_L$ and $m < \min\{\lambda_L(1 - \lambda_L)(c_H - c_L), \lambda_H(1 - \lambda_H)(c_H - c_L)\}$, or $m < m_H^\dagger$.

associated with the rise in the equilibrium price. Hence, while high-risk applicants fare worse than low-risk ones on average, they do not necessarily face a higher probability of rejection.

In the U.S. and numerous other countries it is illegal to price discriminate with respect to characteristics such as gender, race, or age in many markets (e.g., credit, housing, or labor markets). It is interesting, therefore, to investigate the welfare consequences arising from a prohibition on price discrimination. Let $\overline{p}_M$ denote the equilibrium price that obtains when price discrimination is prohibited. Also, suppose that firms must substantiate rejection decisions by providing verifiable evidence that rejected applicants are actually unqualified.[13]

**Proposition 5 (Prohibiting Price Discrimination).** *Banning price discrimination raises the equilibrium expected utility of high-risk applicants, lowers the equilibrium expected utility of low-risk applicants, and induces firms to investigate high-risk applicants more intensively than low-risk ones.*

This result says that banning price (or wage) discrimination does unambiguously raise the *ex ante* welfare of high-risk consumers (or workers) and reduce the *ex ante* welfare of low-risk ones. Interestingly, it also says that prohibiting price discrimination accentuates quantity discrimination.[14]

In particular, high-risk applicants are subjected to more intense scrutiny and suffer disproportionately high rejection rates, $\overline{n}_H(\overline{p}_M) > \overline{n}_L(\overline{p}_M)$. The intuition underlying this is clear. Because $\overline{p}_M \in (\overline{p}_L, \overline{p}_H)$, the net cost to a firm of misallocating the good to a high-risk applicant rises and the net cost of misallocating to a low-risk applicant falls when price discrimination is prohibited. Hence, firms have stronger incentives to investigate high-risk applicants and weaker incentives to investigate low-risk ones. Moreover, because the cost of misallocating the good is $c_H - \overline{p}_M$ for both groups, and because there is a larger fraction of high-cost consumers in the high-risk group, the

[13] With out this assumption, the equilibrium price would be $\overline{p}_M = \overline{p}_L$, and firms would reject all applications by high-risk consumers without acquiring any information about them.

[14] There is a large empirical literature documenting quantity discrimination in markets where price discrimination by sex, race, or age is illegal. See, for example, Munnell *et al* (1996).

incentive to investigate high-risk applicants is unambiguously higher when price discrimination is banned.

## 8 Demand for Multiple Units

To this point, it has been assumed that consumers wish to purchase only a single unit of the good which they value at $v$. It is instructive, however, to consider 'smoother' preferences.

Suppose that each individual receives gross surplus of $v(q)$ from consuming $q \geq 0$ units of the good, where $v(\cdot)$ is increasing, strictly concave, and bounded. A firm that sells $q$ units to a consumer incurs cost of $c_H q$ or $c_L q$ depending on the consumer's type. In this context, the assumption that it is efficient to serve only low-cost consumers is written as $c_H > v'(0) > c_L$. At the beginning of the game, firms simultaneously announce contracts of the form $(p, q)$.[15] In other words, they commit to supply a qualified applicant with $q \geq 0$ units in exchange for a total payment of $p$. (Note that $p$ is a fixed tariff not a price per unit, although the results obviously hold under either specification.) All other aspects of the model are as in Section 2.

Analogous with (1), define the cost per accepted application to be

$$AC(q,n) \equiv \frac{\lambda(1-\alpha)^n c_H q + (1-\lambda)c_L q + kn}{\lambda(1-\alpha)^n + (1-\lambda)}.$$

The problem facing a social planner in this context is

$$\max_{(p,q,n)} U(p,q,n) \equiv \left(\lambda(1-\alpha)^n + (1-\lambda)\right)\left(v(q) - p\right) \quad \text{s.t. } p = AC(q,n). \tag{9}$$

It is easy to check that for $m$ sufficiently small, an interior optimum obtains and is characterized

[15] There is no loss in generality from assuming that each firm offers a single contract because the contract firms offer in equilibrium is the one that maximizes the consumer's continuation payoff. Hence, consumers would not choose a different contract if it were offered.

by the following (rearranged) first-order conditions:

$$\lambda(1-\alpha)^{n^*}(c_H q^* - v(q^*)) = m \tag{10}$$

and

$$v'(q^*) = \frac{\lambda(1-\alpha)^{n^*} c_H + (1-\lambda)c_L}{\lambda(1-\alpha)^{n^*} + (1-\lambda)}. \tag{11}$$

Condition (10) is a straightforward reformulation of condition (4); at $n^*$, the marginal social cost of misallocating $q^*$ units of the good equals the marginal social cost of acquiring more information about the applicant. Condition (11) is similarly intuitive; at $q^*$, the marginal utility of consumption equals the expected marginal cost of production.

Working backward through the game as in Section 4 yields the unique equilibrium outcome in this setting, $(\overline{p}, \overline{q}, \overline{n})$.

**Proposition 6 (The Consumption Distortion).** *In any Type 1 equilibrium,* $\overline{p} = AC(\overline{q}, \overline{n})$ *and the following inequalities are satisfied:*

$$\lambda(1-\alpha)^{\overline{n}}(c_H \overline{q} - v(\overline{q})) < m$$

*and*

$$v'(\overline{q}) > \frac{\lambda(1-\alpha)^{\overline{n}} c_H + (1-\lambda)c_L}{\lambda(1-\alpha)^{\overline{n}} + (1-\lambda)}.$$

The first inequality in Proposition 6 indicates that, given the equilibrium consumption level specified in the contract, firms infringe consumer privacy too much. In other words, holding fixed $q = \overline{q}$, it is possible to generate and *ex ante* Pareto improvement by reducing the search intensity from $\overline{n}$ to some value $\tilde{n}$ and increasing the price to $p = AC(\overline{q}, \tilde{n})$. This inefficient infringement of privacy stems from the lack of commitment and competitive pressure discussed above in Section 5.

The second inequality in Proposition 6 indicates that, given the equilibrium search intensity, consumers demand too few units of output. In other words, the marginal utility of consumption is higher than the expected marginal cost of production. So, holding fixed $n = \overline{n}$, it is possible to generate an *ex ante* Pareto improvement by increasing the consumption level specified in the contract from $\overline{q}$ to some level $\tilde{q}$ and raising the price to $p = AC(\tilde{q}, \overline{n})$.[16] This distortion also stems from the non-contractibility of $n$. In particular, consumers understand that if they demand less output, then the firms will investigate them less intensively. In an effort, therefore, to preserve privacy, consumers purchase an inefficiently small quantity of the good. By demanding a smaller amount of output, consumers reduce the cost to a firm of mistakenly selling to a high-cost applicant and thereby soften the incentives for information acquisition.

## 9 Searching for Good News

In this section, a variant of the model is studied in which firms have access to a different type of information acquisition technology. Specifically, prior to this point it has been assumed that firms search for 'bad news' in the sense that a single negative piece of information (e.g., a criminal conviction) reveals an applicant to be type $c_H$ with certainty. Suppose to the contrary that a single piece of 'good news' (e.g., a positive reference) reveals an applicant to be type $c_L$. That is, consider $n$ conditionally independent Bernoulli signals, $Y_1, \ldots, Y_n$, where

$$\Pr\{Y_i = 1|c\} = \begin{cases} \alpha, & \text{if } c = c_L, \\ 0, & \text{if } c = c_H \end{cases}$$

[16] Note that this implies that the contract $(\overline{p}, \overline{q})$ is not renegotiation-proof because once a firm judges an applicant to be qualified, both parties prefer a larger consumption level than $\overline{q}$. Renegotiation-proof contracts are undoubtedly relevant in some important markets, but space constraints do not permit their treatment here.

This information structure corresponds to searching for good news in the sense that it admits no false positives. Hence, firms collect information on their applicants and approve them if and only if they observe at least one favorable signal ($Y_i = 1$).

The expected payoff to an applicant in this setting is

$$\widetilde{U}(p, n) \equiv (1 - \lambda)\left(1 - (1 - \alpha)^n\right)(v - p). \tag{12}$$

As before, the first term in this expression is the probability of having his application approved and the second term is the surplus from purchasing the good. The cost to a firm per accepted application is

$$\widetilde{AC}(n) = c_L + \frac{kn}{(1 - \lambda)\left(1 - (1 - \alpha)^n\right)}.$$

When $m$ is sufficiently small, the socially efficient search intensity is characterized by the first-order condition

$$(1 - \lambda)(1 - \alpha)^{n^{**}}(v - c_L) = m.$$

It is useful to compare this with the condition defining the optimal sample size when searching for bad news, (4). When searching for bad news, the marginal social benefit of information acquisition derives from identifying a type $c_H$ applicant and denying him the product (saving surplus of $c_H - v$). When searching for good news, by contrast, the marginal social benefit of information acquisition derives from identifying a type $c_L$ applicant and allocating him the product (generating surplus of $v - c_L$).

A firm that posts price $p$ selects its search intensity, $\tilde{n}(p)$, according to the first-order condition

$$(1 - \lambda)(1 - \alpha)^{\tilde{n}(p)}(p - c_L) = m. \tag{13}$$

**Proposition 7 (Insufficient Information Acquisition).** *The unique equilibrium outcome when firms search for good news involves positive information acquisition* iff *the market is* ex ante *viable and* $m < \lambda(1-\lambda)(c_H - c_L)$*, or the market is not* ex ante *viable and* $m < (1-\lambda)(v - c_L)$*. The equilibrium price is*

$$\tilde{p} = c_L + \sqrt{\frac{m(v - c_L)}{1-\lambda}}.$$

*Moreover,* $\tilde{n}(\tilde{p}) < n^{**}$ *(i.e., firms acquire too little information about their applicants).*

Proposition 3 of Section 5 shows that if firms search for bad news about their applicants, then they collect too much information in equilibrium. By contrast, Proposition 7 shows that if firms search for good news, then they collect too little information. The upshot in either case is that too few applicants are approved.

The reason firms collect too little information when searching for good news is easily understood. As noted above, mistakes in this environment involve not identifying some of the low-cost consumers. The social cost of each mistake is, therefore, $v - c_L$. The private cost to a firm, however, is $\tilde{p} - c_L$. Since $\tilde{p} < v$, the private cost of making a mistake is smaller than the social cost and firms, therefore, acquire too little information.

An interesting feature of the equilibrium outcome characterized in Proposition 7 is that firms earn strictly positive profit. The reason competition fails in this setting is easily explained. When firms search for good news, a consumer's expected utility in the continuation equilibrium, $\widetilde{U}(p, \tilde{n}(p))$, is not monotone decreasing. In particular, it is increasing for prices less than $\tilde{p}$ and decreasing for higher prices. Hence, a firm setting a price less than $\tilde{p}$ will attract no applicants. Low prices induce low levels of information acquisition and, therefore, result in very low probability of acceptance.

## 10 Adverse Selection

In this section, the original setting in which firms search for bad news is modified by supposing that rejected applicants remain in the market and reapply to other firms. This requires some modification of the basic model presented in Section 2. In particular, it is necessary to add a dynamic component and (for the sake of tractability) to suppose that each firm is small relative to the market.

Consider a time horizon running from $-\infty$ to $+\infty$. In each period $t$, a continuum of consumers with total measure equal to one enters the market. Each consumer wishes to purchase one unit of the good which he values at $v$. A fraction $\lambda$ of the entering consumers have cost $c_H > v$ and the complementary fraction have cost $c_L < v$. Consumers who purchase the good exit the market. Those who do not purchase the good exit the market and receive payoffs of zero with probability $(1-\rho) \in (0,1)$ in each period.

There is a continuum of identical firms with total measure greater than $1/(1-\rho)$ (the largest possible measure of consumers in the market). Each firm has production capacity of one unit per period.[17]

At the beginning of each period $t$, the firms simultaneously post prices. Next, the consumers in the market simultaneously decide whether to apply to one of the firms or to remain idle in the current period. If a consumer applies to buy the good, then the firm to which he applies acquires information and either approves or rejects his application. If the firm approves his application, then trade takes place and the consumer exits the market. If the firm rejects the consumer's application, then the consumer remains in the market with probability $\rho$ in which case he may apply to buy the good from a different firm in the next period. Firms do not share information about rejected

[17] It is important that there is always excess capacity in the market so that a firm that posts a price above the competitive level does not attract an applicant.

applications. All parties possess discount factor $\delta < 1$.

For notational convenience, define

$$\phi \equiv 1 - (1 - \alpha)^n$$

to be the probability of detecting a high cost consumer (called the screening intensity). Let $\mu_t$ denote the firms' belief about the fraction of high-cost consumers in the applicant pool at the beginning of period $t$. The solution concept is defined as follows.

**Definition 3 (Steady-State Markov Equilibrium).** A steady-state Markov equilibrium consists of a price function, $\hat{p}(\mu_t)$, a screening-intensity function, $\hat{\phi}(p, \mu_t)$, and beliefs by market participants satisfying the following conditions.

**(i)** It is optimal for all firms to post the price $\hat{p}(\mu_t)$ and screen according to $\hat{\phi}(p, \mu_t)$ in every period.

**(ii)** It is optimal, given his beliefs, for a consumer to apply for the good at the lowest price posted in every period.

**(iii)** On the path of play, beliefs are correct and stationary; i.e., there exists $\hat{\mu} \in [0, 1]$ such that $\mu_t = \hat{\mu}$ for all $t$.

The first thing to note is that the actions of an individual firm in period $t$ have negligible impact on the composition of the applicant pool and, therefore, do not influence future states $\mu_{t+1}, \mu_{t+2}, \ldots$. Hence, in a Markov equilibrium, firms simply maximize current profit in each period. This means that the screening intensity function of a firm that posts price $p$ is found by solving

$$\max_{\phi \geq 0} \Pi(p, \phi; \mu_t) = (1 - \phi)\mu_t(p - c_H) + (1 - \mu_t)(p - c_L) + m \ln(1 - \phi).$$

The first-order condition yields

$$\hat{\phi}(p,\mu_t) = \begin{cases} 1 - \dfrac{m}{\mu_t(c_H - p)}, & \text{if } m < \mu_t(c_H - p) \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Next, if consumers believe that market prices will remain constant over time (which is true in a steady-state), then they will apply to purchase the good at every opportunity. Since (by Lemma 2) consumers apply to the firms posting the lowest price in any period, equilibrium profits are zero. The equilibrium price function, $\hat{p}(\mu_t)$, is, therefore, defined implicitly by the condition

$$\Pi(\hat{p}(\mu_t), \hat{\phi}(\hat{p}(\mu_t), \mu_t); \mu_t) = 0. \tag{15}$$

On the path of play in a steady-state Markov equilibrium, the applicant pool at the beginning of each period consists of the new arrivals in the market along with all the high-cost consumers who have not previously had their applications approved and have not otherwise exited the market. (Low-cost consumers are always approved in the period when they arrive.) The measure of high-cost consumers in the applicant pool, therefore, is

$$\lambda \left(1 + \rho\hat{\phi} + (\rho\hat{\phi})^2 + \ldots\right) = \frac{\lambda}{1 - \rho\hat{\phi}},$$

where $\hat{\phi} = \hat{\phi}(\hat{p}(\hat{\mu}), \hat{\mu})$. The proportion of high-cost consumers in the applicant pool in a steady-state Markov equilibrium, therefore, is

$$\hat{\mu} = \frac{\lambda}{1 - (1-\lambda)\rho\hat{\phi}}. \tag{16}$$

Observe that adverse selection (i.e., $\hat{\mu} > \lambda$) obtains unless $\rho = 0$ (all consumers exit the market after one period) or $\hat{\phi} = 0$ (applicants are not screened).

Setting $\mu_t = \hat{\mu}$ in (14) and (15) yields three equations in the steady-state equilibrium variables $\hat{p}$, $\hat{\phi}$, and $\hat{\mu}$. A steady-state Markov Equilibrium corresponds to a solution to this system of equations satisfying $\hat{p} < v$. In order to characterize such a solution, substitute from (14) into (16) to get the function

$$\mu(p, \rho) \equiv \frac{\lambda(c_H - p) - (1 - \lambda)\rho m}{(1 - (1 - \lambda)\rho)(c_H - p)}, \tag{17}$$

for $p < p^\dagger$ and $\mu(p, \rho) \equiv \lambda$ for $p \geq p^\dagger$, for all $p \in [c_L, c_H]$ and $\rho \in [0, 1]$. Now, for a given value of $\rho$, consider the function

$$\gamma(p) \equiv \Pi(p, \hat{\phi}(p, \mu(p, \rho)); \mu(p, \rho)), \quad p \in [c_L, c_H].$$

Any value $\hat{p} < v$ for which $\gamma(\hat{p}) = 0$ constitutes a steady-state Markov Equilibrium price. The corresponding equilibrium proportion of high-cost consumers in the applicant pool is $\mu(\hat{p}, \rho)$ and the equilibrium screening intensity is $\hat{\phi}(\hat{p}, \mu(\hat{p}, \rho))$.

**Proposition 8 (Existence and Characterization). (i)** *If $\gamma(v) > 0$ and $m < \lambda(1-\lambda)(c_H - c_L)$, then there exists a unique steady-state Markov equilibrium outcome. In particular, firms screen their applicants and sell only to the ones who appear qualified.*

**(ii)** *If $\gamma(v) > 0$ and $m \geq \lambda(1 - \lambda)(c_H - c_L)$, then there exists a unique steady-state Markov equilibrium outcome. In particular, firms do not screen their applicants; i.e., $\hat{\phi} = 0$, $\hat{\mu} = \lambda$, and $\hat{p} = \lambda c_H + (1 - \lambda)c_L$.*

**(iii)** *If $\gamma(v) \leq 0$, then no steady-state Markov equilibrium exists.*

Consider the parameter $\rho$, the probability that a rejected applicant remains in the market. If $\rho = 0$, then the steady-state Markov equilibrium outcome characterized in Proposition 8 corresponds exactly to the equilibrium outcome of the static game analyzed in section 4. As $\rho$ rises, however, adverse selection becomes increasingly problematic if the equilibrium involves information acquisition because the stock of rejected (high-cost) consumers in the applicant pool grows.

**Proposition 9 (Adverse Selection).** *If* $\rho = 0$*, then* $\hat{p} = \overline{p}$*,* $\hat{\mu} = \lambda$*, and*

$$\hat{\phi} = 1 - (1 - \alpha)^{\overline{n}(\overline{p})}.$$

*Moreover, if* $m < \lambda(1 - \lambda)(c_H - c_L)$*, then the following comparative statics obtain for all* $\rho \in [0, 1]$*:*

**(i)** $\partial\hat{p}/\partial\rho > 0$*,*

**(ii)** $\partial\hat{\mu}/\partial\rho > 0$*,*

**(iii)** $\partial\hat{\phi}/\partial\rho > 0$*.*

This result is intuitive. It says that when $\rho > 0$, systematic differences between a steady-state Markov equilibrium with information acquisition and a Type 1 equilibrium of the static game emerge. Specifically, $\rho > 0$ implies a higher proportion of high-cost consumers in the applicant pool, a higher screening intensity, and a higher price in equilibrium. In particular, the direct effect of a rise in $\rho$ is to raise the stock of rejected consumers who remain in the applicant pool. This, in turn, increases incentives for firms to screen. Finally, the price rises to account both for the rise in information-acquisition costs and the fact that more high-cost applicants are (in spite of the increased screening intensity) mistakenly allocated the good.

In order to highlight the impact of adverse selection, consider the limiting situation in which $\rho \to 1$ . In this case, the zero-profit condition (15) can be recast as

$$\hat{p} = \lambda c_H + (1 - \lambda)c_L - m \ln(1 - \hat{\phi}) \left( \frac{\lambda}{1 - \hat{\phi}} + (1 - \lambda) \right).$$

This is disturbing. First of all, it implies that a steady-state Markov equilibrium with information acquisition does not exist unless the market is *ex ante* viable, and even when it exists, it is very wasteful. Specifically, the equilibrium price equals the unconditional expected cost of selling to a new consumer plus the cost of screening all applicants. To see why this holds, note that all $(1-\lambda)$ of the low-cost consumers purchase the good as soon as they enter the market. Now, the stock of high-cost consumers in the market at the beginning of each period is $\lambda/(1-\hat{\phi})$, and the measure of these who are mistakenly allowed to purchase the good is $\lambda$. Hence, the total revenue earned in the market, $\hat{p}$, equals the total cost of production, $\lambda c_H + (1-\lambda)c_L$, plus the cost of screening all applicants, $-m\ln(1-\hat{\phi})\left(\lambda/(1-\hat{\phi})+(1-\lambda)\right)$. Observe, however, that if every firm stopped screening, then the competitive price in the new steady state would be $\lambda c_H + (1-\lambda)c_L$. The problem, of course, is that it is not an equilibrium for all firms to stop acquiring information. Hence, the equilibrium outcome has a Prisoners'-Dilemma flavor in which all firms devote resources to screening applicants and yet each firm makes it share of mistakes and sells to just as many high-cost consumers as in a setting where no firm collected any information at all.

The problem here stems from a classical externality. When a firm chooses its screening intensity, it does not account for the adverse impact of its decision on the other firms in the industry. Specifically, when a firm learns that an applicant is high-cost and rejects him, it returns him to the applicant pool where he will continue to apply to other firms. Hence, when rejected consumers remain in the market, there is an even sharper divergence between the social and private benefit of information acquisition. In particular, when $\rho$ is sufficiently high, then it is socially optimal to acquire no information at all. Nevertheless, firms possess incentives to screen applicants and dump their rejects back into the applicant pool.

This discussion points to an important distinction between the acquisition and the sharing of information. While it often is efficient to preserve privacy by inducing firms to collect less information, it may be important to allow them to 'share' the information they collect.[18] Hence, it may make sense to concentrate consumer data in a few key repositories with easy access by all firms in the industry. The question is, just how much personal data should be available at such a site.

## 11 Conclusion

A theory of privacy and information acquisition in competitive markets based on incomplete contracts was explored. The model does not involve an inherent preference for privacy on the part of individuals (although this would strengthen the findings). Rather, firms were assumed to demand cost-relevant information about the consumers applying to purchase their output and to use this information to decide which of them are qualified. The consumers, themselves, also possess some uncertainty about their own characteristics, and they, therefore, face a trade-off. Specifically, the price a consumer pays for the good – conditional on being judged qualified to buy it – initially decreases in the amount of information firms acquire about him. On the other hand, the probability of being judged unqualified is increasing in the level of information acquisition. There is typically a unique efficient level of privacy that is characterized by equality between the marginal social cost of misallocating the good and the marginal social cost of acquiring more information.

It was shown that if firms search for bad news about applicants in a setting where information acquisition levels are non-contractible, then they will compete 'too aggressively' in the sense that they post the lowest price consistent with zero economic profit. Unfortunately, this low price gives them incentives to acquire excessive amounts of information. In other words, all consumers would be better off *ex ante* if the firms posted higher prices and acquired less information. This inefficient infringement of privacy arises because firms do not account for the consumer surplus earned by

[18] Pagano and Jappelli (1993) stress the importance of information sharing in credit markets.

high-cost applicants who are mistakenly sold the good at the competitive price. Hence, there is a divergence between the social and private benefit of information acquisition. In situations where the efficient level of information acquisition is low, it may even be socially beneficial to grant consumers the right to complete privacy rather than suffer the excessive information acquisition that would otherwise result. It was also shown that consumers may preserve some privacy by demanding inefficiently low levels of output.

Economic discrimination was investigated in a setting with two groups of consumers, a high-risk group and a low-risk group. In the absence of regulation, members of the high-risk group face a higher equilibrium price than members of the low-risk group. Banning price discrimination accentuates quantity discrimination by leading to a situation in which high-risk applicants are subjected to more scrutiny and suffer disproportionately high rejection rates, although their overall welfare rises.

In a setting where firms search for good news about applicants, prices are excessively high and information-acquisition levels too low in equilibrium. This inefficiency occurs because firms do not account for the consumer surplus earned by low-cost applicants who are mistakenly rejected. Again, there is a divergence between the social and private benefit of information acquisition.

Finally, a setting in which rejected applicants remain in the market and apply to firms unaware of their earlier rejections was considered. The resulting adverse selection was shown to be potentially very severe, either causing the market to shut down or to generate a situation in which information acquisition is entirely wasteful.

There are, of course, many aspects of privacy that were not considered here: the nuisance associated with telemarketing calls, junk mail and spam; the hassles arising from inaccuracies in credit reports and government files; and the potentially devastating effects of identity theft, to name a few. Privacy is a complex concept involving basic notions of human dignity, fairness and freedom. It is probably not possible to capture all its facets in a single model, and no attempt was made to do so here. Rather, this investigation was focused on a single – but important – aspect of privacy, the incentives for collection of personal information by participants in a competitive industry. A multitude of other important privacy issues awaits future work.

# Appendix

This appendix contains the proofs of all results presented in the text.

### Lemma 1

*Proof.* Suppose that (3) holds and substitute from (4) into $W(n)$ to obtain the welfare from implementing Policy 1

$$W^* = -m + (1-\lambda)(v - c_L) + m \ln\left(\frac{m}{\lambda(c_H - v)}\right). \tag{A1}$$

Differentiating this with respect to $m$ yields

$$\frac{\partial W^*}{\partial m} = \ln\left(\frac{m}{\lambda(c_H - v)}\right).$$

This is strictly negative for $m \in (0, \lambda(c_H - v))$. Moreover, evaluating $W^*$ at $m = \lambda(c_H - v)$ reveals

$$W^* = v - \lambda c_H - (1-\lambda)c_L.$$

This is obviously non-positive if and only if the market is not *ex ante* viable. Moreover,

$$\lim_{m \to 0} W^* = (1-\lambda)(v - c_L) > 0.$$

Hence, $W^*$, which is continuous in $m$, is positive at $m = 0$, decreases monotonically, and is non-positive at $m = \lambda(c_H - v)$. ■

### Proposition 1

*Proof.* Each part is proven in turn:

**(i)** First, suppose that the market is *ex ante* viable and (3) holds. It was shown in the text that if (3) holds, then Policy 1 strictly dominates Policy 2. Moreover, the welfare from implementing Policy 2 is $v - \lambda c_H - (1-\lambda)c_L$, which is positive. Hence, Policy 2 dominates Policy 3.

Next, suppose that the market is not *ex ante* viable and $m < m^\dagger$. Since the market is not *ex ante* viable, Policy 3 dominates Policy 2. However, by Lemma 1, $W^* > 0$, so Policy 1 dominates Policy 3.

**(ii)** Suppose that the market is *ex ante* viable and (3) fails. It was shown in the text that if (3) fails, then Policy 2 dominates Policy 1. Moreover, the welfare from implementing Policy 2 is $v - \lambda c_H - (1-\lambda)c_L$, which is positive. Hence, Policy 2 dominates Policy 3 in this case.

**(iii)** Suppose that the market is not *ex ante* viable and $m \geq m^\dagger$. The welfare from implementing Policy 2 is $v - \lambda c_H - (1-\lambda)c_L$, which is non-positive. Hence, Policy 3 dominates Policy 2. Moreover, Lemma 1 indicates that $W^* \leq 0$, so Policy 3 dominates Policy 1 as well.

■

### Lemma 2

*Proof.* Suppose that $p < p^\dagger$ and substitute from (6) into (A2) to get

$$U(p, \overline{n}(p)) = \left( \frac{m}{c_H - p} + (1 - \lambda) \right) (v - p). \tag{A2}$$

Differentiation yields

$$\frac{dU(p, \overline{n}(p))}{dp} = \frac{m(v - c_H)}{(c_H - p)^2} - (1 - \lambda).$$

This is negative because $c_H > v$. For $p \geq p^\dagger$, $U(p) = v - p$, which is obviously decreasing. ■

### Lemma 3

*Proof.* First, suppose that the market is not *ex ante* viable and $m \geq m^\dagger$. By way of contradiction, suppose that the set of relevant prices is non-empty. By Proposition 1, it is efficient to abandon the market and obtain first-best welfare of zero. If a firm alone sets the lowest relevant price $p$, then all the consumers apply to it and earn aggregate consumer surplus of

$$U(p, \overline{n}(p)) = \left( \lambda(1 - \alpha)^{\overline{n}(p)} + (1 - \lambda) \right) (v - p) > 0.$$

The total surplus earned in the market cannot exceed first-best welfare. Hence,

$$U(p, \overline{n}(p)) + \Pi(p, \overline{n}(p)) \leq 0.$$

It follows, therefore, that $\Pi(p, \overline{n}(p)) < 0$, which contradicts the supposition.

Next, suppose that the market is *ex ante* viable or that $m < m^\dagger$. Applying the Envelope Theorem to (8) gives

$$\frac{\partial \Pi(p, \overline{n}(p))}{\partial p} = \lambda(1 - \alpha)^{\overline{n}(p)} + (1 - \lambda) > 0.$$

Also,

$$\Pi(c_L, \overline{n}(c_L)) = \lambda(1 - \alpha)^{\overline{n}(c_L)}(c_L - c_H) - k\overline{n}(c_L) < 0.$$

Hence, $\Pi(p, \overline{n}(p))$ is strictly increasing and negative at $p = c_L$. Since it is evidently continuous, the result follows from observing that $\Pi(v, n) = W(n)$ and $W(n^*) > 0$. ■

### Proposition 2

*Proof.* For cases (i) and (ii), Lemma 3 reveals that there is a unique zero-profit price $\overline{p} < v$. Standard Bertrand-style arguments (see Tirole 1988 pp. 209–11) then establish that in any equilibrium at least two firms post a price of $\overline{p}$ and no firm posts a lower price, and such a constellation of prices is an equilibrium. Consumers apply to the low price firms by Lemma 2. Firms acquire information according to (6).

For case (iii) observe that the set of relevant prices is empty. In other words, there exists no price that yields applicants positive expected utility in the continuation equilibrium. ■

### Proposition 4

*Proof.* Differentiate (1) to get

$$AC'(n) = \frac{\lambda \ln(1-\alpha)(1-\alpha)^n((1-\lambda)(c_H - c_L) - kn) + k(\lambda(1-\alpha)^n + (1-\lambda))}{(\lambda(1-\alpha)^n + (1-\lambda))^2}. \quad \text{(A3)}$$

Evaluating this at $n = 0$ gives

$$AC'(0) = -\ln(1-\alpha)(m - \lambda(1-\lambda)(c_H - c_L)).$$

This is obviously negative when $m < \lambda(1-\lambda)(c_H - c_L)$. Suppose, therefore, that the market is not *ex ante* viable but that $m < m^\dagger$. Because the market is not *ex ante* viable and because $0 < m^\dagger < \lambda(c_H - v)$, it follows that

$$(1-\lambda)(v - \lambda c_H - (1-\lambda)c_L) + m^\dagger \ln\left(\frac{m^\dagger}{\lambda(c_H - v)}\right) < 0.$$

Rearranging this gives

$$(1-\lambda)(v - c_L) + m^\dagger \ln\left(\frac{m^\dagger}{\lambda(c_H - v)}\right) - \lambda(1-\lambda)(c_H - c_L) < 0.$$

Substituting for the first two terms from the definition of $m^\dagger$ (i.e., set the right side of (A1) equal to zero) gives

$$m^\dagger - \lambda(1-\lambda)(c_H - c_L) < 0.$$

Since $m < m^\dagger$ it follows that $AC'(0) < 0$. Hence, $AC(n)$ is initially decreasing. Moreover,

$$\lim_{n\to\infty} AC(n) = +\infty.$$

So, $AC$ attains a global minimum at some critical point in $\mathbb{R}_+$ where $AC' = 0$.

Next, rewrite (A3) in the form

$$AC'(n) = \left(\frac{\lambda \ln(1-\alpha)(1-\alpha)^n}{(\lambda(1-\alpha)^n + (1-\lambda))^2}\right)\left((1-\lambda)(c_H - c_L) - kn - m\left(1 + \frac{(1-\lambda)}{\lambda(1-\alpha)^n}\right)\right). \quad \text{(A4)}$$

The first term is evidently negative. Hence, the second term must equal zero at a critical point. Moreover, at any critical point, the sign of $AC''$ must equal the sign of

$$\frac{d}{dn}\left(kn + m\frac{(1-\lambda)}{\lambda(1-\alpha)^n}\right) = k + k\left(\frac{(1-\lambda)}{\lambda(1-\alpha)^n}\right),$$

which is positive. Hence, there is a single critical point, $\tilde{n}$, at which $AC'(\tilde{n}) = 0$, and it corresponds to the global minimum of $AC$.

Finally, set the second term in (A4) equal to zero and multiply through by

$$-\frac{\lambda(1-\alpha)^{\tilde{n}}}{\lambda(1-\alpha)^{\tilde{n}} + (1-\lambda)}$$

to get

$$m - \lambda(1-\alpha)^{\tilde{n}} \left( \frac{(1-\lambda)(c_H - c_L) - k\tilde{n}}{\lambda(1-\alpha)^{\tilde{n}} + (1-\lambda)} \right) = 0,$$

or

$$m - \lambda(1-\alpha)^{\tilde{n}} \left( c_H - \frac{\lambda(1-\alpha)^{\tilde{n}} c_H + (1-\lambda)c_L + k\tilde{n}}{\lambda(1-\alpha)^{\tilde{n}} + (1-\lambda)} \right) = 0,$$

or

$$\lambda(1-\alpha)^{\tilde{n}}(c_H - AC(\tilde{n})) = m.$$

Since firms earn zero profit in equilibrium we have that $\overline{p} = AC(\overline{n}(\overline{p}))$. Substituting into (6) gives

$$\lambda(1-\alpha)^{\overline{n}(\overline{p})}(c_H - AC(\overline{n}(\overline{p}))) = m.$$

Hence, $\overline{n}(\overline{p}) = \tilde{n}$. ■

### Proposition 3

*Proof.* First, suppose the market is *ex ante* viable and $\lambda(c_H - v) \leq m < \lambda(1-\lambda)(c_H - c_L)$. Then $\overline{n}(\overline{p}) > 0$ by Lemma 4, while $n^* = 0$ by Proposition 1.

Next, suppose either that the market is *ex ante* viable and $m < \lambda(v - c_H)$ or that the market is not *ex ante* viable and $m < m^\dagger$. Then, the efficient level of information acquisition is given in (4) and the equilibrium level is given in (6). Comparing these equations reveals $\overline{n}(v) = n^*$. By Lemma 3, $\overline{p} < v$. The result then follows from the fact that $\overline{n}(p)$ is strictly decreasing. ■

### Proposition 4

*Proof.* If the market is *ex ante* viable and consumers have complete privacy, then Bertrand competition will clearly result in equilibrium welfare of $v - \lambda c_H - (1-\lambda)c_L$.

**(i)** Suppose $m \in [\lambda(c_H - v), \lambda(1-\lambda)(c_H - c_L))$. In this case, Proposition 2 indicates that a Type 1 equilibrium with $\overline{n}(\overline{p}) > 0$ will prevail. Observe, however, that Proposition 1 indicates that Policy 2 (acquire no information and allocate the good to all consumers) is socially optimal.

**(ii)** The welfare deriving from a Type 1 equilibrium is

$$\overline{W} \equiv \left(\lambda(1-\alpha)^{\overline{n}(\overline{p})} + (1-\lambda)\right)(v - \overline{p}).$$

Substituting from (6) renders this as

$$\overline{W} \equiv \left(\frac{m}{c_H - \overline{p}} + (1-\lambda)\right)(v - \overline{p}).$$

Observe that

$$\lim_{m \to 0} \overline{W} = (1-\lambda)(v - c_L) > v - \lambda c_H - (1-\lambda)c_L.$$

Since $\overline{W}$ is evidently continuous in $m$, this establishes the claim.

■

### Lemma 5

*Proof.* For a general value of $\lambda$, the price in a Type 1 equilibrium is found by substituting (6) into the zero-profit condition

$$\Pi(\overline{p}, \overline{n}(\overline{p})) = -m + (1-\lambda)(\overline{p} - c_L) + m \ln\left(\frac{m}{\lambda(c_H - \overline{p})}\right) = 0. \tag{A5}$$

Implicit differentiation yields

$$\frac{\partial \overline{p}}{\partial \lambda} = \frac{(c_H - \overline{p})\,(\lambda(\overline{p} - c_L) + m)}{\lambda\,((1-\lambda)(c_H - \overline{p}) + m)} > 0. \tag{A6}$$

Hence, $\lambda_H > \lambda_L$ implies $\overline{p}_H > \overline{p}_L$. Next, differentiate (A2) with respect to $\lambda$ to get

$$\frac{\partial U(\overline{p}, \overline{n}(\overline{p}))}{\partial \lambda} = -(v - \overline{p}) + \frac{dU(\overline{p}, \overline{n}(\overline{p}))}{d\overline{p}} \frac{\partial \overline{p}}{\partial \lambda}.$$

This is negative by Lemma 2 and (A6). ■

### Proposition 5

*Proof.* Lemma 5 reveals $\overline{p}_L < \overline{p}_H$. Hence, the result will follow from (6) and from Lemma 2 if it can be shown that $\overline{p}_M \in (\overline{p}_L, \overline{p}_H)$. Expected equilibrium profit to a typical firm is

$$\begin{aligned}&\theta\left(\lambda_H(1-\alpha)^{\overline{n}_H}(\overline{p}_M - c_H) + (1-\lambda_H)(\overline{p}_M - c_L) - k\overline{n}_H\right)\\ &+(1-\theta)\left(\lambda_L(1-\alpha)^{\overline{n}_L}(\overline{p}_M - c_H) + (1-\lambda_L)(\overline{p}_M - c_L) - k\overline{n}_L\right).\end{aligned}$$

Substituting for $\overline{n}_H$ and $\overline{n}_L$ from (6) and setting profit equal to zero implicitly defines the equilibrium price

$$\begin{aligned}&\theta\left[-m + (1-\lambda_H)(\overline{p}_M - c_L) + m\ln\left(\tfrac{m}{\lambda_H(c_H - \overline{p}_M)}\right)\right]\\ &+(1-\theta)\left[-m + (1-\lambda_L)(\overline{p}_M - c_L) + m\ln\left(\tfrac{m}{\lambda_L(c_H - \overline{p}_M)}\right)\right] = 0.\end{aligned}$$

Both terms in square brackets are evidently increasing in $\overline{p}_M$. Moreover, the first term is zero when $\overline{p}_M = \overline{p}_H$ and the second term is zero when $\overline{p}_M = \overline{p}_L$. Hence, the average of the two terms is zero only if $\overline{p}_M \in (\overline{p}_L, \overline{p}_H)$. ■

### Proposition 6

*Proof.* Suppose a firm posts the contract $(p, q)$. Then, it will acquire information about its applicants in accordance with the (familiar) first-order condition

$$\lambda(1-\alpha)^n(c_H q - p) = m. \tag{A7}$$

The following three claims are needed to prove the result.

**Claim 1.** In a Type 1 equilibrium, $\overline{p} = AC(\overline{n}, \overline{q})$. To see this, suppose by way of contradiction that $\overline{p} > AC(\overline{n}, \overline{q})$, and let $\Sigma > 0$ be aggregate equilibrium profit. The least profitable firm earns profit no greater than $\Sigma/2$. However, by choosing $\epsilon > 0$ sufficiently small, it could earn profit arbitrarily close to $\sigma$ by offering the contract $(\overline{p} - \epsilon, \overline{q})$. In particular, it would acquire information about applicants in accordance with (A7) and all consumers would apply to it by Lemma 2.

**Claim 2.** In a Type 1 equilibrium, $(\overline{n}, \overline{p}, \overline{q})$ solves (9) subject to $p = AC(n,q)$ and (A7). To see this, suppose by way of contradiction that it is not true and that $(\hat{n}, \hat{p}, \hat{q})$ solves the maximization problem. By Claim 1, firms earn zero profit at $(\overline{n}, \overline{p}, \overline{q})$, and they acquire information in accordance with (A7). In other words, $(\overline{n}, \overline{p}, \overline{q})$ satisfies the constraints but does not maximize the objective; i.e.,

$$\left(\lambda(1-\alpha)^{\overline{n}} + (1-\lambda)\right)(v(\overline{q}) - \overline{p}) < \left(\lambda(1-\alpha)^{\hat{n}} + (1-\lambda)\right)(v(\hat{q}) - \hat{p}).$$

Consider a firm that deviates by offering the contract $(\hat{p}+\epsilon, \hat{q})$ for some $\epsilon > 0$. Let $\hat{n}_\epsilon$ be the search intensity satisfying (A7) under this contract. Then, for $\epsilon$ sufficiently small

$$\left(\lambda(1-\alpha)^{\overline{n}} + (1-\lambda)\right)(v(\overline{q}) - \overline{p}) < \left(\lambda(1-\alpha)^{\hat{n}_\epsilon} + (1-\lambda)\right)(v(\hat{q}) - \hat{p} - \epsilon).$$

Hence, all consumers will apply to the deviating firm which will make positive profit because (by the Envelope Theorem)

$$\frac{d}{dp}\left(\lambda(1-\alpha)^{\hat{n}}(\hat{p} - c_H\hat{q}) + (1-\lambda)(\hat{p} - c_L\hat{q}) - k\hat{n}\right) = \lambda(1-\alpha)^{\hat{n}} + (1-\lambda) > 0.$$

**Claim 3.** In a Type 1 Equilibrium, it cannot be the case that $\overline{n} = n^*$ and $\overline{q} = q^*$. To see this, suppose by way of contradiction that $\overline{n} = n^*$ and $\overline{q} = q^*$. Then, $\overline{p} = p^*$ by Claim 1. In a Type 1 equilibrium, however, $v(q^*) > p^*$. But, inspection of (10) and (A7) shows that this implies $\overline{n} > n^*$.

Claim 2 indicates that $(\overline{n}, \overline{q})$ is found by solving the Lagrangian

$$\max_{(n,q,\gamma)} \lambda(1-\alpha)^n(v(q) - c_Hq) + (1-\lambda)(v(q) - c_Lq) - kn + \gamma(m - \lambda(1-\alpha)^n(c_Hq - AC(n,q))).$$

Substituting the solution into the first-order condition for $n$ and rearranging gives

$$\lambda(1-\alpha)^{\overline{n}}(c_H\overline{q} - v(\overline{q})) = m - \overline{\gamma}(\lambda(1-\alpha)^{\overline{n}}(c_H\overline{q} - AC(\overline{n}, \overline{q})) + z),$$

where

$$z \equiv -\frac{\lambda(1-\alpha)^{\overline{n}}}{\ln(1-\alpha)}\frac{\partial AC(\overline{n}, \overline{q})}{\partial n}.$$

By the same argument as in the proof of Proposition 4, the function $AC(n, \overline{q})$ is U-shaped and attains its minimum at $n = \overline{n}$. Hence,

$$\frac{\partial AC(\overline{n}, \overline{q})}{\partial n} = 0.$$

Also,

$$c_H\overline{q} > \overline{p} = AC(\overline{n}, \overline{q}).$$

Finally, Claim 3 implies $\overline{\gamma} > 0$. Together these observations yield the first inequality in the proposition.

Substituting the solution into the first-order condition for $q$ and rearranging gives

$$v'(\overline{q}) = \frac{\lambda(1-\alpha)^{\overline{n}}c_H + (1-\lambda)c_L + \overline{\gamma}\lambda(1-\alpha)^{\overline{n}}(c_H - (\partial AC(\overline{n}, \overline{q})/\partial q))}{\lambda(1-\alpha)^{\overline{n}} + (1-\lambda)}.$$

Note that

$$c_H - \frac{\partial AC(\overline{n}, \overline{q})}{\partial q} = \frac{(1-\lambda)(c_H - c_L)}{\lambda(1-\alpha)^{\overline{n}} + (1-\lambda)} > 0.$$

This observation yields the second inequality in the proposition. ■

## Proposition 7

*Proof.* The result is proven in three steps.

**Step 1.** Substitute from (13) into (12) to get

$$\widetilde{U}(p, \tilde{n}(p)) = \left((1-\lambda) - \frac{m}{p - c_L}\right)(v - p).$$

Differentiation yields

$$\frac{d\widetilde{U}(p, \tilde{n}(p))}{dp} = \frac{m(v - c_L)}{(p - c_L)^2} - (1 - \lambda)$$

and

$$\frac{d^2\widetilde{U}(p, \tilde{n}(p))}{(dp)^2} = -\frac{2m(v - c_L)}{(p - c_L)^3} < 0.$$

Hence, $\widetilde{U}(p, \tilde{n}(p))$ is maximized at $\tilde{p}$.

**Step 2.** Suppose the market is *ex ante* viable. For $m \geq \lambda(1-\lambda)(c_H - c_L)$, the equilibrium outcome evidently involves pricing at $p^0 = \lambda c_H + (1-\lambda)c_L$ and accepting all applications. Next, suppose $m < \lambda(1-\lambda)(c_H - c_L)$. By Step 1, the unique equilibrium outcome will involve all firms pricing at $\tilde{p}$ and acquiring information optimally if

$$\tilde{p} < v \;\Leftrightarrow\; m < (1-\lambda)(v - c_L).$$

Simple algebra reveals

$$v > \lambda c_H + (1-\lambda)c_L \;\Leftrightarrow\; (1-\lambda)(v - c_L) > \lambda(1-\lambda)(c_H - c_L),$$

from which the result follows.

**Step 3.** Suppose the market is not *ex ante* viable. For $m \geq (1-\lambda)(v - c_L)$, no equilibrium in which the market is active exists. Specifically, if firms price less than $v$ and acquire no information, then they will clearly reject all applications. To see that no equilibrium with positive information acquisition exists either, note from (13) that a firm will acquire information about an applicant *iff*

$$p > c_L + \frac{m}{1-\lambda}.$$

But

$$c_L + \frac{m}{1-\lambda} \geq v.$$

Hence, consumers will not apply to purchase the good at any price that induces positive information acquisition. Now consider $m < (1-\lambda)(v - c_L)$. Simple algebra shows that this condition is equivalent to $\tilde{p} < v$. Hence, it remains only to show that firms make non-negative profit by pricing at $\tilde{p}$ and approving qualified applicants. The profit per applicant from this strategy is

$$\begin{aligned}\widetilde{\Pi} &= \left(1-(1-\alpha)^{\tilde{n}(\tilde{p})}\right)(\tilde{p}-c_L) - k\tilde{n}(\tilde{p}) \\ &= \sqrt{m(1-\lambda)(v-c_L)} - m + m\ln\left(\sqrt{m/((1-\lambda)(v-c_L))}\right) \\ &= m\left(z^{-1} - 1 + \ln(z)\right),\end{aligned}$$

where

$$z = \sqrt{\frac{m}{(1-\lambda)(v-c_L)}}.$$

Note that $z=1$ implies $\widetilde{\Pi}=0$. Moreover,

$$\frac{d}{dz}\left(z^{-1}-1+\ln(z)\right) = z^{-1} - z^{-2}.$$

This is evidently negative *iff* $z<1$. Hence, $m \in (0,(1-\lambda)(v-c_L))$ implies $\widetilde{\Pi}>0$. At $m=0$, the equilibrium involves perfect information; i.e., firms screen all applicants at zero cost and set the competitive price of $c_L$.

■

## Proposition 8

*Proof.* By definition

$$\gamma(p) = \begin{cases} -m + (1-\mu(p,\rho))(p-c_L) + m\ln\left(\frac{m}{\mu(p,\rho)(c_H-p)}\right), & \text{if } p<p^\dagger \\ p - \lambda c_H - (1-\lambda)c_L, & \text{if } p \geq p^\dagger. \end{cases} \qquad p \in [c_L, c_H].$$

First, observe that $\gamma(p)$ is continuous. In particular,

$$-m + (1-\lambda)(p^\dagger - c_L) + m\ln\left(\frac{m}{\lambda(c_H-p^\dagger)}\right) = p^\dagger - \lambda c_H - (1-\lambda)c_L.$$

Next, observe that $\gamma(p)$ is clearly increasing for $p \geq p^\dagger$. For $p<p^\dagger$,

$$\gamma'(p) = (1-\mu(p,\rho)) - \mu_p(p,\rho)(p-c_L) + m\left(\frac{1}{c_H-p} - \frac{\mu_p(p,\rho)}{\mu(p,\rho)}\right).$$

This is positive because

$$\mu_p(p,\rho) = -\frac{(1-\lambda)\rho m}{(1-(1-\lambda)\rho)(c_H-p)^2} \leq 0.$$

Hence, there exists at most one value $\hat{p} \in [c_L, c_H]$ for which $\gamma(\hat{p})=0$. Moreover, if such a $\hat{p}$ exists, then $v>\hat{p}$ *iff* $\gamma(v)>0$.

Suppose $m < \lambda(1-\lambda)(c_H-c_L)$. Then $c_L < p^\dagger$. Substitution yields

$$\gamma(c_L) = -m + m\ln\left(\frac{m}{\mu(c_L,\rho)(c_H-c_L)}\right).$$

To see that this is negative, observe that

$$m < \lambda(1-\lambda)(c_L - c_H) \;\Leftrightarrow\; \lambda < \mu(c_L, \rho).$$

And hence

$$m < \lambda(1-\lambda)(c_H - c_L) < \lambda(c_H - c_L) < \mu(c_L, \rho)(c_H - c_L).$$

Next, observe that

$$\gamma(p^\dagger) = (1-\lambda)(c_H - c_L) - \frac{m}{\lambda}.$$

This is clearly positive *iff* $m < \lambda(1-\lambda)(c_H - c_L)$. Hence, there exists a unique steady-state equilibrium price $\hat{p} \in (c_L, p^\dagger)$, and it gives rise to positive screening and adverse selection.

Finally, assume $m \geq \lambda(1-\lambda)(c_H - c_L)$. There are two cases to consider. First, if $p^\dagger \geq c_L$, then the above argument establishes that $\gamma(p^\dagger) \leq 0$. On the other hand, if $p^\dagger < c_L$, then

$$\gamma(c_L) = \lambda(c_L - c_H) < 0.$$

In either case,

$$\gamma(c_H) = (1-\lambda)(c_H - c_L) > 0.$$

Hence, there exists a unique steady-state equilibrium price $\hat{p} \in (c_L, c_H)$, and the fact that $\hat{p} > p^\dagger$ implies that the equilibrium involves no screening ($\hat{\phi} = 0$) and no adverse selection ($\hat{\mu} = \lambda$). ■

## Proposition 9

*Proof.* If $\rho = 0$, then (17) reveals that $\hat{\mu} = \lambda$. The claim then follows from the zero-profit condition (15) and the first-order condition (14).

Next, suppose $m < \lambda(1-\lambda)(c_H - c_L)$. In this case, Proposition 8 shows that $\hat{p} < p^\dagger$. This implies that the zero-profit condition (15) can be written

$$\pi = -m + (1-\hat{\mu})(\hat{p} - c_L) + m \ln\left(\frac{m}{\hat{\mu}(c_H - \hat{p})}\right) = 0$$

and (17) can be written

$$\hat{\mu} = \mu(\hat{p}, \rho).$$

Differentiating these with respect to $\rho$ yields respectively

$$\pi_p \frac{\partial \hat{p}}{\partial \rho} + \pi_\mu \frac{\partial \hat{\mu}}{\partial \rho} = 0$$

and

$$\frac{\partial \hat{\mu}}{\partial \rho} = \mu_p \frac{\partial \hat{p}}{\partial \rho} + \mu_\rho.$$

Solving these gives

$$\frac{\partial \hat{p}}{\partial \rho} = -\frac{\pi_\mu \mu_\rho}{\pi_p + \pi_\mu \mu_p}$$

and

$$\frac{\partial \hat{\mu}}{\partial \rho} = \frac{\mu_\rho \pi_p}{\pi_p + \pi_\mu \mu_p}.$$

These are both positive because

$$\pi_p = (1-\hat{\mu}) + \frac{m}{c_H - \hat{p}} > 0,$$

$$\pi_\mu = -(\hat{p} - c_L) - \frac{m}{\hat{\mu}} < 0,$$

$$\mu_p = -\frac{(1-\lambda)\rho m}{(1-(1-\lambda)\rho)(c_H - \hat{p})^2} < 0,$$

and

$$\mu_\rho = \frac{(1-\lambda)(\lambda(c_H - \hat{p}) - m)}{(1-(1-\lambda)\rho)^2(c_H - \hat{p})} > 0.$$

Finally,

$$\hat{\phi} = 1 - \frac{m}{\hat{\mu}(c_H - \hat{p})}.$$

Hence,

$$\frac{\partial \hat{\phi}}{\partial \rho} > 0$$

*iff*

$$\frac{m}{\left(\hat{\mu}(c_H - \hat{p})\right)^2}\left((c_H - \hat{p})\frac{\partial \hat{\mu}}{\partial \rho} - \hat{\mu}\frac{\partial \hat{p}}{\partial \rho}\right) > 0,$$

or

$$(c_H - \hat{p})\frac{\partial \hat{\mu}}{\partial \rho} - \hat{\mu}\frac{\partial \hat{p}}{\partial \rho} > 0,$$

or

$$(c_H - \hat{p})\pi_p + \hat{\mu}\pi_\mu > 0,$$

or

$$\hat{p} - \hat{\mu}c_H - (1-\hat{\mu})c_L < 0.$$

The last line holds because a firm could otherwise make non-negative profit by pricing at $\hat{p}$ and selling the good without screening, contrary to Proposition 8. ■

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