Weitzman's R & D Problem*

LaTeX file: *weitzman* — Daniel A. Graham, August 14, 2007

A firm is considering two "research and development" projects. The rate of interest is 10% per period and the projects have the following attributes:

Project	# 1		# 2	
Cost	\$15		\$20	
Duration	1 period		2 periods	
Reward	\$100	\$55	\$240	\$0
Probabilities	.5	.5	.2	.8
Expected Present Value	\$55.5[a]		\$19.7[b]	

[a] $55.5 = -15 + \frac{1}{1.1}[.5 \times 100 + .5 \times 55]$
[b] $19.7 = -20 + \frac{1}{1.1^2}[.2 \times 240 + .8 \times 0]$

Note that it takes 1 period to develop #1 and 2 periods to develop #2. The costs of \$15 and \$20, respectively, are measured in "\$ at the beginning of the development period". At the end of the development period the reward for the project is revealed and a decision can be made to accept this reward or to develop the other project. This reward is the payoff of an "accepted project" and is measured in "\$ at the time of acceptance". The projects are mutually exclusive in the sense that it is not possible to obtain a reward from #1 and #2. Thus #1 could be researched and, depending upon the discovered value of #1's reward, a decision could be made regarding whether or not to research #2. Having researched both #1 and #2 a decision could be made to accept the largest of the (now known) rewards, et cetera.

- Which project should be researched first? Is this the project with the highest expected present value?

  The expected values of researching the two projects are

$$\begin{aligned} E_1 &= -15 + (1/1.1)[.5(100) + .5(55)] \\ &= 55.5 \\ E_2 &= -20 + (1/1.1)^2[.2(240) + .8(0)] \\ &= 19.7 \end{aligned}$$

  The expected value of researching 2 if 1 is researched first and turns out to be worth 55 is

$$-20 + (1/1.1)^2[.2(240) + .8(55)] = 56$$

  Since this is greater than 55 it pays to research 2 in the event that 1 pays 55. Since

$$-20 + (1/1.1)^2[.2(240) + .8(100)] = 85.8$$

  it would not pay to research 2 if 1 pays 100. The expected value of 1 first is thus

*This example and the optimal rule are due to Weitzman, "Optimal Search", *Econometrica*, 47 (May, 1979) pages 641–54.

$$-15 + \frac{1}{1.1}\left\{.5(100) + .5\left(-20 + \frac{1}{1.1^2}[.2(240) + .8(55)]\right)\right\} = 55.9$$

and the expected value of researching 2 first is

$$-20 + \frac{1}{1.1^2}\left\{.2(240) + .8\left(-15 + \frac{1}{1.1}[.5(100) + .5(55)]\right)\right\} = 56.3$$

The greatest expected payoff is thus associated with researching 2 first.

Consider the following shadow prices. v_1 is a sure payment which would make the firm indifferent between accepting v_1 and researching the first project if these were the only two alternatives and if v_1 could be accepted either before or after the reward from the first project is discovered. Let v_2 be the corresponding shadow price for the second project. Thus

$$v_1 = -15 + \frac{1}{1.1}[.5 \times 100 + .5 \times v_1]$$
$$v_2 = -20 + \frac{1}{1.1^2}[.2 \times 240 + .8 \times v_2]$$

Solving gives

$$v_1 = 55.83$$
$$v_2 = 58.049$$

Note that the following rule for researching projects is optimal: research that project first which has the largest shadow price. Stop if the actual reward from the project researched first exceeds the shadow price of the other project; otherwise research the other project and accept the largest discovered reward.