

# Index Volatility Futures in Asset Allocation: A Hedging Framework

*Jai Jacob, Portfolio Manager/Analyst, Lazard Asset Management*

*Emma Rasiel, PhD, Assistant Professor of the Practice of Economics, Duke University*

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In March 2004, the Chicago Board Options Exchange introduced futures contracts on the S&P 500 Volatility Index (VIX Index), offering investors, for the first time, the opportunity to make *pure play* trades on the direction of implied volatility. We examined the contribution of futures on the VIX Index (VX Futures) as an additional asset in both passive and actively managed portfolios containing a diversified U.S. equity index and U.S. Treasury bonds.

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As we will show, a relatively small allocation to VX Futures significantly improves a portfolio's risk-return performance over both short and long horizons:

- When included as a third asset, VX Futures significantly improve the efficient frontier of optimal equity-bond portfolios—and more effectively so than other traditional hedge assets such as gold or oil.
- In a portfolio with a fixed allocation between the S&P 500 Index and bonds, reweighting part of the bond allocation to VX Futures generated similar returns, with consistently lower volatility and an improved Sharpe ratio, in almost every quarter from the first quarter of 1986 to the fourth quarter of 2007.
- Opportunistically reweighting a proportion of an S&P 500 Index portfolio into VX Futures provided a significant hedge against expected negative performance in the S&P 500 Index.

We will demonstrate that the positive contribution of VX Futures to well-diversified equity portfolios derives from its relatively stable, and consistently negative, correlation with the S&P 500 Index. Further, because there is always increased demand for equity options as a hedging tool in falling equity

markets, we believe it is reasonable to assume that VX Futures will continue to perform well in this regard in the future. Consequently, VX Futures provide increased rates of return precisely when equity portfolios are under-performing.

## VX Futures

Prior to 2004, it would have been complex and costly to include equity-implied volatility as a steady component of an investment portfolio. Exposure to volatility was available only by buying options, which included two considerable downsides:

- Options have exposure not just to volatility, but also to market direction. To maintain directional neutrality requires frequent updating of a *delta hedge* position on the underlying asset.
- Options are a *wasting* asset (their value inevitably decreases simply with the passage of time). Therefore, options cannot be included as an asset in a buy-and-hold portfolio.

With the introduction of VX Futures, the Chicago Board Options Exchange established a pure play on volatility that is both neutral to directional market moves and can be maintained for the long run. Further, VX Futures can be *rolled* from one contract to the next as each contract expires through a rela-

tively low-frequency, low-cost transaction that mirrors those used in long-term speculative positions in many commodities markets.

## The VIX Index

Though VX Futures were introduced as a tradable asset only in 2004, we analyzed the historical performance of the VIX Index going back to 1986, when it was first listed as an indicator of demand for options on the S&P 500 Index. (Later in this discussion we will examine the correlation between the VIX Index and tradable VX Futures contracts, to determine whether VX Futures are a reasonable proxy for the VIX Index.)

The VIX Index is a measure of the *implied volatility* used to price options. The higher the implied volatility, the higher the option price. Since options, especially put options, function as short-term insurance against negative market moves, increased demand for downside protection in uncertain or hostile markets tends to drive up options prices, and this is reflected in an increase in implied volatility. High levels of the VIX Index (which is sometimes labeled “the fear index”) are seen as an indication that market participants are nervous about the future direction of the underlying S&P 500 Index.

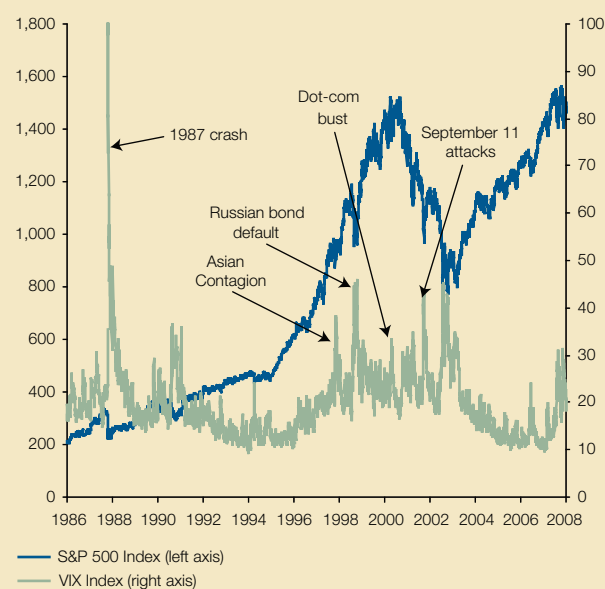
## An Ideal Hedging Tool for a Diversified Equities Portfolio

An important characteristic of the VIX Index is its tendency to spike upwards in response to negative market events.

Exhibit 1 clearly displays the massive jump in volatility in response to the 1987 Crash, the smaller, but noticeable, upward spikes during the Asian Contagion of summer 1997, the Russian debt crisis a year later, and the dot-com bust and September 11 attacks in the early part of this decade. The current credit crunch and related equity sell off can also be seen to have boosted the level of the VIX Index in the last quarter of 2007.

The reason why the VIX Index performs so well as a hedge to a long equity portfolio can be attributed to its negative correlation with the S&P 500 Index. In fact, the VIX Index has the unique distinction of having been negatively correlated with the S&P 500 Index for every one-year period since 1986. The VIX Index tends to underperform other assets during positive markets, but it outperforms significantly in negative equity markets, making it an ideal hedging component of a well-diversified equity portfolio. Despite a relatively low average annual rate of return over this time period (approximately 1.5%, versus 10% on the S&P 500 Index, 5% on oil, and 3.5% on gold), the VIX Index provides outsize returns exactly when they are needed.

EXHIBIT 1: S&P 500 INDEX AND VIX INDEX, 1986-2008



As of 4 January 2008

Source: www.Economagic.com, Chicago Board Option Exchange (CBOE)

Table 1 provides annualized correlations between S&P 500 Index returns, corporate bonds, various commodities, and currency markets. Notice in particular the correlations between these assets and the S&P 500 Index for the years in which the equity markets had negative returns (1990, 1994, 2000, 2001 and 2002, highlighted in the table). The VIX Index was the only asset with a substantially negative correlation (i.e., strong positive returns) in every one of the abovementioned years. Other assets that sometimes show strong negative correlations with the S&P 500 Index (such as the Moody’s BAA Corporate Bond Index, the Goldman Sachs Commodity Index and oil) were nonetheless positively correlated with the S&P 500 Index in 2002. Long positions in those assets would have exacerbated, rather than dampened, the portfolio’s negative returns in this case.

As the table shows, the VIX Index had higher negative correlation with the S&P 500 Index than any of the other assets in every single year. This makes it an excellent contributor to a passively-managed portfolio, as it will always tend to smooth returns during negative markets. Unlike other assets that may have periods of both positive and negative correlation, we believe that the VIX Index’s unique relationship to the S&P 500 Index suggests that these two indices will *always* be negatively correlated.

TABLE 1: CALENDAR YEAR CORRELATIONS BETWEEN THE S&amp;P 500 INDEX AND VARIOUS ASSETS

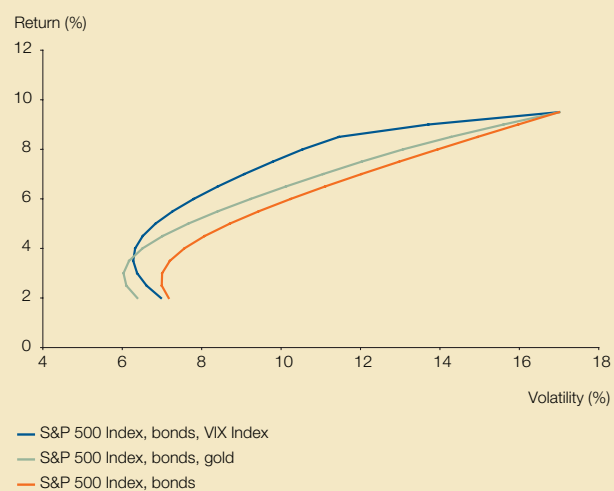
	S&P 500 Index Annual Returns	Moody's BAA Corp. Bond Index	Goldman Sachs Commodities Index	Oil Spot Price	Gold	Silver	\$/Euro (formerly \$/DM)	VIX Implied Volatility Index
1986	14.56%	-0.32	-0.06	-0.14	0.04	0.04	0.04	-0.32
1987	2.34%	-0.32	0.11	0.08	-0.28	-0.19	-0.23	-0.83
1988	12.43%	-0.32	-0.03	-0.05	-0.06	0.10	-0.30	-0.69
1989	28.25%	-0.17	0.05	-0.02	-0.10	0.04	-0.05	-0.66
1990	-8.97%	-0.28	-0.40	-0.28	-0.08	0.07	-0.10	-0.54
1991	27.63%	-0.36	-0.26	-0.24	-0.21	0.01	0.21	-0.56
1992	5.19%	-0.15	0.03	0.06	-0.04	0.03	-0.08	-0.55
1993	7.21%	-0.35	0.02	-0.04	-0.17	-0.03	0.03	-0.51
1994	-2.06%	-0.55	-0.14	-0.16	-0.18	-0.04	-0.20	-0.72
1995	33.18%	-0.44	0.00	0.03	-0.12	-0.07	-0.23	-0.45
1996	22.11%	-0.56	0.03	0.03	-0.10	-0.16	-0.17	-0.68
1997	24.07%	-0.35	-0.16	-0.12	0.00	-0.05	-0.33	-0.70
1998	31.56%	0.21	0.08	0.01	0.02	0.04	-0.17	-0.82
1999	18.54%	-0.30	-0.04	-0.02	0.03	-0.07	-0.38	-0.80
2000	-13.28%	0.00	-0.08	-0.06	-0.03	-0.07	-0.08	-0.78
2001	-9.08%	0.01	0.00	0.01	-0.08	-0.15	-0.22	-0.82
2002	-22.42%	0.50	0.20	0.17	-0.31	-0.11	-0.32	-0.82
2003	20.70%	0.29	-0.24	-0.24	-0.20	0.06	-0.40	-0.66
2004	11.95%	0.04	-0.08	-0.12	0.14	0.09	0.08	-0.75
2005	5.54%	-0.04	-0.06	-0.07	-0.02	0.05	-0.04	-0.82
2006	10.99%	-0.11	0.04	0.01	-0.05	-0.05	0.12	-0.82
	minimum	-0.56	-0.40	-0.28	-0.31	-0.19	-0.40	-0.83
	maximum	0.50	0.20	0.17	0.14	0.10	0.21	-0.32

Source: www.Economagic.com, Chicago Board Option Exchange (CBOE)

## An Improved Efficient Frontier

Exhibit 2 demonstrates that inclusion of the VIX Index in a long-only diversified portfolio of equities and bonds substantially improves the efficient frontier of risk/return tradeoffs. It is important to note that adding any asset with less-than-perfect correlation to other assets in the portfolio will tend to improve the efficient frontier. Gold is negatively correlated with the S&P 500 Index for much of the time as well; therefore, we have also shown the improvement provided by gold as a third asset. But as Exhibit 2 shows, the VIX Index lifted the efficient frontier demonstrably more than gold, reflecting the higher magnitude of its negative correlation in all time periods.

EXHIBIT 2: EFFICIENT FRONTIER FOR TWO- AND THREE-ASSET PORTFOLIOS, JANUARY 1986-JULY 2007



Source: www.Economagic.com, Chicago Board Option Exchange (CBOE), Lazard Asset Management

TABLE 2: 1986 – 2007 AVERAGE RETURNS AND VOLATILITIES

Asset allocations			Return			Volatility		
S&P 500 Index	Bonds	VIX Index	Average	Minimum	Maximum	Average	Minimum	Maximum
100%	0%	0%	10.45%	-23.38%	34.08%	15.79%	7.81%	33.74%
80%	20%	0%	8.59%	-16.75%	30.86%	12.86%	6.95%	27.02%
80%	10%	10%	9.25%	-15.70%	28.56%	9.41%	4.96%	16.02%
60%	40%	0%	7.14%	-10.13%	27.67%	10.15%	6.33%	20.61%
60%	30%	10%	7.59%	-9.08%	25.35%	7.95%	5.25%	12.90%
40%	60%	0%	5.49%	-8.41%	24.47%	7.93%	5.15%	14.91%
40%	55%	5%	5.94%	-6.60%	22.15%	7.77%	5.93%	13.38%
20%	80%	0%	3.83%	-10.77%	21.26%	6.75%	4.45%	11.09%
20%	75%	5%	4.06%	-9.87%	20.10%	6.50%	4.89%	11.27%

Source: www.Economagic.com, Chicago Board Option Exchange (CBOE), Lazard Asset Management

## The VIX Index in a Passive Asset Allocation Framework

In order to examine the VIX Index's performance in the context of a passive allocation strategy, we first evaluated historic returns and volatilities of portfolios with various allocations between equities and bonds. We calculated average, maxi-

imum, and minimum returns and volatilities for four different portfolios with:

1. 80% allocation to equities, 20% to bonds
2. 60% allocation to equities, 40% to bonds
3. 40% allocation to equities, 60% to bonds
4. 20% allocation to equities, 80% to bonds

EXHIBIT 3A: ANNUAL RETURNS

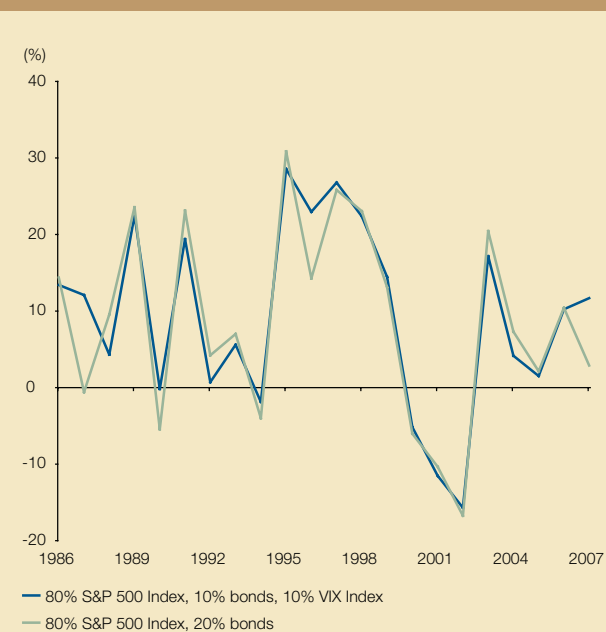
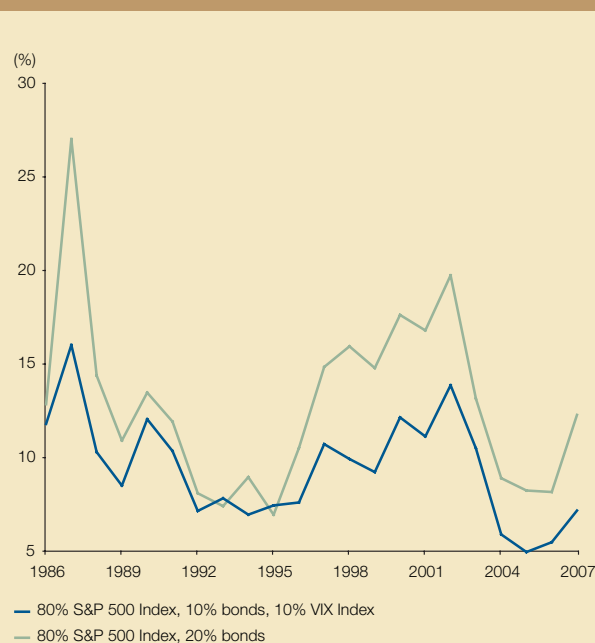


EXHIBIT 3B: ANNUAL VOLATILITY



Then, we compared the returns and volatilities of these portfolios with those of portfolios with the same allocations to equities, but where between 5% and 10% of the bond allocation was transferred into the VIX Index.

We found that re-allocating small amounts of the portfolio in this way increased average annual returns by between 20 and 70 basis points, with the additional benefit of decreasing the portfolio volatility. For a portfolio with an 80% equity allocation, the volatility reduction was greater than 3%. Even portfolios with a relatively low equity allocation showed reduced volatility with just small allocations to the VIX Index. Lower portfolio volatility translates into higher flexibility in terms of adding or removing funds from the portfolio. The need to *time the market* is reduced when daily and monthly changes in returns are smaller. Table 2 summarizes the results of this analysis, and shows that the benefits provided by the re-allocation to the VIX Index were bigger for portfolios with a higher equity exposure.

We then looked more closely at a portfolio with 80% equity and 20% bonds (the 80-20 portfolio) versus a portfolio in which 10% of the bond allocation was moved to the VIX Index (the 80-10-10 portfolio.) An investor who maintained the 80-20

portfolio over the entire period from 1986 to 2007 would have earned an average annual return of 8.59%, with 12.86% average annual volatility. By re-allocating to the 80-10-10 portfolio, the investor's average annual return would have increased by 66 basis points, while the average annual volatility would have been reduced by more than 3%. Exhibits 3A and 3B plot a comparison of annual returns and volatilities on these two portfolios over time.

We observed that the return improvements and the volatility reductions in negative quarters were typically higher than in positive quarters—a further indication of the excellent hedging properties of the VIX Index. To demonstrate this, we separated the sample period into quarters, and extracted the quarters with negative returns on equity (There were 22 such quarters.) We then examined the 80-20 portfolio versus the 80-10-10 and the S&P 500 Index for those 22 quarters, or the sample period (see Exhibits 4A and 4B). The average improvement in returns due to allocating 10% of the portfolio to the VIX Index was greater than 2% quarterly (approximately 9% annualized), while the average volatility reduction was a little over 2% quarterly (approximately 4% annualized).

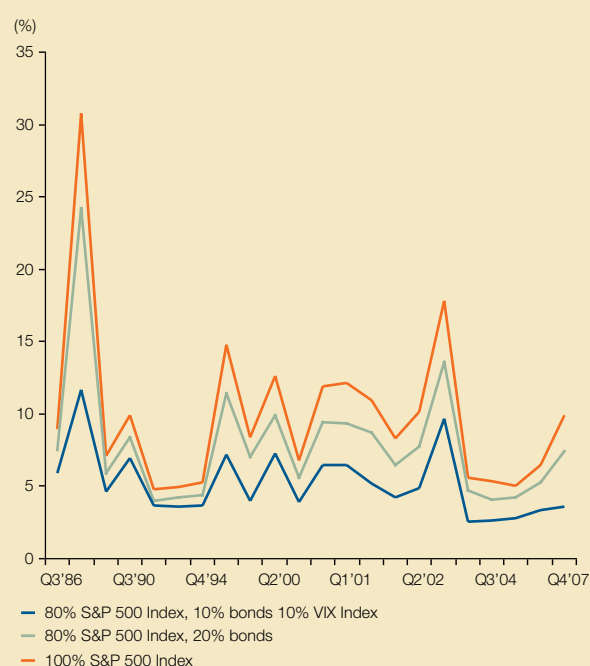
EXHIBIT 4A: QUARTERLY RETURNS WHEN EQUITY RETURNS ARE NEGATIVE



As of 3 January 2008

Source: [www.Economagic.com](http://www.Economagic.com), Chicago Board Option Exchange (CBOE), Lazard Asset Management

EXHIBIT 4B: QUARTERLY VOLATILITY WHEN EQUITY RETURNS ARE NEGATIVE



As of 3 January 2008

Source: [www.Economagic.com](http://www.Economagic.com), Chicago Board Option Exchange (CBOE), Lazard Asset Management

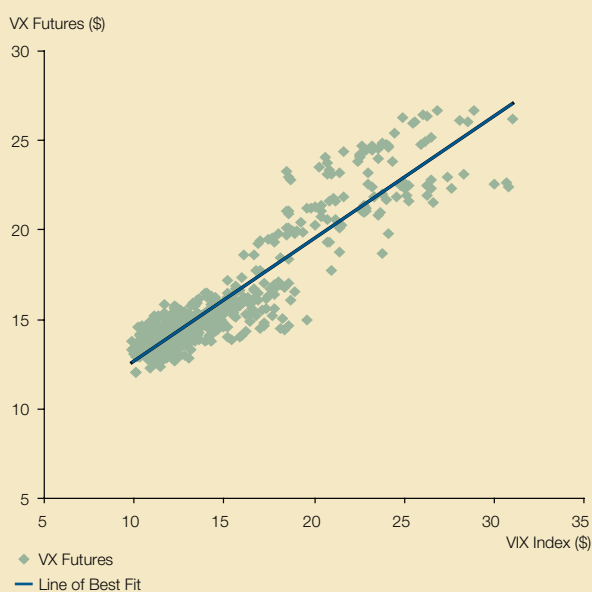
## How Well Do VX Futures Mirror the VIX Index?

Having established the hedging properties of the VIX Index, we examined whether one could, in fact, realize reductions in volatility by trading VX Futures as a proxy for the VIX Index itself.

Exhibit 5 shows the values of the VIX Index and of VX Futures, starting in late 2004 (at which point VX Futures were beginning to show reasonable levels of volume and liquidity). To generate a rolling VX Futures time series, we used front-month VX Futures values on the February-May-August-November quarterly cycle.<sup>1</sup> In each quarter, we rolled from the near contract to the next when the near contract had one month left before expiration (this is the time when trading volume typically begins to decrease relative to the next contract).

As is clear from this exhibit, VX Futures are not perfectly correlated with the underlying VIX Index. Typically, VX Futures exhibit lower volatility, meaning returns are not as high when equity returns are negative. To more rigorously test whether VX Futures are a good proxy for VIX Index, therefore, we regressed the VX Futures on the VIX Index for the three-year period in which we have VX data. Exhibit 6 shows a scatter plot of VX Futures vs. the VIX Index, as well as the straight line of “best fit” that we can use to extrapolate *model* VX

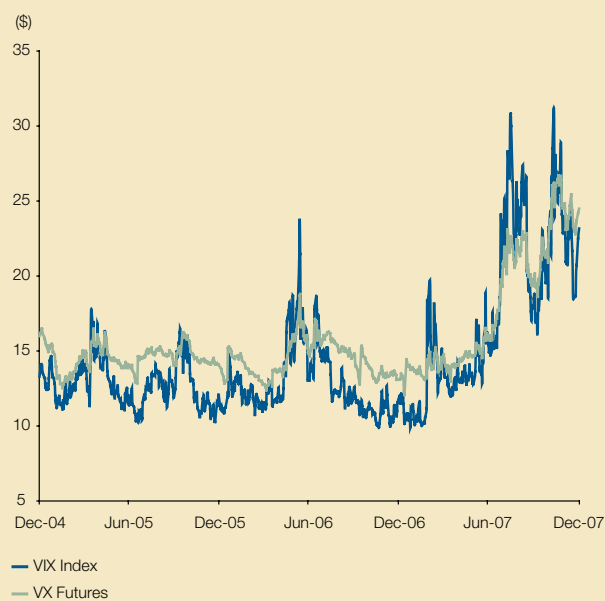
EXHIBIT 6: VIX INDEX VS. VX FUTURES, JANUARY 2005 – DECEMBER 2007



As of 2 January 2008

Source: CBOE Futures Exchange (CFE), Lazard Asset Management

EXHIBIT 5: THE VIX INDEX VS. VX FUTURES ON THE INDEX, JANUARY 2005 – DECEMBER 2007



As of 2 January 2008

Source: Chicago Board Option Exchange (CBOE), CBOE Futures Exchange (CFE)

Futures for the time period prior to when real VX Futures became available (March 2004.)

The correlation between VX Futures and the VIX Index is a very high 92%, which gives us confidence that the linear extrapolation could be a reasonable proxy for VX Futures. Exhibit 7 shows the VIX Index levels over the entire sample period,<sup>2</sup> together with the extrapolated model VX (which we will call MVX to distinguish it from the true VX Futures series.)

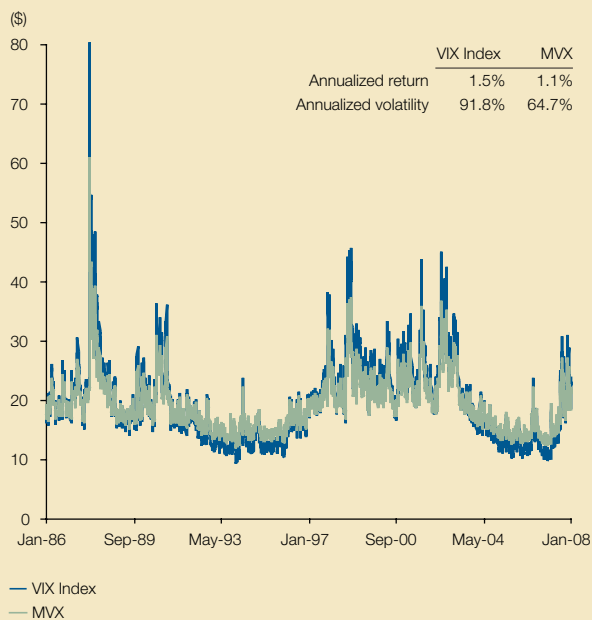
We note that both the annualized returns and volatility of the MVX over the 20-year sample period were lower than those of the VIX. This is what we might have expected based on Exhibit 5.

Using the MVX, we then replicated the analysis of the portfolios with varying allocations to equity and bonds, to determine whether the MVX offers similar hedging properties versus the VIX Index.

### MVX Versus the VIX Index: Improvement in a Static Allocation

Exhibits 8A and 8B illustrate the results of this analysis. The returns on the portfolio with 10% allocation to MVX (80-10-10M) were slightly lower than those of the portfolio with 10% allocation to the VIX Index (80-10-10); the mean return was approximately 35 basis points lower. However, the 80-10-10M

EXHIBIT 7: VIX INDEX LEVELS VS. MVX



As of 4 January 2008

Source: Chicago Board Option Exchange (CBOE), Lazard Asset Management

portfolio still performed significantly better on average than the 80-20 portfolio. Meanwhile, the volatility of the 80-10-10M was distinctly lower than that of the 80-20 portfolio and slightly lower than that of the 80-10-10.

## The MVX and Active Portfolio Hedging

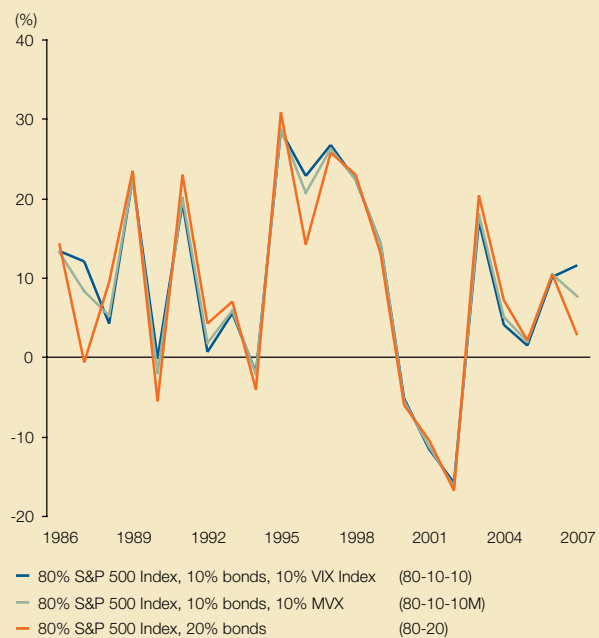
Having established that the proxy MVX “index” is indeed a contributor to performance in a passive portfolio context, we now want to test whether there is an optimal level of VX Futures exposure to hedge against an anticipated fall in the equity markets.

Assuming one owns a diversified equity portfolio, the problem can be posted this way:

**“If one believes that the S&P 500 Index is going to sell off by  $X\%$  over the next  $Y$  months, how much of the portfolio should one re-allocate from equity to  $VX$  Futures to reduce that negative return by  $Z\%$ ?”**

In a Capital Asset Pricing Model (CAPM) framework, we know that we can reduce a portfolio’s market-related risk (or *beta* risk) by allocating a percentage of a well-diversified portfolio into an asset that has a low or negative beta relative to

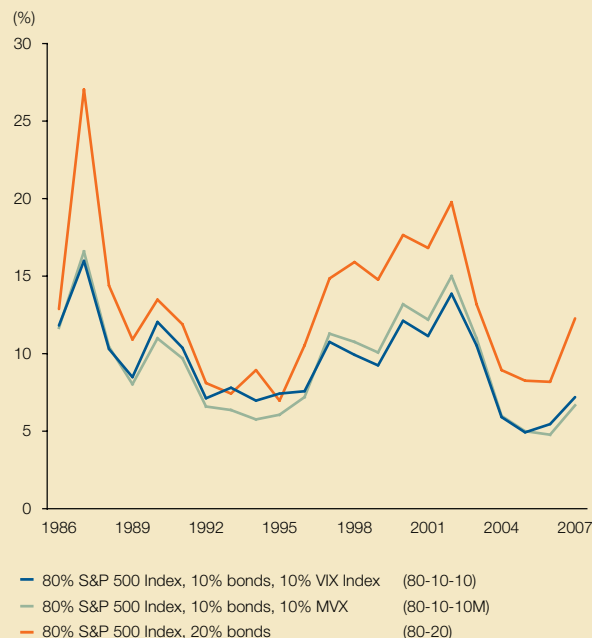
EXHIBIT 8A: ANNUAL RETURNS



As of 31 December 2007

Source: www.Economagic.com, Chicago Board Option Exchange (CBOE), Lazard Asset Management

EXHIBIT 8B: ANNUAL VOLATILITY



As of 31 December 2007

Source: www.Economagic.com, Chicago Board Option Exchange (CBOE), Lazard Asset Management

that portfolio. Moreover, we can estimate an asset's beta by comparing its returns over time against the returns of the well-diversified portfolio.

Exhibit 9 shows a scatter plot of monthly returns of the MVX over the sample period (1988 – 2007) relative to the monthly returns on the S&P 500 Index.<sup>3</sup> The slope of the blue *trend curve* on this chart represents the beta of the MVX (or weighted expected return relative to the S&P 500 Index). Commonly, an asset's beta is assumed to be constant for all levels of the underlying market index (in this case the S&P 500 Index). That is, the trend curve is usually assumed to be a straight line. Here, however, it is clear that a curved (quadratic) relationship is a better fit to this data.<sup>4</sup>

The implication of such a non-linear trend curve is that the MVX's beta varies depending on the current level of the S&P 500 Index. Negative returns with a higher magnitude on the S&P 500 Index are associated with larger MVX betas (or a steeper slope on the trend curve). Further, all of the betas are negative (meaning the trend curve always has a negative slope). This positive convexity works in favor of the portfolio when hedging the S&P 500 Index's exposure with the MVX. When the S&P 500 Index returns are particularly large and negative, the MVX's beta is also large. This allows for a hedge of the S&P 500 Index with a relatively small allocation to the MVX. Conversely, if an anticipated negative downturn in the S&P 500 Index is hedged using the MVX, the impact of an error (i.e., if the S&P 500 Index rallies rather than falls), will be minimal. This is true for two reasons:

1. The MVX hedge trade was reasonably small (as explained above), and
2. When the S&P 500 Index rallied, the MVX beta was relatively low (albeit still negative), resulting in the MVX returns having lower correlation to the S&P 500 Index in positive markets.

Thus, a small positive allocation to MVX helped considerably to hedge a portfolio in hostile markets, while having minimal negative impact in favorable markets.

Exhibit 10 illustrates the expected returns of several portfolios that are composed of varying weights in the MVX and in two specific S&P 500 Index return scenarios. The blue line represents the range of expected portfolio returns assuming a 10% monthly loss in the S&P 500 Index. As part of the portfolio was reallocated to the MVX, the portfolio return increased rapidly. The MVX had a beta of approximately  $-2.7$  in this environment, so a 27% allocation of the portfolio to the MVX completely offset the negative S&P 500 Index return (where the blue line crosses the x-axis). A higher allocation to the MVX resulted in a positive expected return of the portfolio.

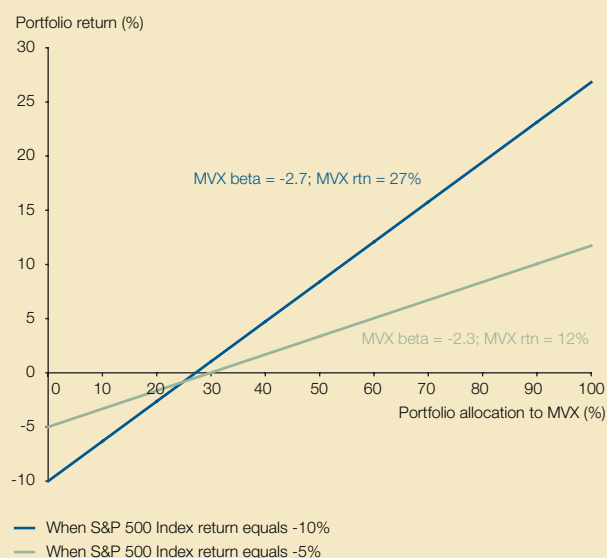
EXHIBIT 9: MONTHLY S&P 500 INDEX RETURNS VS. MVX RETURNS



As of 4 January 2008

Source: www.Economagic.com, Lazard Asset Management

EXHIBIT 10: MONTHLY RETURNS FOR VARYING PORTFOLIO ALLOCATIONS TO MVX



As of 4 January 2008

Source: www.Economagic.com, Lazard Asset Management

The green line paints a similar picture, this time in a 5% loss scenario for the S&P 500 Index. In this case, the MVX had a beta of approximately  $-2.3$ , so a larger allocation (around 30%) of the portfolio to the MVX was required to fully offset the negative performance of the S&P 500 Index. Exhibit 10 demonstrates that any allocation to MVX, however small, helped to offset negative returns in the S&P 500 Index to some extent. For example, in the scenario in which the S&P 500 Index had a 10% loss, one could have reduced that loss to 5% with just a 15% allocation to MVX.

We generalized this result to determine how much MVX allocation is needed to hedge  $Z\%$  of a negative return on the S&P 500 Index over various time horizons. Assuming an expected  $X\%$  loss on the S&P 500 Index, this translates into creating a portfolio with an expected return of  $r_p = (1 - z)x$  (where  $z$  and  $x$  equal to  $0.01Z$  and  $0.01X$ , respectively, to keep units aligned).

In order to do so, we estimated the function representing the quadratic curve shown in Exhibit 9 as:

$$v_n = a_n x_n^2 + b_n x_n + c_n$$

Here,  $v_n$  and  $x_n$  represent  $n$ -month returns on the MVX and the S&P 500 Index, respectively, while coefficients  $a_n$ ,  $b_n$ , and  $c_n$  were derived using a best fit non-linear least squares optimization technique. Using monthly returns data over the last 20 years, for example, we estimated the following one-month coefficients for the function:

$$a_1 = 3.36; b_1 = -2.01; c_1 = 0.01$$

By carrying out a similar analysis for a range of different time periods, we estimated the coefficients  $a_n$  and  $b_n$  over various time horizons  $n$ . These coefficients are shown in Table 3.

	1mo	2mo	3mo	4mo	5mo	6mo	9mo
$a$	3.36	2.92	2.42	1.78	1.42	0.93	0.76
$b$	-2.01	-1.78	-1.55	-1.33	-1.13	-0.97	-0.59
$c$	0.01	0.02	0.02	0.03	0.03	0.04	0.03

Source: www.Economagic.com, Lazard Asset Management

The expected return  $r_p$  for a portfolio with weight  $w$  in MVX, and  $(1 - w)$  in the S&P 500 Index is provided by the following formula:

$$\begin{aligned} r_p &= (1 - w) x_n + w v_n \\ &= (1 - w) x_n + w (a_n x_n^2 + b_n x_n + c_n) \end{aligned}$$

For our hedged portfolio, we set  $r_p = (1 - z) x_n$  to obtain:

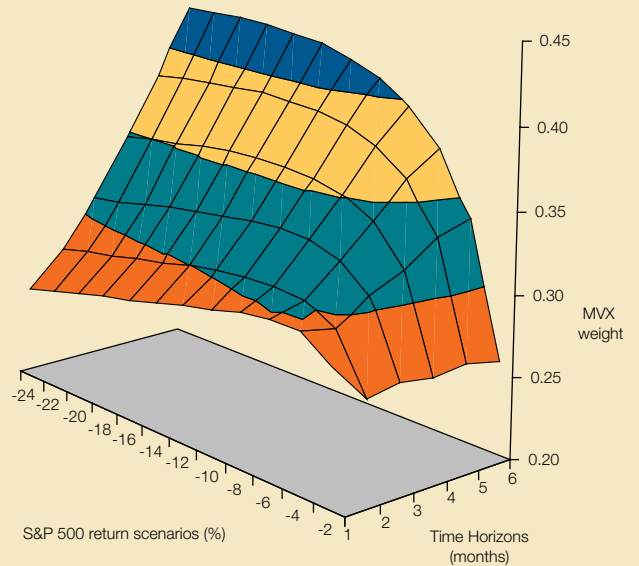
$$\begin{aligned} (1 - z) x_n &= (1 - w) x_n + w (a_n x_n^2 + b_n x_n + c_n) \\ x_n - z x_n &= x_n - w x_n + w (a_n x_n^2 + b_n x_n + c_n) \\ -z x_n &= w [a_n x_n^2 + (b_n - 1) x_n + c_n] \\ w &= -z x_n / [a_n x_n^2 + (b_n - 1) x_n + c_n] \end{aligned}$$

Should we wish to fully hedge the portfolio,  $z = 1$  (hence  $r_p = 0$ ). Thus:

$$w = -x_n / [a_n x_n^2 + (b_n - 1) x_n + c_n]$$

Exhibits 11 and 12 provide the  $w$  values (i.e., the required percentage allocations to MVX) when  $z = 1$  and  $z = 1/2$ , respectively, under various negative return estimates and time horizons for the S&P 500 Index. The three-dimensional charts show that a longer time horizon of negative returns on the S&P 500 Index requires a higher MVX allocation. However, the lower the absolute value of the negative return of the S&P 500 Index, the more stable the required MVX allocation over different time horizons.

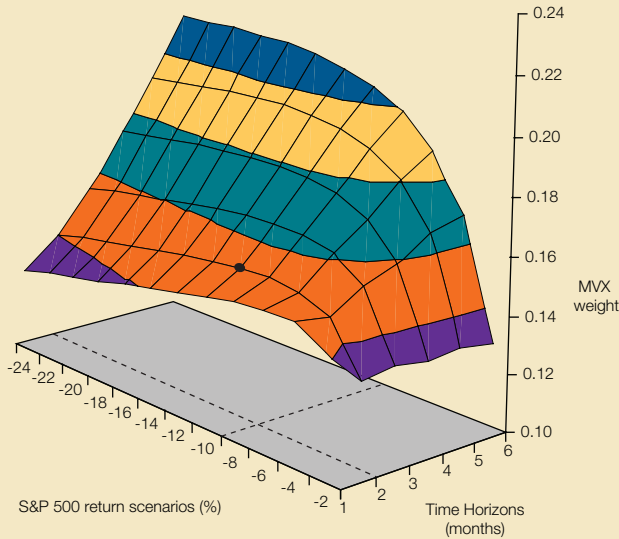
EXHIBIT 11: PERCENTAGE ALLOCATION TO MVX FOR  $z = 1$  UNDER VARYING S&P500 INDEX NEGATIVE RETURN SCENARIOS AND TIME HORIZONS



As of 4 January 2008

Source: www.Economagic.com, Lazard Asset Management

EXHIBIT 12: PERCENTAGE ALLOCATION TO MVX FOR  $z = 1/2$  UNDER VARYING S&P500 INDEX NEGATIVE RETURN SCENARIOS AND TIME HORIZONS



As of 4 January 2008

Source: www.Economagic.com, Lazard Asset Management

Suppose that one anticipates a 10% sell off in the S&P 500 Index over the next two months. In this case,  $a_2 = 2.92$ ,  $b_2 = -1.78$ , and  $c_2 = 0.02$  (from Table 3). Let's say one would like to reduce the portfolio's negative return by half; that is, only a 5% reduction in the portfolio's value over the next two months will be tolerated.

Hence  $z = 1/2$ . In this case,

$$\begin{aligned} w &= -(0.5)(-0.1) / [2.92(0.1)^2 + (-1.78 - 1)(0.1) + 0.02] \\ &= 0.15 \end{aligned}$$

This scenario corresponds to the black point in Exhibit 12.

Thus, the answer to the question

**"If one believes that the S&P 500 Index is going to sell off by 10% over the next 2 months, how much of her diversified equity portfolio should one re-allocate from equity to VX Futures to reduce that negative return by half?"**

is: 15%.

## Conclusions

Traders and hedge fund managers intuitively know that VX Futures protect during periods of market turmoil. In this paper, we quantitatively confirmed this relationship and illustrated its non-linearity. The convexity of VX Futures returns, when plotted against the S&P 500 Index, implies a decreasing marginal hedge (i.e., the more severe the equity correction, the fewer incremental VX Futures are required to hedge). Thus, the portfolio allocation to VX Futures remains low, and relatively stable, across a broad range of negative return scenarios for the S&P 500 Index. The hedging framework we proposed is relevant to active risk-takers in financial markets, because it could allow portfolio managers to mitigate risk more precisely, by taking into account the anticipated magnitude and time horizon of market corrections.



**NOTES:**

- 1 There are VX Futures contracts with maturities in all 12 months of the year, but liquidity on the serial months is low relative to these quarterly contracts.
- 2 The 5-day period around the Crash of 1987 represents an outlier for the VIX. This was the first test of a true market dislocation for the VIX, which some have attributed to program trading relating to equity options. We have excluded that period from our analysis.
- 3 Exhibit 9 and all of the following estimations use data starting from 1988, rather than 1986. This is because the 1987 crash had an extreme impact on the VIX Index, driving it up to triple figure values that have not been seen since (see exhibit 1). We have, therefore, excluded that period of data from the subsequent analysis, since it is not representative of VIX Index performance for the last 20 years.
- 4 The quadratic form is about a 5% better fit than a linear regression in this case.

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