

ENERGY METHODS AND CASTIGLIANO'S THEOREMS
CE 130 — Structural Design and Optimization
Spring, 2006

- Strain Energy: $U = \frac{1}{2} \int_V \{\sigma\}^T \{\epsilon\} dV$
 - External Work: $W_E = U = \int F dD$; Complementary Work: $W_E^* = U^* = \int D dF$
 - Superposition: $N = N_o + \sum v_i F_i$; $M = M_o + \sum m_i F_i$; $V = V_o + \sum v_i F_i$; etc.
 - Castigliano's First Theorem: $F_i = \partial U / \partial D_i$
 - Castigliano's Second Theorem: $D_i = \partial U^* / \partial F_i$
 - Linear Elastic Systems: $U = U^*$
-

Mechanical Loads

Axial $U = \frac{1}{2} \int_l \frac{N^2}{EA} dl = \frac{1}{2} \sum \frac{N^2 L}{EA}$ $n_i = \partial N / \partial F_i$ $D_i = \int_l \frac{N n_i}{EA} dl = \sum \frac{N n_i L}{EA}$

Bending $U = \frac{1}{2} \int_l \frac{M^2}{EI} dl$ $m_i = \partial M / \partial F_i$ $D_i = \int_l \frac{M m_i}{EI} dl$

Shear $U = \frac{1}{2} \int_l \frac{V^2}{G(A/\alpha)} dl$ $v_i = \partial V / \partial F_i$ $D_i = \int_l \frac{V v_i}{G(A/\alpha)} dl$

Torsion $U = \frac{1}{2} \int_l \frac{T^2}{GJ} dl$ $t_i = \partial T / \partial F_i$ $D_i = \int_l \frac{T t_i}{GJ} dl$

Temperature Loads

Axial $U = \sum N \alpha \Delta T L$ $\frac{\partial U}{\partial F_i} = \sum \frac{\partial N}{\partial F_i} \alpha \Delta T L$

Bending $U = \int_l M \alpha \left[\frac{\Delta T_b - \Delta T_t}{h} \right] dl$ $\frac{\partial U}{\partial F_i} = \int_l \frac{\partial M}{\partial F_i} \alpha \left[\frac{\Delta T_b - \Delta T_t}{h} \right] dl$

Statically Indeterminate Structures and Superposition

1. Remove I redundants, R_i , $i = 1, \dots, I$, where I is the degree of indeterminacy.
2. Solve for the internal forces, M_o , N_o , V_o , in the resulting statically determinate structure (without the redundants), due to the real applied loads.
3. Now, remove all of the real applied loads, and apply I unit virtual loads to the structure in the direction of the redundants, one at a time.
4. Solve for I sets of internal forces, m_i , n_i , v_i , in each of the I different statically determinate systems.
5. Apply superposition for moments, axial forces, and shears.

$$M = M_o + \sum_{i=1}^I R_i m_i \qquad N = N_o + \sum_{i=1}^I R_i n_i \qquad V = V_o + \sum_{i=1}^I R_i v_i$$

6. Write I statements of Castigliano's Theorem, one for each virtual system, and enforce compatibility with respect to support settlement, and relative positions, and solve for the redundants, R_i .
-