

SUMMARY OF THE PRINCIPLE OF VIRTUAL WORK

CE 130 — Structural Design and Optimization

Spring, 2003

	Strain Energy U	Internal Virtual Work \bar{W}_I
Axial	$\frac{1}{2} \int_l \frac{N^2}{EA} dl = \frac{1}{2} \sum \frac{N^2 L}{EA}$	$\int_l \frac{nN}{EA} dl = \sum \frac{nNL}{EA}$
Bending	$\frac{1}{2} \int_l \frac{M^2}{EI} dl$	$\int_l \frac{mM}{EI} dl$
Shear	$\frac{1}{2} \int_l \frac{V^2}{G(A/\alpha)} dl$	$\int_l \frac{vV}{G(A/\alpha)} dl$
Torsion	$\frac{1}{2} \int_l \frac{T^2}{GJ} dl$	$\int_l \frac{tT}{GJ} dl$

Principle of Virtual Work:

Virtual work is the work done by a *real force* acting through a *virtual displacement* ...

$$\bar{W}_E = \sum F_i \bar{D}_i = \bar{W}_I = \int_V \{\sigma\}^T \{\bar{\epsilon}\} dV$$

... or a *virtual force* acting through a *real displacement*.

$$\bar{W}_E = \sum \bar{F}_i D_i = \bar{W}_I = \int_V \{\bar{\sigma}\}^T \{\epsilon\} dV$$

Temperature: (Statically Determinate Structures)

Axial:
$$\bar{W}_I = \int_l n \alpha \left[\Delta T_t - \left(\frac{\Delta T_t - \Delta T_b}{h} \right) h_2 \right] dl = \sum n \alpha \Delta T L$$

Bending:
$$\bar{W}_I = \int_l m \alpha \left[\frac{\Delta T_b - \Delta T_t}{h} \right] dl$$

Statically Indeterminate Structures and Superposition:

1. Remove I redundants, R_i , $i = 1, \dots, I$, where I is the degree of indeterminacy.
2. Solve for the internal forces, M_0 , N_0 , V_0 , in the resulting statically determinate structure (without the redundants), due to the real applied loads.
3. Now, remove all of the real applied loads, and apply I unit virtual loads to the structure in the direction of the redundants, one at a time.
4. Solve for I sets of internal forces, m_i , n_i , v_i , in each of the I different statically determinate systems.
5. Apply superposition for moments, axial forces, and shears.

$$M = M_0 + \sum_{i=1}^I R_i m_i \qquad N = N_0 + \sum_{i=1}^I R_i n_i \qquad V = V_0 + \sum_{i=1}^I R_i v_i$$

6. Write I statements of the principle of virtual work, one for each virtual system, and enforce compatibility with respect to support settlement, and relative positions, and solve for the redundants, R_i .