

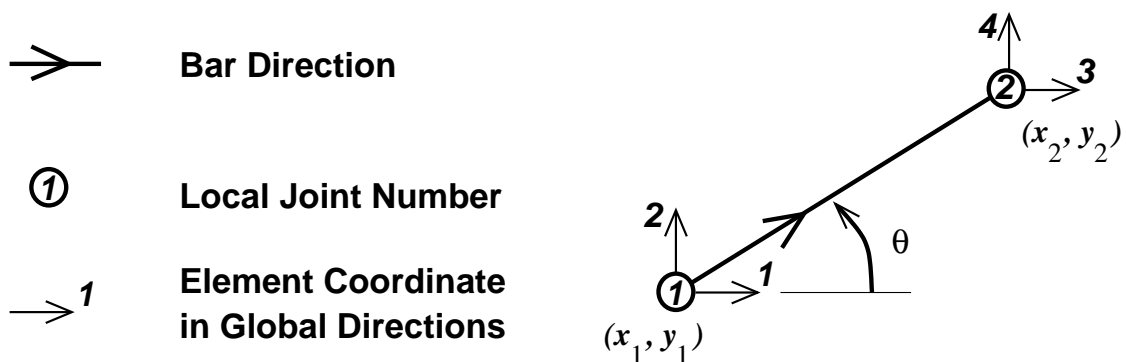
THE DIRECT STIFFNESS METHOD FOR PLANAR TRUSSES

CE 131L. Matrix Structural Analysis

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Method

1. Number all of the joints and all of the elements.
2. Identify the Structural Degrees of Freedom in Global Directions.
Number all the structural degrees of freedom in your truss. In a planar truss, each joint can have a maximum of two degrees of freedom: one in the global X -direction and one in the global Y -direction. If a degree of freedom is restrained by a reaction, then it doesn't get a number.
3. Joint Locations.
Write the (x, y) coordinates of each joint using units consistent with E and A . In other words, if E and A are given in kN/cm^2 and cm^2 , write the (x, y) coordinates in terms of centimeters.
4. Define each element.
Draw each element of your truss individually and draw the local coordinates in the global directions. For example if element number N is a diagonal truss element, and the global directions are X : horizontal and Y : vertical, draw element number N like this:



where 1,2,3,4 are the element coordinates of the truss bar in the global directions. The local coordinates are always numbered 1,2,3,4 with 1 and 3 pointing in the global X direction (to the right) and with 2 and 4 pointing in the global Y direction (up). Some or all of these four coordinates will line up with the structural degrees of freedom that you identified in step 2., above. The angle θ is the counter-clockwise angle from element coordinate 1 to the truss bar.

8. Deflections, \mathbf{d} .

Find the deflections by inverting the stiffness matrix and multiplying it by the load vector. You can do this easily in matlab: $\mathbf{d} = \mathbf{K} \backslash \mathbf{p}$

9. Internal bar forces, n .

Again, recall how the global degrees of freedom line up with each element's coordinates (1,2,3,4). For example, in element number N from step 6., the local element deflections in the global directions, v_1, v_2, v_3, v_4 line up with the structural deflections d_3, d_4, d_7, d_8 . The internal bar forces can be computed from:

$$n = \frac{EA}{L}[c(v_3 - v_1) + s(v_4 - v_2)] = \frac{EA}{L}[c(d_7 - d_3) + s(d_8 - d_4)]$$

where c and s are the direction cosine and sine for the element from step 5.

You should be able to derive this equation.

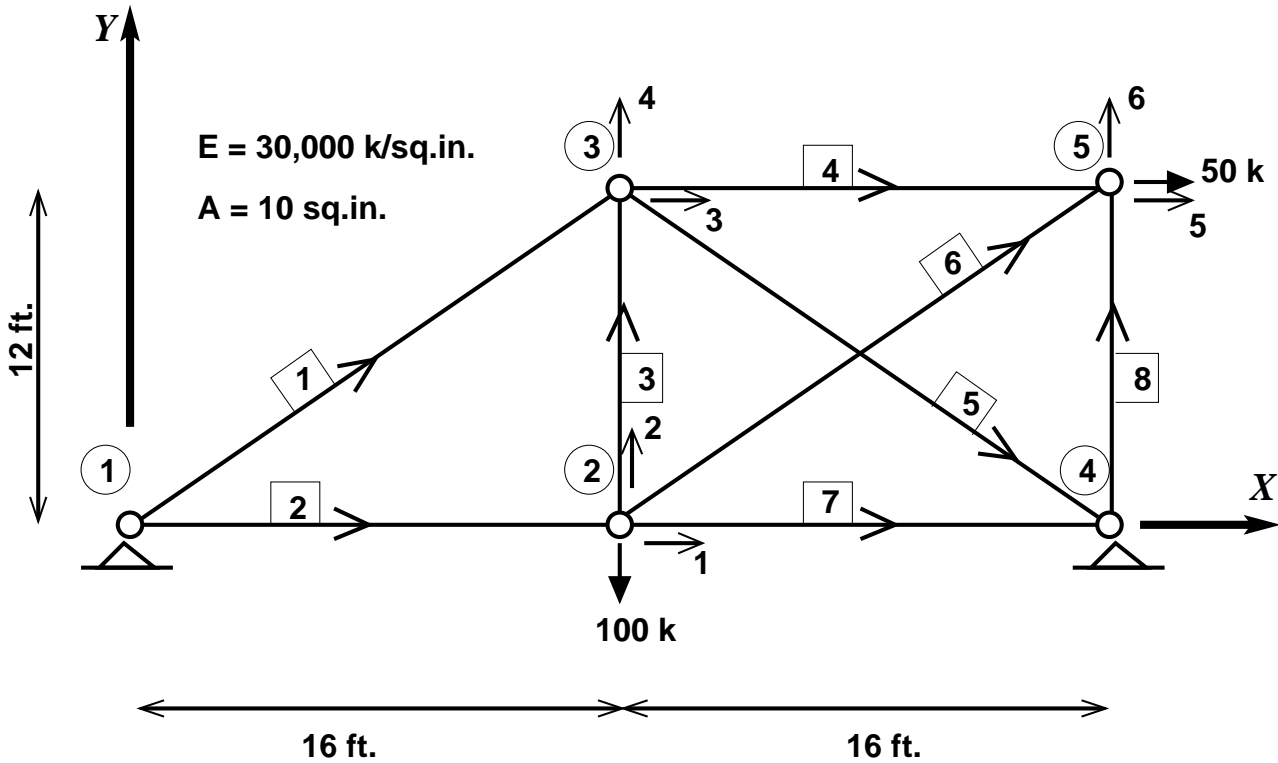
Knowing what each bar force is, the reactions can be easily computed with equilibrium equations.

Notation

- \mathbf{u} = Element deflection vector in the Local coordinate system
- \mathbf{q} = Element force vector in the Local coordinate system
- \mathbf{k} = Element stiffness matrix in the Local coordinate system
... $\mathbf{q} = \mathbf{k} \mathbf{u}$
- \mathbf{T} = Coordinate Transformation Matrix (orthonormal)
... $\mathbf{T}^{-1} = \mathbf{T}^T$
- \mathbf{v} = Element deflection vector in the Global coordinate system
... $\mathbf{u} = \mathbf{T} \mathbf{v}$
- \mathbf{f} = Element force vector in the Global coordinate system
... $\mathbf{q} = \mathbf{T} \mathbf{f}$
- \mathbf{K} = Element stiffness matrix in the Global coordinate system
... $\mathbf{K} = \mathbf{T}^T \mathbf{k} \mathbf{T}$
- \mathbf{d} = Structural deflection vector in the Global coordinate system
- \mathbf{p} = Structural load vector in the Global coordinate system
- \mathbf{K}_s = Structural stiffness matrix in the Global coordinate system
... $\mathbf{p} = \mathbf{K}_s \mathbf{d}$

Coordinate System	Local	Global
Element Deflection	\mathbf{u}	\mathbf{v}
Element Force	\mathbf{q}	\mathbf{f}
Element Stiffness	\mathbf{k}	\mathbf{K}
Structural Deflection	-	\mathbf{d}
Structural Loads	-	\mathbf{p}
Structural Stiffness	-	\mathbf{K}_s

Example



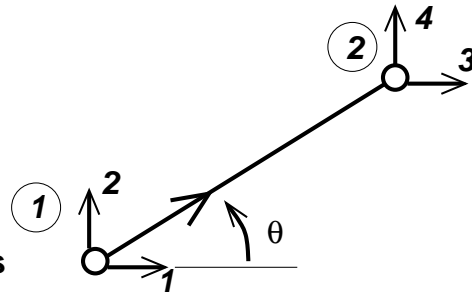
- ① Global Joint Number
- ▢ Bar Number
- Bar Direction
- ₁ Structural Degree of Freedom
- Applied Force
- ① Local Joint Number
- ₁ Local Degree of Freedom
- Global Coordinate System

System Equation: $\{f\} = [K]\{d\}$

$\{d\}$: displacement vector
 $= \{ d_1 \ d_2 \ d_3 \ d_4 \ d_5 \ d_6 \}$

$\{f\}$: load vector = $\{ 0 \ -100 \ 0 \ 0 \ 50 \ 0 \}$

Element Coordinates in Global Directions



```

function K = truss( x1,y1,x2,y2,E,A )
% K = TRUSS (X1,Y1,X2,Y2,E,A)
% Returns a 2-D truss element stiffness matrix in global coordinates
%
%       X1,Y1 are the coordiniates of joint 1 of the truss element
%       X2,Y2 are the coordiniates of joint 2 of the truss element
%       E       is the elastic modulus
%       A       is the cross sectional area

L = sqrt( (x2-x1)^2 + (y2-y1)^2 );
c = (x2-x1) / L;
s = (y2-y1) / L;

K = (E*A/L) * [ c^2    c*s   -c^2   -c*s ;
               c*s    s^2   -c*s   -s^2 ;
               -c^2   -c*s    c^2    c*s ;
               -c*s   -s^2    c*s    s^2 ];

% ----- TRUSS

hudson17% matlab

>> help truss

K = TRUSS (X1,Y1,X2,Y2,E,A)
Returns a 2-D truss element stiffness matrix in global coordinates

       X1,Y1 are the coordiniates of joint 1 of the truss element
       X2,Y2 are the coordiniates of joint 2 of the truss element
       E       is the elastic modulus
       A       is the cross sectional area

>> format bank                               % two decimal places after the .
>> E = 3e4;                                   % modulus of elasticity
>> A = 10;                                     % area of cross section

>> K1 = truss(0,0,12*16,12*12,E,A)           % member 1
K1 =

      800.00      600.00     -800.00     -600.00
      600.00      450.00     -600.00     -450.00
     -800.00     -600.00      800.00      600.00
     -600.00     -450.00      600.00      450.00

```

```
>> K2 = truss(0,0,12*16,0,E,A)           % member 2
K2 =
```

```
1562.50    0.00 -1562.50    0.00
   0.00    0.00    0.00    0.00
-1562.50    0.00  1562.50    0.00
   0.00    0.00    0.00    0.00
```

```
>> K3 = truss(12*16,0,12*16,12*12,E,A)   % member 3
K3 =
```

```
0.00    0.00    0.00    0.00
0.00  2083.33    0.00 -2083.33
0.00    0.00    0.00    0.00
0.00 -2083.33    0.00  2083.33
```

```
>> K4 = truss(12*16,12*12,12*32,12*12,E,A) % member 4
K4 =
```

```
1562.50    0.00 -1562.50    0.00
   0.00    0.00    0.00    0.00
-1562.50    0.00  1562.50    0.00
   0.00    0.00    0.00    0.00
```

```
>> K5 = truss(12*16,12*12,12*32,0,E,A)   % member 5
K5 =
```

```
800.00  -600.00  -800.00   600.00
-600.00   450.00   600.00  -450.00
-800.00   600.00   800.00  -600.00
600.00  -450.00  -600.00   450.00
```

```
>> K6 = truss(12*16,0,12*32,12*12,E,A)   % member 6
K6 =
```

```
800.00   600.00  -800.00  -600.00
600.00   450.00  -600.00  -450.00
-800.00  -600.00   800.00   600.00
-600.00  -450.00   600.00   450.00
```

```
>> K7 = truss(12*16,0,12*32,0,E,A)          % member 7
K7 =
```

```
    1562.50    0.00   -1562.50    0.00
         0.00    0.00    0.00    0.00
   -1562.50    0.00    1562.50    0.00
         0.00    0.00    0.00    0.00
```

```
>> K8 = truss(12*32,0,12*32,12*12,E,A)      % member 8
K8 =
```

```
    0.00    0.00    0.00    0.00
    0.00  2083.33    0.00  -2083.33
    0.00    0.00    0.00    0.00
    0.00 -2083.33    0.00   2083.33
```

```
% ----- input the Global Stiffness Matrix by hand ...
```

```
>> Ks = [ 3925    600     0     0   -800   -600   ;
>         600  2533.33    0  -2083.33  -600   -450   ;
>         0     0   3162.5    0  -1562.5    0     ;
>         0  -2083.33    0   2983.33    0     0     ;
>        -800  -600  -1562.5    0   2362.5   600   ;
>        -600  -450     0     0     600  2533.33 ]
```

```
Ks =
```

```
   3925.00    600.00    0.00    0.00   -800.00   -600.00
    600.00   2533.33    0.00  -2083.33   -600.00   -450.00
     0.00     0.00   3162.50    0.00  -1562.50    0.00
     0.00  -2083.33    0.00   2983.33    0.00    0.00
   -800.00   -600.00  -1562.50    0.00   2362.50    600.00
   -600.00   -450.00    0.00    0.00    600.00   2533.33
```

```
>> find(Ks-Ks')           % check to see if Ks is symmetric ...
ans = [](0x0)             % It is!
```

```
>> p = [ 0 -100 0 0 50 0 ]'      % input the load vector
```

```
p =
```

```
    0.00
 -100.00
    0.00
    0.00
    50.00
    0.00
```

```
>> format                 % change formats for more sig. fig's
```

```
>> d = Ks \ p             % compute the joint displacements
```

```
d =
```

```
    0.0146067
   -0.1046405
    0.0027214
   -0.0730729
    0.0055080
   -0.0164325
```