

CE 281: Experimental Systems

Fall, 2000

Lab 2, due October 31, 2000

Photo-elastic analysis of a thick ring under diametric compression.

Analysis

The bending stresses in a thick ring can be approximated by¹

$$\sigma_b(r, \theta) = \frac{M(\theta)(R - r)}{rA(\bar{r} - R)} \quad (1)$$

where $M(\theta)$ is the bending moment, r is the distance from the center of curvature, A is the cross sectional area of the beam, \bar{r} is the radius to the centroid, and R is the radius to the neutral axis,

$$R = \frac{A}{\int_A dA/r}. \quad (2)$$

For a ring under diametric compression, derive expressions for the bending moment $M(\theta)$ as a function of the angle θ from the loading point. Also derive expressions for the shear force $V(\theta)$ and the normal force $N(\theta)$.

If the load is applied along a vertical axis, then the shear force on a horizontal section passing through the center of the ring is zero. Using the expressions above, and those that you derive, determine the stress acting on this horizontal plane as a function of r . Along the inner and outer radii of the ring the shear stress should be zero as well. Again, using your expressions, determine the circumferential stresses at the inner and outer radii.

Knowing the state of stress on these surfaces, and that this is a plane-stress problem, you can determine the maximum shear stress τ_{\max} along these surfaces. Plot the maximum shear stress as a function of r along a radial horizontal cut, and the maximum shear stress profile as a function of θ for the inner and outer surfaces.

Experiment

We will calibrate the photo-elastic analysis using a calibration disk of the same material as the ring under test. The fringe order at the center of the disk, N , is proportional to the diametric load P , the diameter D , and the photoelastic sensitivity f_σ . f_σ has units of [stress/fringe/thickness].

$$f_\sigma = \frac{8P}{\pi DN} \quad (3)$$

Note the dimensions of the calibration disk and the load applied to it.

Obtain a digital image of the calibration disk from <http://www.duke.edu/hpgavin/ce281/> and determine f_σ .

¹Egor P. Popov, *Introduction to mechanics of solids*, Prentice Hall, 1968, pp 208–213.

Knowing the relationship between N and τ_{\max} use the images of the *thin ring only* at

<http://www.duke.edu/~hpgavin/ce281/>

and figure 12.22 of Beckwith, to determine the stresses on a horizontal section parallel to the cut, and the circumferential stresses along the inner and outer radii. For the image in Beckwith, assume that the material and thickness are the same as for the calibration disk. Also assume that the ring in the photocopy distributed in class is full size. Determine the load required to produce the fringe pattern shown.

To interpret the images, first find the zero-order fringe ($N = 0$). Then count fringes moving away from the zero-order fringe. Using a ruler, a protractor, or another method, tabulate the fringe orders (or stresses) as a function of radius and circumference for the cases to be analyzed. Compare these stresses with the analytical prediction of the previous section.

Data Analysis

Type your data into columns of an ASCII file. Plot the data with points and the analytical prediction with lines.

Report

Describe the objective, analysis, design, testing, and data analysis, and conclusions of your experiment in separate sections.

Show the derivation of $N(\theta)$ and $V(\theta)$ in L^AT_EX.

Do the analytical and physical models support one another?

Labs are to be typeset in L^AT_EX. PostScript figures are to be generated using Gnuplot, MATLAB, or another computer plotting package, and included within the text or at the end of the lab report.