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TECHNICAL NOTE

A NOTE ON NON-LINEAR ELASTICITY OF ISOTROPIC OVERCONSOLIDATED CLAYS

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SUMMARY

An empirically established rule of Wroth[1] for the dependence of the shear modulus on the mean effective pressure and the overconsolidation ratio in clays is investigated within the framework of non-linear elasticity. The resulting isotropic–deviatoric coupling is derived and compared to experiments.

INTRODUCTION

The use of linear elastic theory to describe clay behaviour has led to a considerable confusion in the literature. Attempts were made to represent stress–strain curves by quasi-elastic behaviour and select equivalent moduli to predict, for example, the settlement of structures on clay soils. However, because of the strong dependence of the elastic moduli on the mean effective pressure, the increase of stiffness with depth has an important influence on the actual magnitude of soil settlement. This prompted many researchers to investigate how the values of the moduli actually vary with pressure and the overconsolidation ratio, and how this kind of behaviour can affect engineering predictions, Simons.[2]

This paper deals with one of the aspects of non-linearity of clays in overconsolidated range—the dependence of the shear modulus on the effective mean stress and the overconsolidation ratio. Experiments of Baldi *et al.*[3] on Boom clay indicate that its shear modulus may increase up to four times when the mean stress grows to 6 MPa. It is also studied how a particular clay with a fixed maximum preconsolidation stress is affected by the overconsolidation ratio. The value of the shear modulus at an isotropic stress increases with the overconsolidation ratio. Ladd[4] has shown that this increase is linearly related to the logarithm of overconsolidation ratio. Therefore, using a constant shear modulus in problems where there is a meaningful variation of the isotropic stress may lead to significant errors.

In this paper, an empirical equation proposed for clays by Wroth,[1] and also Wroth and Houlsby[5] is chosen to represent the variation of shear modulus. Hyperelasticity imposes constraints on the choice of non-linear elastic moduli resulting from the history independence of strain and energy. These constraints are used to derive a set of consistent stress–strain equations.

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Above all, the shear modulus dependence on the mean effective stress implies a dependence of the volumetric strain on the shear stress. A particular form of such dependence corresponding to Wroth's law is derived. The predictions calculated using the model for different stress paths are compared within a plastic yield surface to the experimental results obtained at ISMES, Bergamo, Italy, for Boom clay. Cross-effects resulting from the volumetric–deviatoric coupling are discussed.

EXPERIMENTS AND EMPIRICAL BACKGROUND

In linear isotropic elasticity, two basic constants—the effective Young's modulus E and the Poisson ratio ν—are required for material description. Alternatively, the effective bulk and shear moduli, K and G, may be used. The above constants apply to dry or drained soil conditions. Under undrained conditions, in terms of the total stress, the bulk modulus is infinite assuming incompressibility of water, and the shear modulus equals to that under drained conditions, i.e. $G_u = G$.

For most clays the elasticity moduli are known not to be constants. Two principal non-linear effects in clays are a decreasing volumetric deformability during increasing isotropic loading and a dependence of the shear modulus on the mean normal stress and the overconsolidation ratio. The results of consolidation tests reported by Namy[6] suggest that the elastic bulk modulus varies linearly with the mean effective pressure. The elastic shear modulus increases with depth in soil deposits, as shown e.g. by Jamiolkowski *et al.*[7]

Figure 1(a) shows the variation of the volumetric strain with isotropic pressure in an isotropic consolidation test of natural Boom Clay from a depth of 240 m. Figure 1(b) shows for the same material the variation of shear modulus with initial isotropic pressure in undrained triaxial

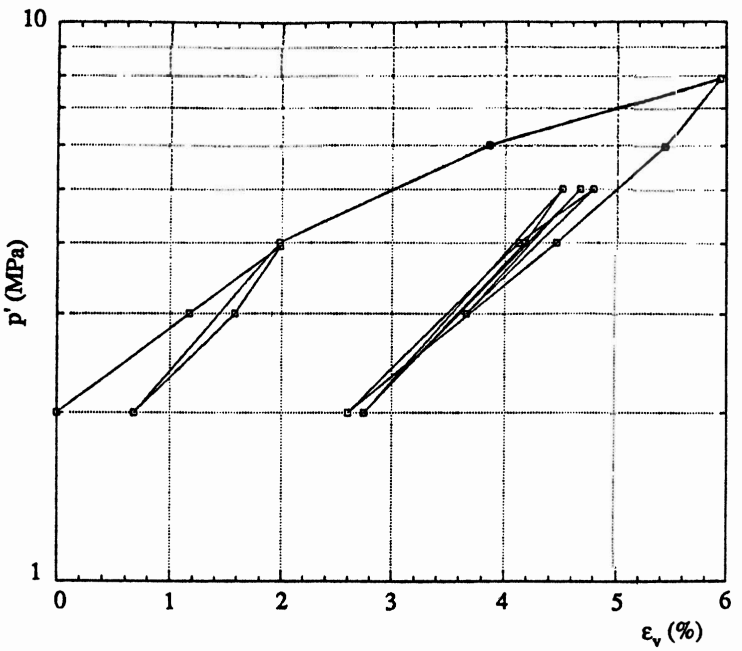

Figure 1(a). Variation of the volumetric strain with isotropic pressure for natural Boom clay, after Baldi et al.[3]

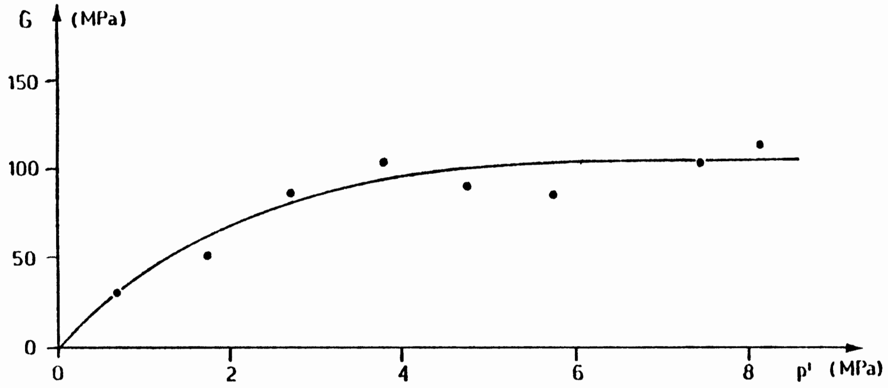

Figure 1(b). Shear modulus G variation with the effective isotropic presusre p', after Baldi et al.[19]

compression test. Boom clay is a dark silty clay, very stiff with 22 per cent smectite, 19 per cent illite, 29 per cent kaolinite and 30 per cent quartz. Its water content ranges from 19–21 per cent. The liquid limit is 59–66 and the plasticity index is 25–27. The specific gravity is 2·70. Figure 2(a) illustrates the effective stress paths obtained from undrained, overconsolidated tests for $p'_0 < p'_c = 6$ MPa and normally consolidated tests ($p'_0 \geq p'_c = 6$ MPa). Figures 2(b) and 2(c) give the corresponding stress–strain and water pressure curves.

As far as the volumetric response is concerned, it is customary to use a logarithmic representation in which the volumetric strain, ε_v is linear in logarithm of isotropic stress. As for the shear modulus, the mean principal effective stress and the overconsolidation ratio are considered to be the primary variables.

Wroth and Houlsby[5] discuss at length the variation of the shear modulus ratio with the mean effective pressure and with the overconsolidation ratio. According to Ladd,[4] this ratio is linearly related to the logarithm of the overconsolidation ratio. This was also indicated by the analysis by Wroth[1] of the tests on undisturbed London clay carried out by Webb.[8]

Wroth[1] and Wroth and Houlsby[5] suggested that the ratio of the shear modulus to the mean effective stress, G/p'_0, obeys in the undrained tests the following relationship:

$$G/p'_0 = (G/p'_0)_{\mathrm{NC}}[1 + C\ln n] \tag{1}$$

where p'_0 is the value of the mean effective normal stress, $(G/p'_0)_{\mathrm{NC}}$ is the value for a normally consolidated clay specimen, C is some dimensionless constant that would need to be measured for any particular soil, and following Wroth,[1] n denotes the overconsolidation ratio in terms of the mean effective normal stress. Wroth[1] also confirmed the validity of equation (1) in drained triaxial tests, which gave the same C. The current mean effective stress p' is then substituted for p'_0 in equation (1).

In most cases, the overconsolidation ratio is usually not known for undisturbed samples. Thus, in equation (1), instead of the overconsolidation ratio, a measurable quantity, i.e. the void ratio, was employed for a given load. An alternative parameter, e_λ, which is proportional to overconsolidation ratio was then introduced:

$$e_\lambda = e + \Lambda \ln p' \tag{2}$$

where e is the void ratio and Λ denotes the gradient of the normal consolidation line. It may be seen that the parameter e_λ has the special property that its value is constant for all states disposed

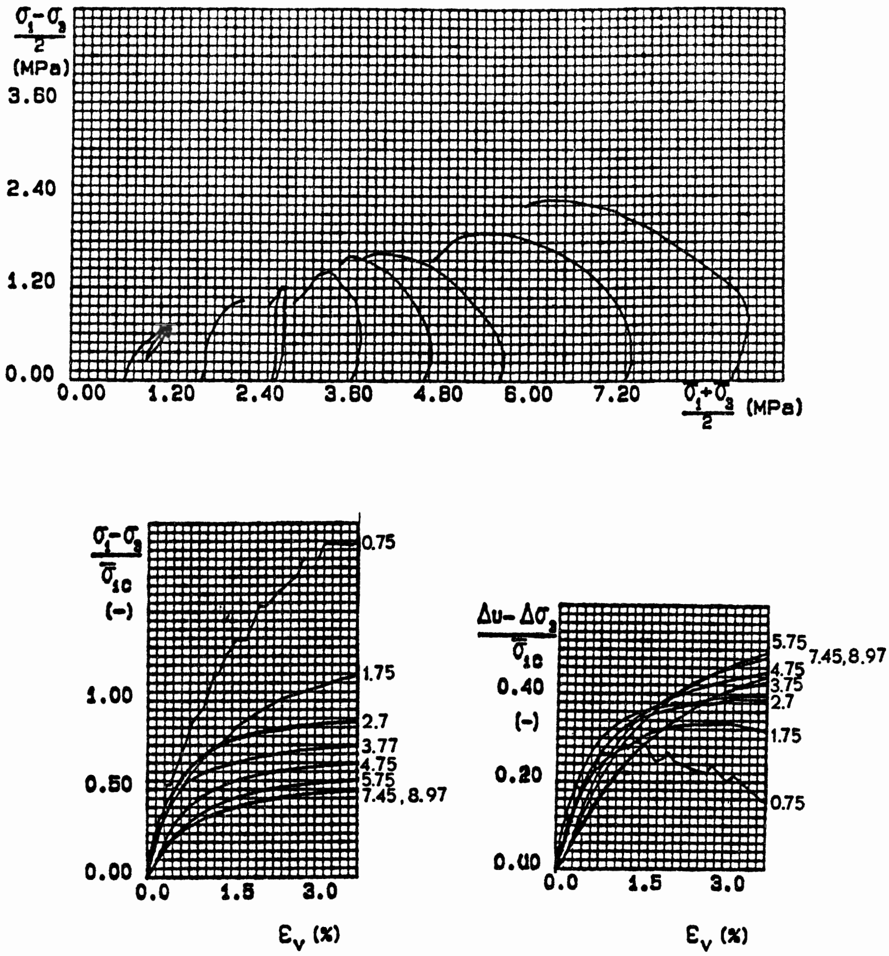

Figure 2(a). The effective stress paths and the corresponding (b) stress–strain and (c) water pressure curves for Boom clay, after Baldi et al.[19]

along any line which is parallel to the normal consolidation line, and thus, linearly related to the logarithm of overconsolidation ratio. Another similar parameter, v_λ, which decreases linearly with the overconsolidation ratio, may be used analogously to e_λ.

The parameters e_λ or v_λ can be easily determined once the constant Λ and the volumetric strain are known. In Figure 3 are shown the processed data for London clay, in the form of G/p'_0 vs. v_λ plots, taken from Wroth and Houlsby.[5] The values for C should be found experimentally using undisturbed samples taken from several depths and were suggested to vary in the range of 0·6–1·4.

THEORETICAL BACKGROUND

Clays exhibit both elastic and plastic properties. They behave elastically, if hysteresis effect is neglected in the initial range of loading and during unloading and reloading, when the stress state is enclosed within the yield surface. These processes may appear strongly non-linear even in the range of small deformations. An elastic process is characterized by its total reversibility. Non-linear reversible processes may be described using hyperelasticity formulation, in which the strain

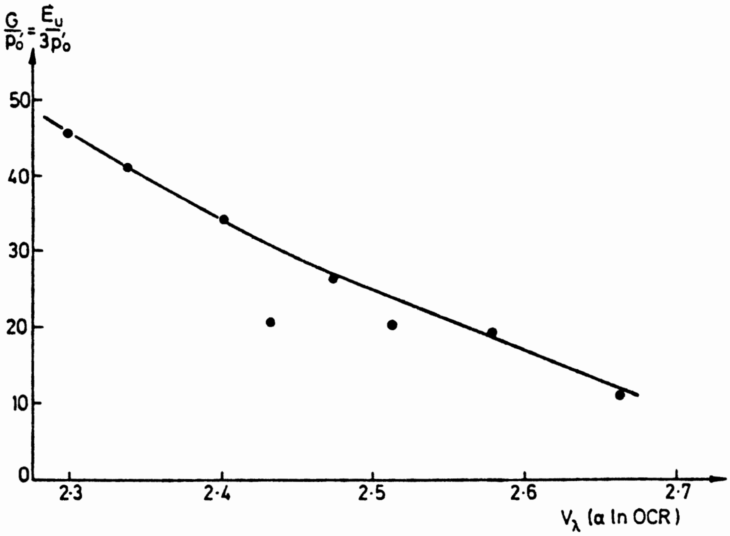

Figure 3. Variation of the shear modulus of London clay with pressure and OCR, after Wroth and Houlsby[5]

ε_{ij} is determined uniquely through the current effective stress σ'_{ij} and is derived as the gradient of a stress potential function (i.e. complementary energy function), V, as

$$\varepsilon_{ij} = A_{ijkl}\sigma'_{kl} = \frac{\partial V(\sigma'_{ij})}{\partial \sigma'_{ij}} \tag{3}$$

where A_{ijkl} is the elastic compliance tensor.

For an isotropic material, the stress potential function V can be expressed in terms of any three independent invariants of the effective stress tensor σ'_{ij}.

In what follows only a tensorially linear relation is considered:

$$\varepsilon_{ij} = \frac{1}{3K}\delta_{ij}p' + \frac{1}{2G}s_{ij} \tag{4a}$$

or

$$p' = K\varepsilon_{kk} \tag{4b}$$

$$s_{ij} = 2Ge_{ij} \tag{4c}$$

where $K = K(\sigma'_{ij})$ is the secant bulk modulus and $G = G(\sigma'_{ij})$ is the secant shear modulus, s_{ij} is the deviatoric stress tensor and e_{ij} is the deviatoric strain tensor.

Since the elastic potnetial is path-independent, the following expressions must be partial derivatives:

$$\varepsilon_{kk} = \frac{\partial V}{\partial p'} \quad \text{and} \quad e_{ij} = \frac{\partial V}{\partial s_{ij}} \tag{5}$$

The reciprocity theorem for second-order, mixed, partial derivatives requires thus that the following relationship must be satisfied at all states of stress

$$\frac{\partial}{\partial p'}\left(\frac{\partial V}{\partial s_{ij}}\right) = \frac{\partial}{\partial s_{ij}}\left(\frac{\partial V}{\partial p'}\right) \quad \text{or} \quad \frac{\partial e_{ij}}{\partial p'} = \frac{\partial \varepsilon_{kk}}{\partial s_{ij}} \; . \tag{6}$$

For non-linear forms of equations (4), both bulk and shear moduli being functions of stress invariants, conditions (6) lead to the requirement that

$$\frac{\partial\left(\dfrac{s_{ij}}{2G(p', s_{ij})}\right)}{\partial p'} = \frac{\partial\left(\dfrac{p'}{K(p', s_{ij})}\right)}{\partial s_{ij}} \tag{7}$$

This means that if there is an experimental evidence on the variation of the moduli K and G with stress, this variation must fulfil equation (7) for the behaviour to be qualified as elastic. For instance, if $K = K(p')$ only, the shear modulus G cannot be a function of pressure.

It should be underlined that especially in the modelling of elastoplastic behaviour, which implies the existence of plastic strain, fulfilment of equation (7) is essential. Otherwise, a generic loading–unloading cycle, although meant as elastic, may generate an irreversible strain, which makes the definition and determination of plastic strain non-unique and conceptually confusing.

Conditions (7) and other similar ones were previously discussed by Mroz,[9] Hueckel and Drescher,[10] Hueckel[11] and Boyce.[12] Zytynski *et al.*[13] used the experimental evidence of Namy[6] suggesting that both bulk modulus and shear modulus are proportional to p', and concluded that such a model is non-conservative and not viable over the whole of the elastic region. Several non-linear elastic equations complying with condition (7) were proposed for sand, or granular media in general.[11,12,14,15] Houlsby[16] discussed separately the dependence of the shear modulus of clays on pressure and the preconsolidation pressure.

A hyperelastic model discussed in this paper is based on the logarithmic volumetric response to isotropic compression and on Wroth's[1] equation (1) for variable shear modulus. This makes it different from the previous works, in that it takes into account simultaneously the dependence of the moduli on the effective stress and the overconsolidation ratio. The philosophy adopted by the earlier authors is followed here. This means that accepting a form of dependence of the shear modulus on the mean normal stress, an appropriate expression is seeked through reciprocity conditions for the bulk modulus dependence on the shear stress.

The volumetric strain response in isotropic processes shall be assumed proportional to the logarithm of the mean effective stress:

$$\varepsilon_v = \frac{\kappa}{1 + e_0} \ln(p'/p'_0) \tag{8}$$

where κ is a material constant (slope of the isotropic unloading or swelling line), e_0 is the reference void ratio and p'_0 is a reference mean effective stress.

The deviatoric strain in equation (4c) is limited to a mean effective-pressure-dependent shear modulus as

$$e_{ij} = \frac{s_{ij}}{2G(p')} \tag{9}$$

In order to satisfy the reciprocity requirement (6), the volumetric strain due to non-isotropic load is proposed to be a function of a deviatoric stress invariant as follows:

$$\varepsilon_v = \frac{\kappa}{1 + e_0} \ln(p'/p'_0) + F(s_{ij}s_{ij}) \tag{10}$$

It is thus clear from this equation that the pressure dependence of the shear modulus G implies also a deviatoric–isotropic coupling in volumetric strain. Function $F(s_{ij}, s_{ij})$ can be obtained, if the strain deviator e_{ij} is known through equation (9) as a function of the mean effective stress p'. Then the partial derivative of e_{ij} with respect to p' is also known and, by virtue of equation (6), the volumetric strain ε_v is obtained by integrating the derivative with respect to the deviatoric stress tensor s_{ij}.

The functional form [equation (1)] of the shear modulus G is chosen after Wroth[1] and Wroth and Houlsby.[5] It can be rewritten for a general state of stress as

$$G = G_n(p'/p'_c)[1 + C\ln(p'_c/p')] \tag{11}$$

or

$$G = G'_n(p'/p'_0)\{1 + C[\ln(p'_c/p'_0) - \ln(p'/p'_0)]\} \tag{12}$$

where G_n is the shear modulus at normal consolidation and taken as a material constant in this theory, p'_c is the maximum isotropic preconsolidation pressure and C is Wroth's constant from equation (1). In reality their values depend on plastic strain; thus an elastoplastic coupling theory[11] should be employed to include this effect as in Houlsby.[16] In this paper, for a given material, they will be considered as constants. p'_0 is an isotropic stress in the initial state, while G'_n is a constant defined as

$$G'_n = G_n(p'_0/p'_c) \quad \text{and} \quad p'_0 < p'_c, \quad p' < p'_c \tag{13}$$

In equation (1), the overconsolidation ratio n has been substituted by the ratio of maximum isotropic past pressure to the current isotropic pressure, p'_c/p', following Wroth.[1] This is different from the usual definition, in which OCR is a ratio of the maximum vertical past pressure to the *in situ* overburden pressure; see for e.g. Reference 17. The two definitions are, in general, not equivalent. It should be noted that while n and OCR *in situ* are characteristics of soil, during a loading process they become variable and change with p'.

The reciprocity condition (6) may now be used as

$$\frac{\partial e_{ij}}{\partial p'} = \frac{\partial\left(\dfrac{s_{ij}}{2G(p', s_{ij})}\right)}{\partial p'} = -\frac{s_{ij}}{2G^2}\frac{\partial G}{\partial p'}$$

$$= -\frac{G'_n}{2G^2 p'_0}[L_2 - C\ln(p'/p'_0)]s_{ij} \tag{14}$$

where the constant L_2 is

$$L_2 = 1 + C[\ln(p'_c/p'_0) - 1] \tag{15}$$

Integrating the above equation with respect to s_{ij} according to (6), one obtains

$$\varepsilon_v = F_0(p') - \frac{G'_n}{4G^2 p'_0}(L_2 - C\ln(p'/p'_0))s_{ij}s_{ij} \tag{16}$$

where the function $F_0(p')$ is already defined through equation (8).

Hence, the volumetric strain–stress relationship can be written as

$$\varepsilon_v = \left(\frac{\kappa}{1 + e_0} + \frac{C}{L_2}\frac{1}{4G_1^2}s_{ij}s_{ij}\right)\ln(p'/p'_0) - \frac{1}{4G_1^2}s_{ij}s_{ij}, \quad G_1^2 = \frac{G^2 p'_0}{L_2 G'_n} \tag{17}$$

or

$$\varepsilon_v = \frac{\kappa}{1 + e_0}\ln(p'/p'_0) - \frac{F_1}{4G^2}q^2 \tag{18}$$

where

$$F_1 = \tfrac{2}{3}G'_n/p'_c[L_2 - C\ln(p'/p'_0)], \quad q = \sqrt{(\tfrac{3}{2}s_{ij}s_{ij})}$$

A similar equation for the volumetric strain is given by Mroz and Norris[18] for an arbitrary $G = G(p')$.

To invert equation (17), the substitution of $s_{ij}s_{ij} = 4G^2 e_{ij}e_{ij}$ results in the following equation:

$$p'/p'_0 = \exp\left(\frac{\dfrac{\varepsilon_v}{e_{ij}e_{ij}} + g_1}{\dfrac{\kappa}{1+e_0}\dfrac{1}{e_{ij}e_{ij}} + g_2}\right) \tag{19}$$

where

$$g_1 = \frac{L_2 G'_n}{p'_0} \quad \text{and} \quad g_2 = \frac{G'_n C}{p'_0}$$

are constants for a given natural material and initial state p'_0.

SPECIALIZATION TO SELECTED STRESS PATHS

In this section, the above model is specialized for cases of triaxial undrained compression and drained extension tests, which include (i) elastic response in terms of p' in an undrained test ($\varepsilon_v = 0$), in which q is increased and (ii) elastic response to a drained isotropic loading, $q = q_0$ = constant test, where ε_v and ε_q are given in terms of p'.

Under triaxial conditions,

$$s_{ij}s_{ij} = \tfrac{2}{3}q^2, \qquad e_{ij}e_{ij} = \tfrac{3}{2}\varepsilon_q^2 \quad \text{and} \quad q = 3G\varepsilon_q \tag{20}$$

Undrained triaxial compression test ($\varepsilon_v = 0$)

In this type of test, uncoupled elasticity predicts that the mean effective pressure is constant. However, due to the isotropic–deviatoric coupling in equation (19), the effective stress paths are expected to be deviated from straight lines at large deviatoric stresses.

The effective stress path equation can be obtained from equation (19) by setting $\varepsilon_v = 0$,

$$\ln(p'/p'_0) = \left(\frac{g_1}{\dfrac{\kappa}{1+e_0}\dfrac{6G^2}{q^2} + g_2}\right) \tag{21}$$

or, substituting the variable modulus G from equation (12) and solving for q, the equation of the effective stress path is

$$q = \sqrt{\left(\frac{A(p'/p'_0)^2\{B\ln(p'/p'_0) - D[\ln(p'/p'_0)]^2 + C^2[\ln(p'/p'_0)]^3\}}{G_1 - G_2\ln(p'/p'_0)}\right)} \tag{22}$$

where

$$A = \frac{6G'^2_n\kappa}{(1+e_0)}$$

$$B = (1 + 2CL_c + C^2L_c^2)$$

C = the original model parameter defined by Wroth[1]

$$D = (2C^2L_c + 2C)$$

and

$$L_c = \ln(p'_c/p'_0)$$

The deviatoric strain ε_q in terms of q and p' is obtained from equations (12) and (20);

$$\varepsilon_q = \frac{q}{3G'_n(p'/p'_0)\{1 + C[L_c - \ln(p'/p'_0)]\}} \tag{23}$$

where q is given through equation (22).

Drained, constant shear, extension test ($dq = d\sigma'_1 - d\sigma'_3 = 0$)

In this test (at a constant value of deviatoric stress q), the cell pressure in the triaxial apparatus and the vertical stress are changed at the same rate and the corresponding changes in the deviatoric and volumetric strains are observed. Thus, rewriting equation (19) for constant $q = q_0$, and substituting for ε_v yields

$$\varepsilon_v = \frac{\kappa}{1+e_0}\ln(p'/p'_0) + \frac{G_2\ln(p'/p'_0) - G_1}{\dfrac{6G_n'^2}{q_0^2}(p'/p'_0)^2\{B - D\ln(p'/p'_0) + C^2[\ln(p'/p'_0)]^2\}} \tag{24}$$

where the constants B, C and D are given by equation (22).

The deviatoric strain can be exressed as follows:

$$\varepsilon_q = \frac{q_0}{3G'_n(p'/p'_0)\{1 + C[L_c - \ln(p'/p'_0)]\}} \tag{25}$$

Thus, a non-zero deviatoric strain occurs as the isotropic pressure changes at a constant shear stress.

DETERMINATION OF MATERIALS CONSTANTS

The experimental data needed to simulate the model behaviour were obtained from the tests performed on homogeneous specimens of Boom clay at ISMES, Bergamo, Italy, Baldi.[3,19] The material constants are found to be the following:

(a) Initial void ratio, $e_0 = 0{\cdot}589$.
(b) Slope of the isotropic swelling line, $\kappa = 0{\cdot}031$.
(c) Maximum effective isotropic past pressure, $p'_c = 6$ MPa.
(d) The value of shear modulus at normal consolidation, $G_n = 100$ MPa, see Figure 1(b).

To find the parameter C, the curve for shear modulus dependence on the mean effective pressure p' [Figure 1(b)] is replotted as G/p' against the logarithm of the overconsolidation ratio in (Figure 4). The constant C is then obtained from Wroth's equation (11) as 0·982.

NUMERICAL SIMULATIONS AND DISCUSSION

Undrained triaxial compression

A series of undrained tests ($\varepsilon_v = 0$) on Boom clay were performed at ISMES with different values of the initial isotropic effective stress, see Figure 2.

The plastic yield function defining the elastic domain is assumed to be an ellipse, the equation of which is given as follows:

$$f = \left(\frac{2p'}{p'_c} - 1\right)^2 + \left(\frac{2q}{p'_c M}\right)^2 - 1 = 0 \tag{26}$$

where $p'_c = 6$ MPa and $M = 0.87$.

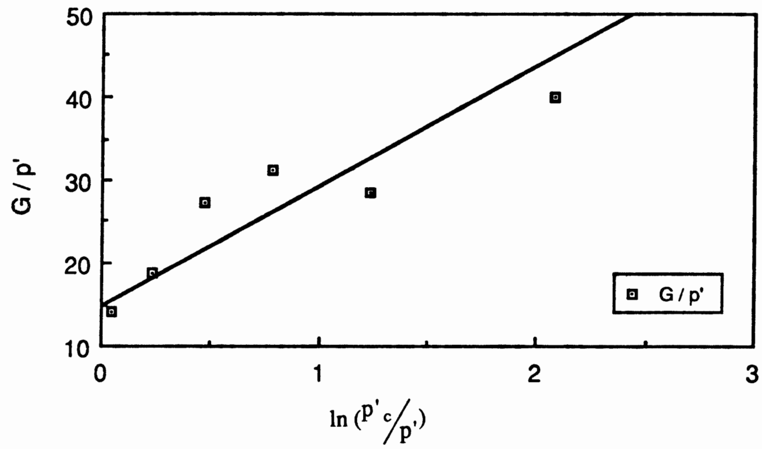

Figure 4. Variation of the shear modulus of Boom clay with pressure and the logarithm of the overconsolidation ratio; $p'_c = 6$ MPa

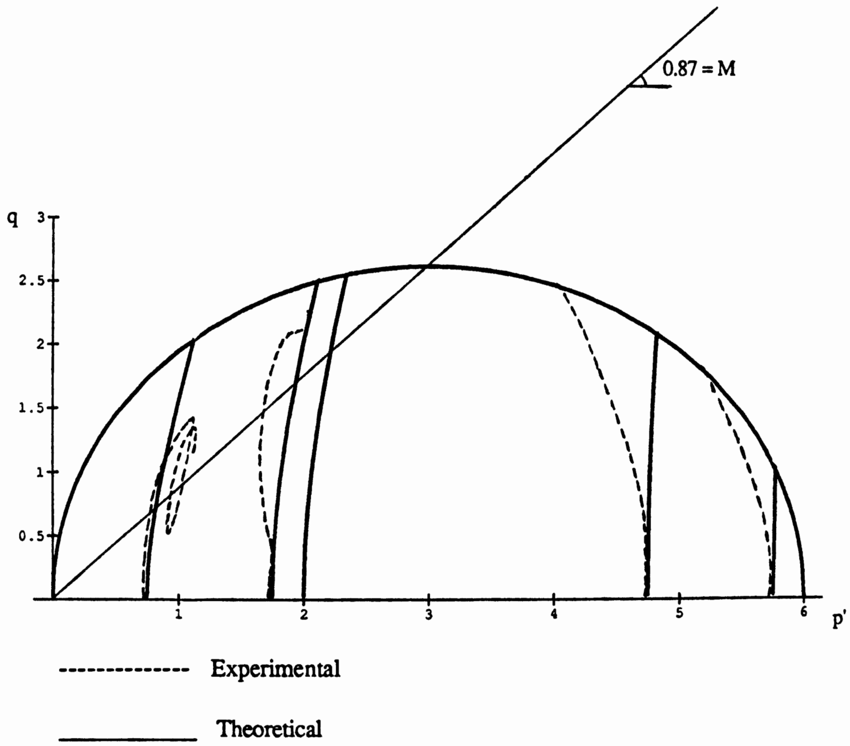

Figure 5. Representation of the theoretical and the experimental stress paths on the yield surface for Boom clay

The theoretical stress paths are calculated for the initial values of the mean effective stress p'_0 equal to 0·75, 1·75, 2·0, 4·75 and 5·75 MPa using equation (22). These stress paths are plotted in Figure 5, where the experimental paths are shown by dotted lines. The plastic yield surface is also shown. Clearly, the simulated paths are to be considered only within the yield surface.

First of all, it may be noted that the simulated effective stress paths markedly deviate from the p' = constant lines. This deviation in all the cases is towards compression, but it decreases with increasing effective mean stress, and close to normal consolidation, is almost imperceptible. At low effective pressures (at $n > 3$), the simulated paths match almost perfectly the experimental results.

Considering that the stress *in situ* for Boom clay is close to the isotropic pressure, equal to 2 MPa, it may be stated that the simulation of *in situ* properties is very good. At low overconsolidation ratios, the tendency of stress paths to deviate towards lower isotropic stresses at increasing q is not reproduced. The paths are very close to vertical. This, however, could be expected, because as seen from Figure 1(b), at isotropic stresses close to maximum pre-consolidation, the value of the shear modulus does not change much. No change in the modulus implies directly the absence of isotropic–deviatoric coupling as clearly seen from equation (7).

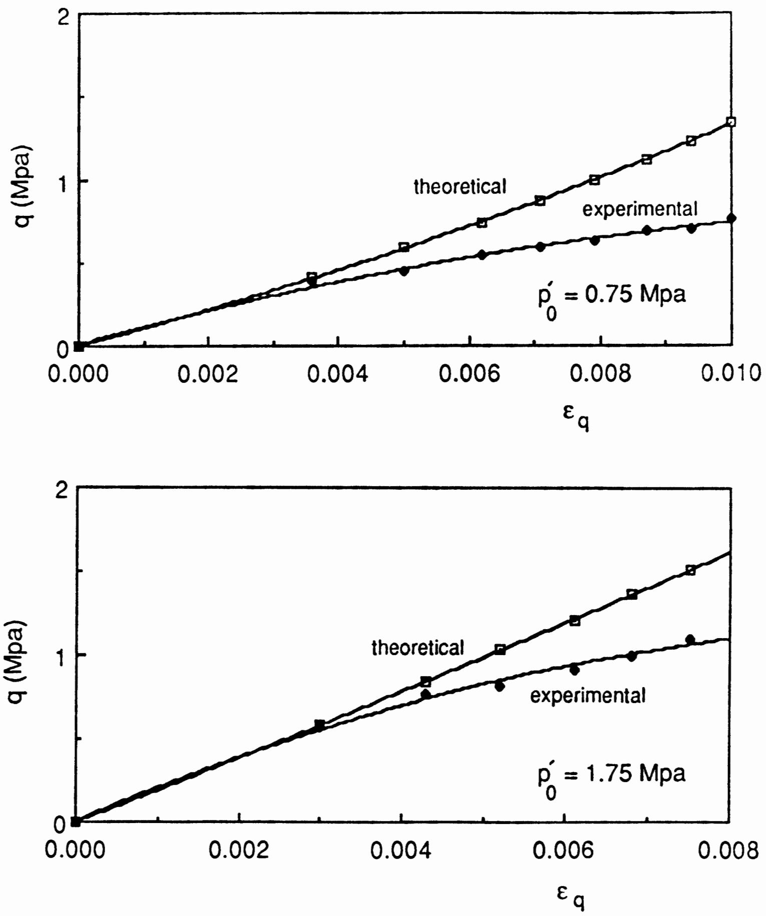

Figure 6. Deviatoric stress–strain curves for $p'_0 = 0{\cdot}75$ MPa, and $p'_0 = 1{\cdot}75$ MPa

Thus, clearly one cannot attribute the observed tendency of stress paths at low values of the overconsolidation ratio to the variability of the shear modulus if this is assumed to be a monotonic function of p'_0. However, a closer inspection of the curve in Figure 1(b) as well as some of the curves published by Ladd[4] indicate that there is a slight tendency of the modulus to drop at values of $n < 1{\cdot}3$. Such a tendency is not, however, confirmed by Webb's results and is not predicted by Wroth's equation (11). The actual significance of such a possible behaviour should be more closely investigated. The tendency of the undrained stress path to decline towards increasing isotropic stress was also obtained by Houlsby;[16] however, his curves are almost parallel for all overconsolidation ratios.

To simulate the deviatoric stress–strain response, equation (23) is used, where for given values of p' the corresponding values of q are calculated from equation (22). Using this procedure, deviatoric stress–strain curves (q vs. ε_q) are obtained for the initial mean effective stress values of 0·75, 1·75, 2·0, 4·75 and 5·75 MPa. Figures 6 and 7 show the theoretical and experimental curves for the above-listed initial mean effective pressures limited to the elastic range. A comparison of the model behaviour with the experimental evidence indicates that the strain is generally underestimated in the simulation, mainly at higher overconsolidation ratios.

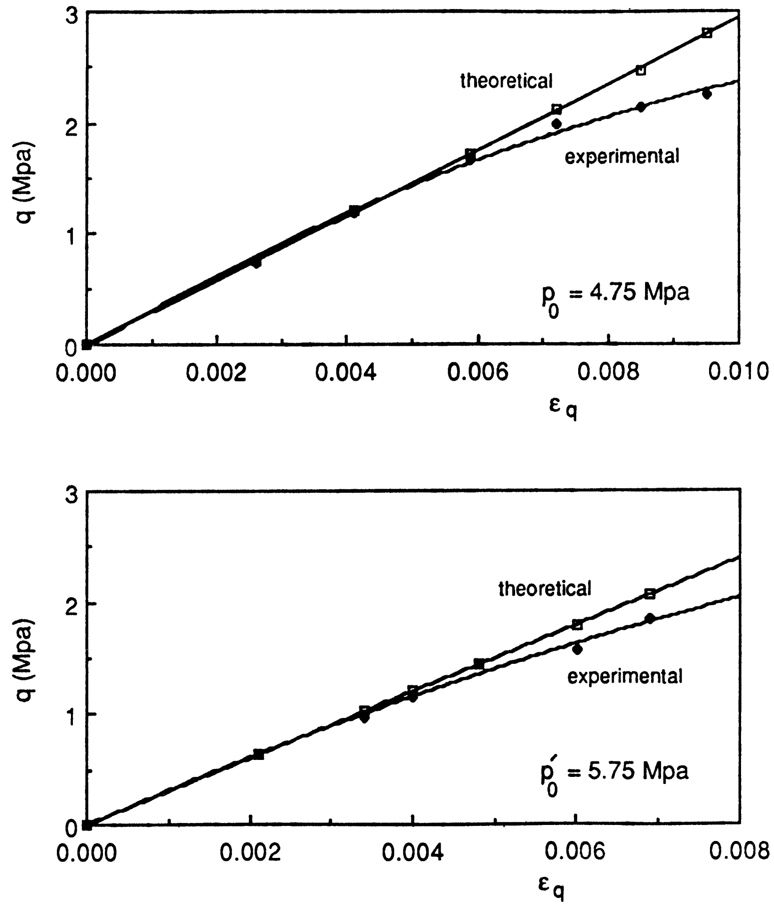

Figure 7. Deviatoric stress–strain curves for $p'_0 = 4{\cdot}75$ MPa and $p'_0 = 5{\cdot}75$ MPa

Drained triaxial extension

In this case, the experimental data obtained from tests on Boom clay at ISMES[3] are discussed to analyse the simulation of the elastic behaviour of the model at constant deviatoric stress $q = q_0 = 1$ MPa. To find whether the process is reversible or not, cyclic unloading and reloading of the mean effective stress was performed with the initial effective stress of $p'_0 = 2$ MPa. The unloading and reloading cycle is as follows (Figure 8):

$$p' = p'_0 = 2\ \text{MPa} \rightarrow p' = 1{\cdot}5\ \text{MPa} \rightarrow p' = 1.2\ \text{MPa} \rightarrow p' = 2.0\ \text{MPa}$$

The volumetric and deviatoric strain equations (24) and (25) are plotted against p' at $q = q_0 = 1$ MPa and $p'_0 = 2$ MPa for this test; see Figures 9(a) and 9(b). The corresponding values of the experimental strains are marked on the graphs for the described unloading and reloading program.

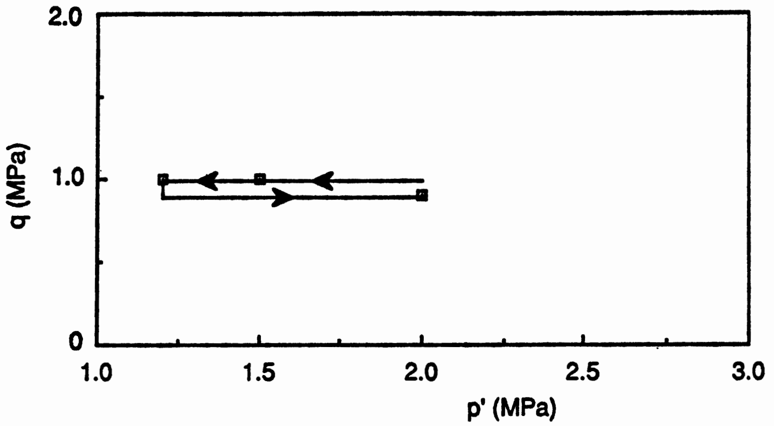

Figure 8. Unloading–reloading cycle of a drained extension test

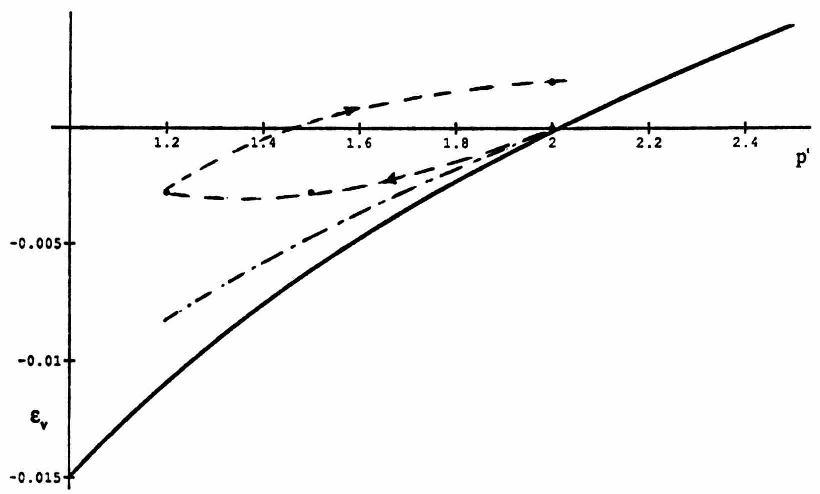

Figure 9(a). Volumetric strain as a function of the isotropic effective pressure with unloading–reloading cycle at $q_0 = 1$ MPa and $p'_0 = 2$ MPa

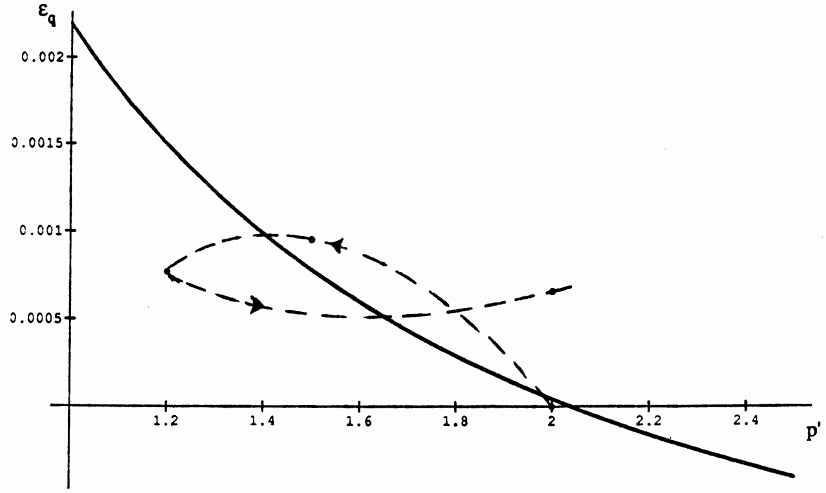

Figure 9(b). Deviatoric strain as a function of the isotropic effective pressure with unloading–reloading cycle at $q_0 = 1$ MPa and $p'_0 = 2$ MPa

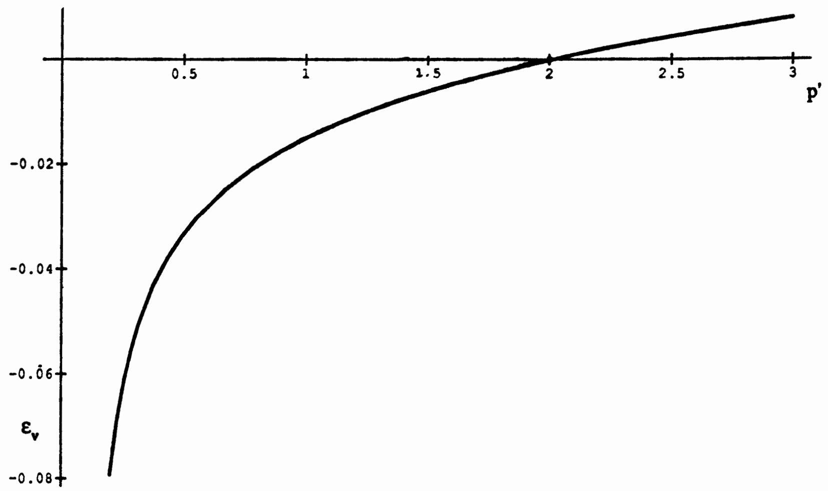

Figure 10. Volumetric strain as a function of the isotropic effective pressure p' at $q_0 = 1$ MPa and $p'_0 = 2$ MPa

In the experiments, the volumetric strains show a visible hysteresis upon unloading and reloading. The prediction is closer to the experimental result in reloading than in unloading. Also shown (dashed line) is the prediction for an uncoupled volumetric strain, following equation (8) in unloading. Both results overestimate the actual strain. However, it must be kept in mind that the modulus is an average value for a large population of samples over a large range of stress, and may be the source of divergence. The deviatoric strains are one order of magnitude smaller than the volumetric strains and the prediction goes through the middle of the experimental hysteresis loop. It appears altogether that the so heavily marked dependence of the shear modulus on pressure, which in the range of interest here, i.e. between 1·2–2·0 MPa, changes over 36 per cent from 50–68 MPa, results in a very modest cross-effect of the rise of deviatoric strain and a corresponding $\sim$ 24 per cent increase in volumetric strain. The latter difference is, however, of the order of the difference from the experimental result.

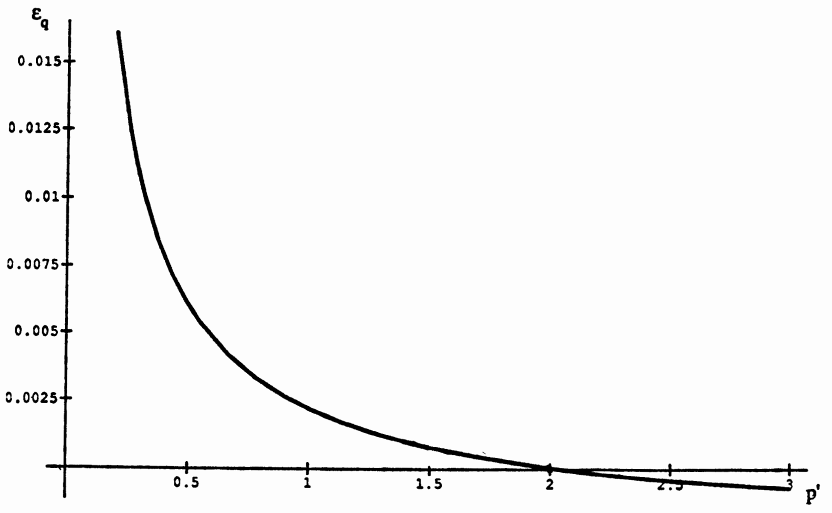

Figure 11. Deviatoric strain as a function of the isotropic effective pressure p' at $q_0 = 1$ MPa and $p'_0 = 2$ MPa

The numerical prediction of the elastic behaviour in a much larger range is shown in Figures 10 and 11. It is visible that the logarithmic effect in volumetric strain becomes important below $p' < 0{\cdot}5$ MPa and that the deviatoric strain follows the same tendency.

CONCLUSIONS

The present study focuses on isotropic–deviatoric coupling in non-linear elasticity of clays induced by a widely observed dependence of the shear modulus on the isotropic pressure. Empirical formula for this dependence proposed by Wroth[1] is incorporated into a theoretically and energetically consistent framework of elasticity. The principal advantage of Wroth's formulation is that it takes into account the influence of the overconsolidation ratio in addition to that of the isotropic pressure itself and requires only one additional constant.

The numerical simulation performed for Boom clay offers the following conclusions. The influence of the shear modulus variation on the stress path in undrained compression is significant and consistent with the experimental evidence at high values of the overconsolidation ratio. Since the shear modulus is approximated as tending to a constant value at lower overconsolidation ratios, the effect of the coupling on the stress path disappears in this range. Some experiments suggest a drop of G for an overconsolidation ratio close to one, which would imply a correct, inverted tendency of the stress path. Anyhow, the monotonic increase of the shear modulus cannot be considered as a cause of the inversion of the undrained stress paths at low overconsolidation ratios and other possible physical causes should be examined. This issue is definitely worth a closer experimental investigation.

The shear modulus sensitivity to p' seems to be consistent with the volumetric strain sensitivity to deviatoric stress, while the cross-effect of the deviatoric strain in isotropic loading at constant shear stress is significantly less important.

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