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## Reactive plasticity for geological materials with a double structure evolving during aging

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### Abstract

Soils subjected to a significant compressive strain *in situ* for a significant period of time under develop a secondary microstructure as a result of reactions of local dissolution/precipitation of less stable minerals. The secondary structure may reach quite a significant compressive strength contributing to a higher overall *in situ* strength. The primary and secondary structure are modeled in terms of reactive plasticity, as they undergo chemical softening and hardening respectively.

### Introduction

*In situ* soil strength and stiffness are notoriously higher from those determined on the basis of laboratory tests. Hypotheses explaining the difference classically focus on the nature and amount of damage to the sample occurring during coring. This paper addresses possible permanent changes in the material *in situ* when it is loaded for a geological scale time periods. It has been suggested (Bjerrum, 1973, Leroueil et al., 1996) that in such conditions soil develops a secondary microstructure through reactions of local dissolution/precipitation of some less stable minerals. The secondary structure may reach quite a significant compressive strength contributing to a higher overall *in situ* strength. However, because its tensile strength is much lower, the secondary microstructure is often damaged during coring operation and unloading, and thus its contribution becomes undetectable or underestimated in laboratory testing.

### Phenomenology

What is called here "secondary microstructure" has been suggested to form *in situ* during unspecified long-term physico-chemical processes producing secondary compression, at constant effective stress, or creep strain. This effect has been attributed to various phenomena termed as aging, thixotropy, delayed compression, age hardening or structuration. While traditionally linked to clays, aging is also known to occur in sands, sandstones, and clayey sands. Aging has been seen to lead also to an increase in stress-strain moduli and cone penetration resistance, even for clean sands (Bjerrum, 1973, Schmertmann, 1991, Mesri et al., 1990).

Aging process may be visualized as loading a soil element in 1-D strain conditions Fig. 1a, up to the *in situ* stress and allowed to reach the end of primary consolidation, that is a state of normal consolidation, as in an experiment by Perret (1995), see Leroueil et al., 1996. Subsequently, a secondary compression develops at constant load for a long period of time, corresponding to period of the geological process of compaction. During the secondary compression soil builds an internal structure resulting into a higher yield limit. As a consequence, when loaded further over an effective stress increment the material exhibits a much higher "apparent pre-compression stress" and initially also a much higher stiffness. However, upon further loading the stiffness reaches that typical of normally consolidated clay.

There is growing evidence that aging process is physico-chemical in nature both for sands and clays. For sand Mitchell and Solymer (1984) attributed the increase of strength during secondary compression to formation of bonds of silica acid gel precipitating from pore solution.

## The mathematical formulation

The considered soil material evolves from a one constituent solid into a two-constituent solid composite. For simplicity pore fluid is not considered here. The evolution starts at the *in situ* stress in the material composed initially from only one (primary) constituent. From then on, there are two constituents: the primary granular material formed by sand grains or granular clay clusters, and the secondary material that constitutes a filler precipitating onto the primary material. The mechanical behavior of the composite is described as that of a mixture of the superimposed continua deforming simultaneously. The total stress in the material is the stress in the primary material until the chemical process of precipitation is terminated, and the secondary material starts to carry a fraction of stress. Any further total stress rate is a weighed sum of the rates of the specific stresses in the constituents

$$\dot{\sigma}_v = \dot{\sigma}_v^{(1)}\alpha + (1-\alpha)\dot{\sigma}_v^{(2)} \quad (1)$$

The coefficient  $\alpha$  is a the contribution coefficient related to the fractions of the two materials in the total volume of solids, considered as constant. Note that the specific stress in the secondary material is defined over the entire initial porous space.

The chemo-mechanical process of the formation of the secondary microstructure consists in the chemical reaction of dissolution of mass from the primary material with the rate  $\dot{\xi}_d$ , followed by precipitation with the rate  $\dot{\xi}_p$ . The precipitation results in a build up of a secondary material in the pores of the primary material. The secondary material is in a stress- and strain-free state during the process of its formation. The dissolution and precipitation are assumed to mechanically weaken the primary material and strengthen secondary material. These processes will be described in terms of chemical softening and chemical hardening plasticity, following Hueckel (1992, 1999). Yielding of each of the two materials occurs when a Cam - clay type yield function and its rate are zero for that material,

$$f^{(i)} = (p^{(i)})^2 - p^{(i)} p_c^{(i)}(\epsilon_v^{p^{(i)}}) + (q^{(i)} / M^{(i)})^2 = 0; \quad \dot{f}^{(i)} = 0 \quad (2)$$

where  $p'$  and  $q$  are first and second deviatoric invariants of specific stresses of the two materials.  $M^{(i)}$  are coefficients related to internal friction angle for each material. The hardening hidden variables,  $p_c^{(i)}$ , describing the apparent maximum past isotropic stress are proposed to be functions not only of the common volumetric plastic strain, but also of the respective degrees of advancement of reactions defined through time integral of the reaction rates  $\dot{\xi}_d, \alpha = d, p$ ,

$$p^a = p^a_{\infty} \exp[(\lambda - \kappa)^{-1} (1 + \epsilon_0) \epsilon_v^{p^a} + R^a \xi_a] \quad (3)$$

where  $R^a$  is the coefficient of chemical softening or hardening. It is assumed that the material element is a closed system. This entails that no mass is exchanged with the external environment during the reactions, therefore described by kinetic rate equations using mass concentration,  $c_a$ , of the dissolved or precipitated minerals. Due to the chemical softening/hardening, the plastic strain rate expressed through the usual associated flow rule becomes a function of reaction rate

$$d\epsilon_v^{p^a} = d\lambda \frac{\partial f}{\partial \sigma_v}; \quad d\lambda = \frac{1}{H} \left( \frac{\partial f}{\partial \sigma_v^{(1)}} d\sigma_v^{(1)} + \frac{\partial f}{\partial \xi_d} d\xi_d \right); \quad H = - \frac{\partial f}{\partial \epsilon_v^{p^a}} \frac{\partial f}{\partial p'} \quad (4)$$

In particular, the resulting secondary compression plastic strain rate occurring during the aging process in the primary material alone at constant stress is

$$d\varepsilon_v^p = - \left( \frac{\partial f}{\partial \xi_d} / \frac{\partial f}{\partial \varepsilon_v^p} \right) d\xi_d \quad (5)$$

In these conditions the plastic strain hardening is induced in the primary material to compensate for the chemical softening to maintain unchanged the stress value. This is a departure from the classical argument that secondary compression is a creep phenomenon, by considering it instead as a result of chemical softening induced by a time dependent kinetic reaction. Eq. (5) indicates that the amount of the secondary compression strain can be used as a measure of the degree of the dissolution reaction advancement, if the ratio in the parenthesis is known.

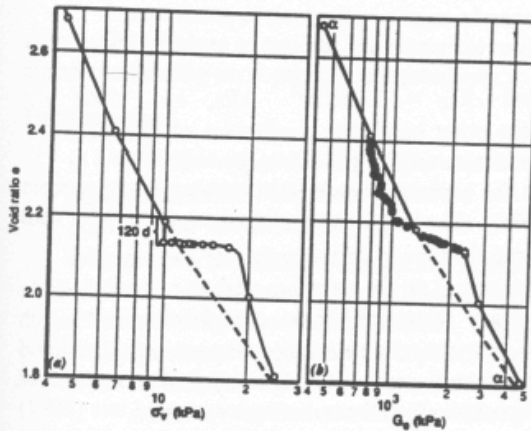
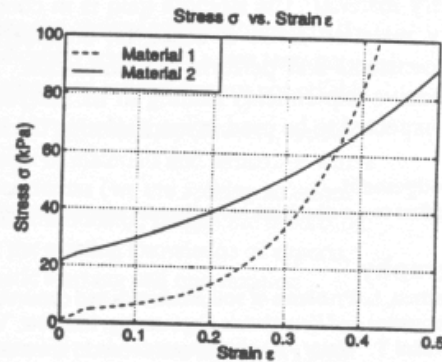
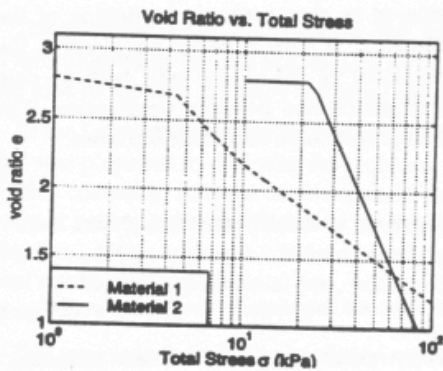
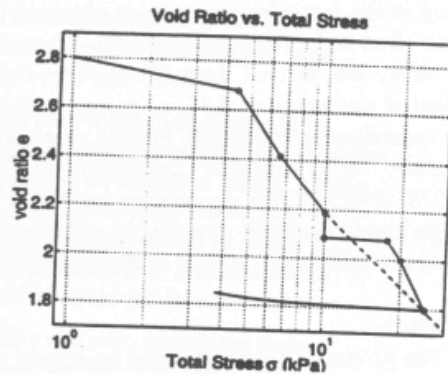


Fig. 1a and b. Stress vs void ratio and elastic shear modulus for Jonquiere clay (from Peret, 1995)

Fig. 2a, b, c. Simulation of the stress void ratio dependence in test in Fig. 1



### Application

In what follows we will focus on the evolution of Jonquiere clay material, artificially sedimented in saline water (Leroueil et al., 1996) Fig.1a. The initial porosity of the material was quite high ( $n=0.68$ ). The material was 15% clay by weight, containing also quartz, amphibole and pyroxene in the form of rock flour  $<2 \mu\text{m}$ . No specific reaction was quantified, but the dissolution of

the rock flour was indicated as a possible cause for the observed structuration. The amount of the secondary compression strain developed during that time was equivalent to 0.024 over 120 days at 10 kPa. Leroueil et al., 1996 proposed to use the evolution of the maximum shear modulus to measure the development of the secondary material structure, shown in Fig. 1b. It is proposed to use this measure to quantify the precipitation reaction rate. It is furthermore proposed that the absolute values of the rates of both reactions are equal,  $\dot{\xi}_p = -\dot{\xi}_d$ , which is to say that all the mass of the dissolved primary material is precipitated to form the secondary material. However, using the change in the elastic shear modulus with respect to the onset of the secondary compression as  $\dot{\xi}_p = -\dot{\xi}_d = \xi_0 \dot{G}_0$ , the coefficient of chemical softening can be found as

$$\bar{R}^d = R^d \xi_0 = \frac{1+e_0}{\lambda-\kappa} \frac{d\varepsilon_v^p}{dG_0} \quad (6)$$

For the numerical values of  $\Delta\varepsilon_v^p = 0.024$  and  $\Delta G_0 = 1.05 \times 10^3$  kPa,  $e_0 = 2.68$ ,  $\kappa = 0.08$ ,  $\lambda = 0.65$ , the value of  $R^d = 0.000148$  kPa $^{-1}$ . On the other hand, the coefficient of chemical hardening in the secondary material is obtained from the estimate of the growth in  $p_c = 21.9$  kPa in the stress-free and strain-free material. Thus, taking for the secondary material the nominal value  $p_{\infty} = 1$  kPa, one arrives from (3) at  $R^p = 0.002938$  kPa $^{-1}$ . The value of the stress contribution coefficient  $\alpha$  was taken as 0.35, as corresponding to the initial fraction of the solids of the material prior to secondary compression. For the whole analysis the lateral stress was assumed as a  $\frac{1}{2}$  of the axial strain. With material constants for the secondary material back calculated to be  $\kappa = 0.0078$ ,  $\lambda = 0.743$ , the experiment by Perret (1995) is reproduced numerically, representing the total logarithmic vertical stress void-ratio curve for the whole material (Fig. 2a) and for the individual materials (Fig. 2b and c) shown in terms of "geotechnical" and mechanical variables.

## Conclusions

The proposed model is based on a long established concept that soil materials gain strength during aging at constant stress, while developing secondary compression strain. As opposed to the traditional approach, this strain is not considered as creep but rather, as a chemo-plastic softening of the primary material. The strength gain is in contrast considered as chemo-plastic hardening of the secondary material. Even if based on engineering experiments, it was possible to identify the material constants and perform the simulation. The development of the model will help to plan comprehensive experiments leading to its independent identification and possible verification. The model is expected to be used in extrapolating the lab results to predict field soil characteristics.

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