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IMPLICATIONS OF THERMAL SENSITIVITY OF THE STATIC INTERNAL FRICTION ANGLE

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ABSTRACT: *Thermal variability of the coefficient of the critical state, or the angle of internal friction is examined in view of experimental results obtained for clays and hard rocks. This sensitivity has several implications for the evolution of what is considered as apparent preconsolidation pressure in constant stress heating tests. Failure conditions in soils at elevated temperatures appear to be strongly dependent on the history of application of stress and temperature, and hence are affected by the thermal sensitivity of the internal friction angle. Consequences to ranges of admissible processes are discussed.*

1 INTRODUCTION

Recent and past experimental evidence suggests that critical state and/or internal friction angle may depend for some geomaterials on temperature. The confusing experimental evidence for clays about the thermal dependence of shear strength points to the possibility of an actual change of the critical state with temperature. This assumption has been discussed before (Hueckel, 1992; Laloui, 1993; Hueckel, 1997; Cekerevac & Laloui, 2004). Foremost, systematic investigations on kaolinite (Cekerevac & Laloui, 2004) have pointed to a very clear growth of M with temperature, as shown in Fig. 1. A thermal variation of the coefficient of critical state, $M(\Delta T)$ has been also observed in experimental data on Boom Clay, as opposed to Pontida clay, which is seemingly insensitive to temperature (Hueckel & Pellegrini, 1991). Other phenomenological implications of such evolution, especially for various failure modes of soils are amply discussed by Hueckel et al. (2009).

A different type of variability of the critical state with temperature is seen for rocks at high temperatures and high pressures. As known for long time, even at constant temperatures rocks exhibit a high non-linearity of the critical state in the stress space (Griffith,1920, Hoek & Brown,1980). As temperature grows, the critical state "gets flatter", to the point that it becomes effective pressure-independent near the melting temperature. The phenomenological aspects of that behavior have been addressed by Hueckel et al. (1994a) and the resulting model has been applied to simulate earthquake inception by a subducting tectonic plate traversing different thermal regimes (Hueckel et al., 1994b).

Therefore, it appears that thermally variable static internal friction angle may be more common that originally thought. In contrast, dynamic friction, and/or internal friction angle occurring during high rate displacement of solid surfaces in contact, or within a soil or rock body along a discontinuity has been known for sometime to decrease because of temperature such friction generates (see e.g. Scholz, 1990 Vardoulakis, 2002, Sulem et al., 2007).

This paper provides a mathematical representation of the constitutive behavior of soils with the static internal friction angle variability with temperature, but not with stress.

2 IMPLICATIONS TO CONSTITUTIVE RULES

The central assumption that results from the observations summarized in Fig. 1 is that the apparent preconsolidation pressure is not the only yield locus characteristics that may evolve with temperature. Indeed, as seen a series of triaxial tests at five different values of the confining pressure, the ultimate triaxial strength is consistently higher at 90°C than at 22°C, by roughly 8-10%.

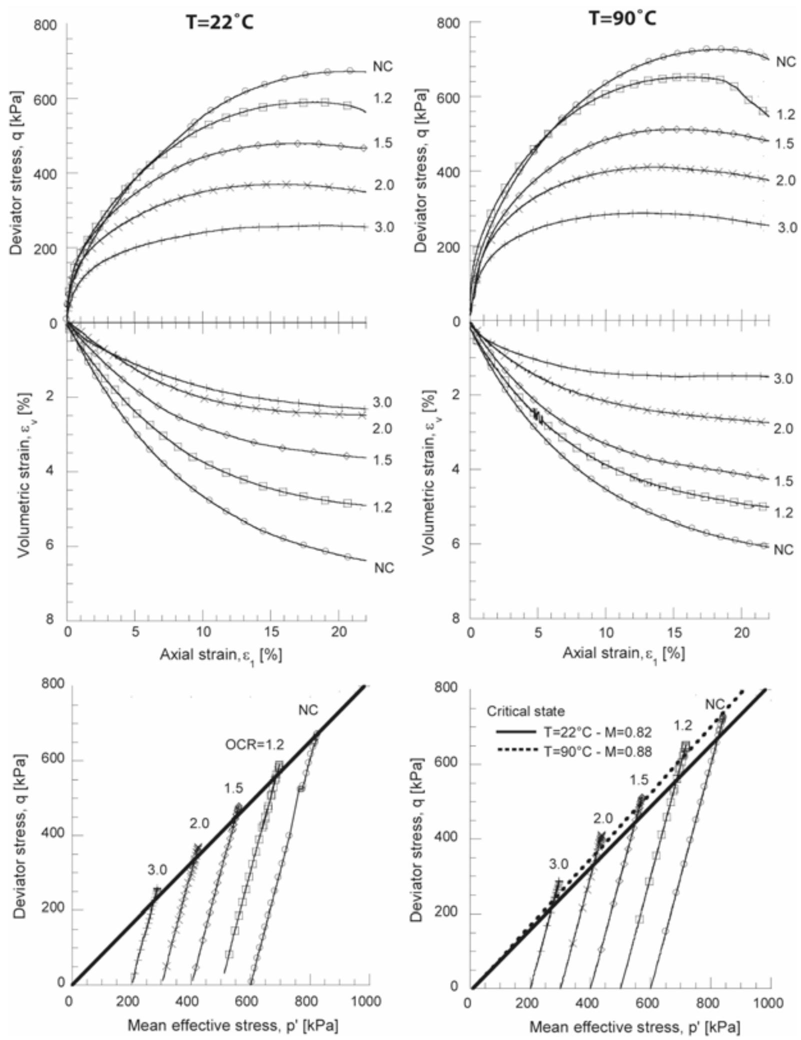

Fig. 1: Results of the triaxial compression tests at 22° and 90°C on Kaolin CH1 (Cekerevac & Laloui, 2004). Representation in the ($q-\varepsilon_1$), ($\varepsilon_v-\varepsilon_1$) and ($q-p'$) plane.

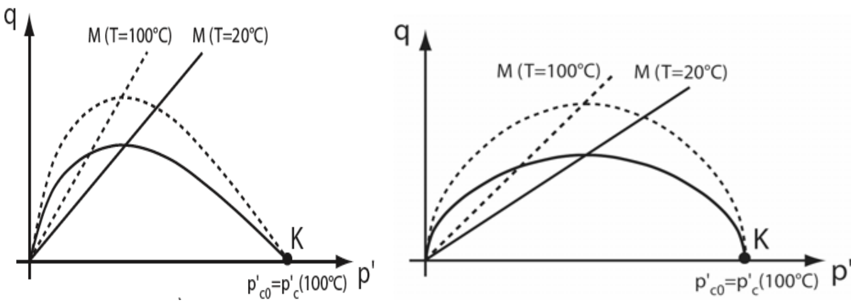

Fig 2: Yield locus evolution induced by the change in the coefficient M: (a) for the original Cam-Clay locus, (b) for modified Cam Clay locus.

The original thermal Cam-Clay model (Hueckel & Borsetto, 1990), in which the elastic domain is enclosed within the yield locus represented via a relationship between the isotropic mean effective stress and the deviatoric stress invariant (p' and q , respectively), either as an ellipse (Modified Cam-Clay locus; Roscoe & Burland, 1968), eq. (1), or a logarithmic function (Original Cam-Clay locus; Schofield & Wroth, 1968), eq. (2), see also Muir Wood (2004), implies that the locus maintains its shape as constant as temperature changes.

In contrast, we presently assume that

$$f = p'^2 - p'p'_c\left(\varepsilon_v^p, \Delta T\right) + \left(q / M\left(\Delta T\right)\right)^2 = 0 \tag{1}$$

$$f = \frac{q}{M\left(\Delta T\right)p'} + \ln\left(\frac{2.718\,p'}{p'_c\left(\varepsilon_v^p, \Delta T\right)}\right) - 1 = 0 \tag{2}$$

The apparent preconsolidation stress p'_c denotes the size of the locus along the isotropic effective stress axis, p' . Its temperature dependence is believed to be the same during heating and cooling; hence, it is postulated to be a one-to-one relationship. Rather than the absolute temperature, the yield locus change is postulated as a function of temperature difference $\Delta T = T - T_0$ relative to a reference temperature value, T_0, at which all parameters, foremost $p'\left(\Delta T = 0\right) = p'_{c0}$, are measured. Hueckel & Borsetto (1990) assumed the following format for p'_c :

$$p'_c = p'_{c0}\exp\left[\frac{1}{\lambda - \kappa}\left(1 - a_0\Delta T\right)\left(1 + e_0\right)\varepsilon_v^p\right] + 2\left(a_1\Delta T + a_2\Delta T^2\right) \tag{3}$$

The last term on the right hand side of Eq. 3 represents the thermal softening function, while λ and κ are the isotropic moduli of the elasto-plastic and elastic incremental compressibility of soil, respectively, whereas e_0 is the initial value of the void ratio. The coefficients a_i , *i = 0, 1,* and *2* are constant. Equation (3) actually represents the thermo-plastic hardening/softening rule. As a result, the plastic compressibility modulus is temperature dependent.

Since the original formulation of the thermal softening function (eq. (3)), a number of alternative representations have been proposed (e.g., Picard, 1994; Hueckel et al., 1994a;

Gera et al., 1996). In this paper, we also use an alternative thermal softening function (Laloui & Cekerevac, 2003), which acts as an amplifier of the plastic strain hardening effect:

$$p'_c = p'_{c0} \exp\left(\frac{1+e}{\lambda-\kappa}\varepsilon_v^p\right)\left(1-\gamma \log\left[1+\frac{\Delta T}{T_0}\right]\right) \tag{4}$$

where γ is a constant material parameter.

As a result, the plastic strain hardening and thermal softening play a multiplicative role on each other. Because of this, the bulk modulus in this formulation is temperature independent.

In the following, both formulations are included. We consider them as alternatives that have the same set of underlying principles.

Under both formulations (i.e. eq. (1) and (3) as well as (2) and (4)) the critical state is described by the locus,

$$q = M\left(\Delta T\right) p' \tag{5}$$

which delimits between dilatant and strain-softening behavior on one side of it and compacting and strain-hardening on the other, if the associative flow rule is assumed. Unfortunately, no data are available to indicate whether thermally induced changes in the critical state are permanent, or not. By the analogy with the yield locus, it is assumed that they are not. This implies that during cooling the friction angle returns to the original value.

The postulate (5) implies, that unlike in the classical thermal Cam Clay, the yield locus shape evolves with temperature, and therefore so does the proportion between the maximum *q* and maximum *p'* of the yield locus, see Fig. 2 a and b for both the original and modified Cam-Clay loci. Clearly, that affects the size and shape of the elasticity domain, and in particular the conditions of shear failure at elevated temperatures. Also it impacts the conditions of thermal failure in undrained heating at a constant total stress. Both issues are addressed in detail by Hueckel et al. (2009).

Specific expressions used for the variation in *M* are:

$$M = M_0 + g\Delta T \tag{6}$$

M_0 and g being material constants, as employed by Laloui (1993), Laloui & Cekerevac (2003) and Laloui & François (2009),

$$M = M_0 \exp\left(\mu_0 \Delta T\right) \tag{7}$$

M_0 and μ_0 being material constants, as proposed by Hueckel (1992) in the context of physico-chemical changes, including additionally ionic concentration and dielectric constant of the pore fluid changes, and finally

$$\tan\varphi = M_0 \left(\frac{\bar{p}}{\sigma_0}\right)^{n(\Delta T)-1} \tag{8}$$

formulated in terms of a nonlinear internal friction angle, φ, for deep rocks using Mohr variables: $\bar{q} = \frac{\sigma_1 - \sigma_2}{2}; \bar{p} = \frac{\sigma_1 + \sigma_2}{2}$ (Hueckel et al., 1994a); σ_0 is a unit stress.

Exponent $n = n_i \left\{1 - \exp\left[\gamma\left(T - T_M\right)\right]\right\}$, n_i being an initial value, γ is a constant, while T_M is a melting temperature.

In the following, we derive governing equations resulting from the constitutive laws and rules.

An interesting novelty results from a new role assumed by the Prager's consistency condition required to maintain the active yielding process:

$$df = \frac{\partial f}{\partial \sigma'_{ij}} d\sigma'_{ij} + \frac{\partial f}{\partial p'_c} dp'_c + \frac{\partial f}{\partial M}\frac{\partial M}{\partial T} dT = 0 \tag{9}$$

If the flow rule is assumed to be associated, that is if:

$$d\varepsilon^p_{kl} = d\lambda \frac{\partial f}{\partial \sigma'_{kl}} \tag{10}$$

As $dp'_c = dp'_c\left(d\varepsilon^p_v, dT\right)$, $\varepsilon^p_v = \varepsilon^p_{kk}$, the amount of the plastic strain rate, controlled by the plastic multiplier $d\lambda$, is affected by the evolution of the internal friction, and hence of M:

$$d\lambda = \frac{1}{H}\left[\frac{\partial f}{\partial \sigma'_{kl}} d\sigma'_{kl} + \left(\frac{\partial f}{\partial p'_c}\frac{\partial p'_c}{\partial T} + \frac{\partial f}{\partial M}\frac{\partial M}{\partial T}\right) dT\right] \geq 0; \quad H = -\frac{\partial f}{\partial p'_c}\frac{\partial p'_c}{\partial \varepsilon^p_v}\frac{\partial f}{\partial p'} \tag{11}$$

as long as $q \neq 0$. Otherwise, this dependence disappears, as for both forms of the yield locus (eqs. (1) and (2)), $\frac{\partial f}{\partial M} = 0$ if $q = 0$. In addition, simultaneously $dp'_c = 0$, i.e. the yield locus ceases evolving.

The conditions for the continuing plastic yielding take an alternative form of a linear complimentary condition

$$d\lambda . df = 0 \tag{12}$$

with the inequalities

$$f = 0;\ d\lambda \geq 0; df \leq 0 \tag{13}$$

The above conditions, together with fact that eq. (9) and inequality (11) contain additional terms resulting from the variability of M alter the ranges where yielding processes at constant stress and at constant temperature are admissible (i.e. satisfying inequalities (13)).

3 IMPLICATIONS TO RANGES OF ADMISSIBLE INCREMENTAL PROCESSES

At any constant effective stress ($d\sigma'_{ij} = 0$), during drained heating along with continuing plastic yielding, there is a specific plastic strain increment generated per increment of temperature:

$$\frac{d\varepsilon^p_v}{dT} = \frac{1}{H}\frac{\partial f}{\partial p'}\left(\frac{\partial f}{\partial p'_c}\frac{\partial p'_c}{\partial T} + \frac{\partial f}{\partial M}\frac{\partial M}{\partial T}\right) \tag{14}$$

Eq. (14) describes a modified compensatory thermo-plastic hardening mode. In this case, as opposed to the M = *const.* case, the plastic strain hardening compensates for both the thermal softening as well as for the thermally induced variation of the yield locus shape. In addition, in the case of heating at a constant, effective stress with a non-zero deviatoric component ($q \neq 0$), the consistency condition (eq. 6) restrains the evolution of p'_c and M during the temperature change:

$$dp'_c = -\left(\frac{\partial f}{\partial M} \bigg/ \frac{\partial f}{\partial p'_c} \right) dM \tag{15}$$

There are several implications of eq. (15), regarding the evolution of the yield locus and admissibility of thermal loads. To start with, let us note that a change in internal friction always implies a non-zero change in the apparent preconsolidation pressure p'_c. As in most cases, both $\frac{\partial f}{\partial M} \leq 0$ and $\frac{\partial f}{\partial p'_c} < 0$, (see e.g. eq.(1)), this means that for an increasing friction angle we have a decrease of the apparent preconsolidation pressure during “plastic heating”, as shown in a sketch in Figs. 3a-d, with the exception for when q = 0 (Fig. 3e). An opposite situation (dM<0) is imaginable, like in Fig. 3f. However, there is no experimental evidence, so far, that in the discussed range of temperatures the static internal friction angle would decrease in the fine grain soils. It needs to be emphasized that for temperature insensitive M, i.e. for $\frac{\partial f}{\partial M} = 0,$ there is evolution of the yield locus during plastic heating at constant effective stress, whatsoever. This is a peculiarity of the thermally variable M response. Figs. 3a–d present four particular cases of the modified Cam Clay yield locus evolution affected by temperature, but constrained to maintain a constant effective stress state, q_K, p'_K at a point K, at yielding. It is seen that depending on the location of the effective stress state during heating the consequences in terms of the evolution of the material strength (yield locus) are substantially different. Most notably, it is seen that the onset of yielding in terms of stress deviator, q, for instance in post heating triaxial loading, may substantially increase at $p' < p'_K$, when heating occurs at stress states, when $p' = p'_K > p'_c/2$, Fig. 3b. It must be emphasized that this type of loading is very important, as it includes heating at a K_0 state, which is a common in situ stress state. To a lesser degree, the same is true for $p'_K < p'_{c0}/2$, Figs. 3c and d. In those cases, there is a much earlier onset of yielding in triaxial paths, for $p' > p'_K$ at elevated temperatures, and clearly much higher ultimate failure stress at critical state.

The stress-less state q = p’= 0 constitutes an exception. While it does satisfy the yield condition, f =0, both $\frac{\partial f}{\partial M} = 0$ and $\frac{\partial f}{\partial p'_c} = 0$, see e.g. (1), leading to a zero thermo-plastic strain rate (see e.g. eq. 14).

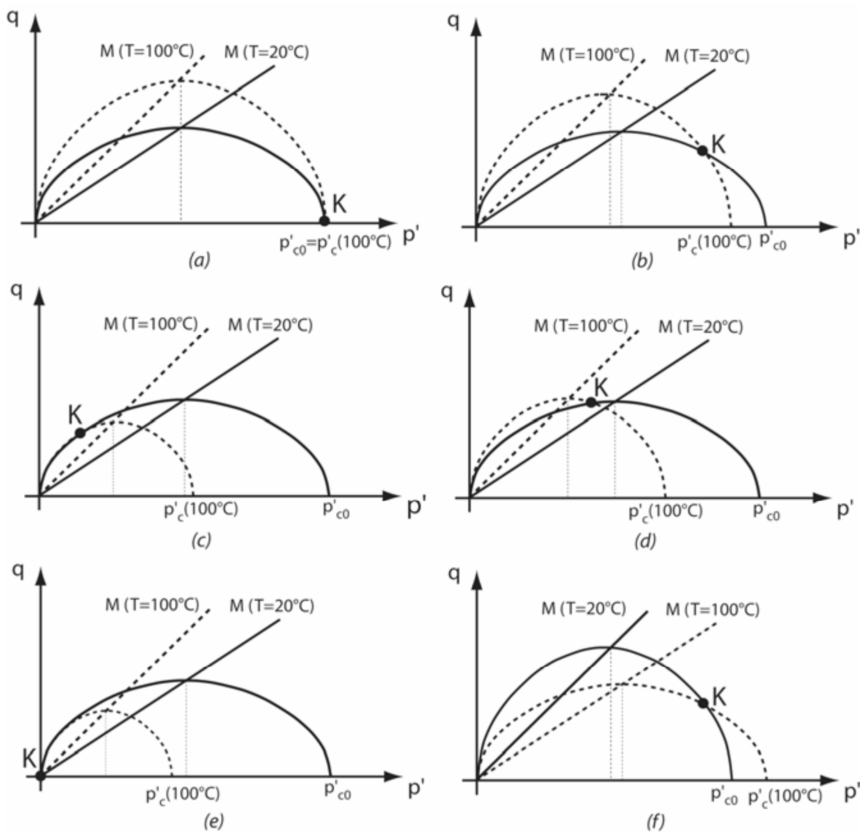

Fig. 3: Examples of evolution of the (modified) Cam-Clay yield locus during heating from 20°C till 100°C at constant stress state, stress point K, at continuous yielding: (a) when stress state is purely isotropic; (b) when the stress state at K has a non zero deviatoric component, $q \neq 0$, and $p' > p'_{c0}/2$; (c) when stress state has a non zero deviatoric component and $p' < p'_c/< p'_{c0}/2$; (d) when stress state has a non zero deviatoric component and $p'_c/2 < p' < p'_{c0}/2$; (e) in the stress-less state, q=p'=0, that is at no yielding; (f).for a decreasing M (ΔT), when the stress state has a non zero deviatoric component, $q \neq 0$, and $p' > p'_{c0}/2$

Thermal variability of the internal friction angle (and coefficient *M*) also affects admissibility ranges of the thermo-plastic heating process. In the classical thermal Cam Clay models (see e.g. Hueckel & Borsetto, 1990) with the associated flow rule (7), for $p' < p'_{c0}/2$, i.e. for *H<0*, drained heating at constant stress states at yielding results in a statically inadmissible process. This reflects the fact that the hardening parameter, which is $d\varepsilon_v^p \leq 0$ produces strain softening, and so does the temperature effecting thermal softening. Given the fact that the constant stress heating *(dT > 0)* requires a plastic strain hardening to compensate for the thermal softening, the stress state becomes unsustainable (see also Palmer et al., 1967 and Maier & Hueckel, 1979). It may be seen, that in such case for $\frac{\partial f}{\partial M} = 0$, and for *H < 0*, $\partial f / \partial p' < 0$, with $\frac{\partial f}{\partial p'_c} < 0$ and $\frac{\partial p'_c}{\partial T} < 0$ for Δ*T>0* equation (14) upon substitution for $d\varepsilon_v^p = d\lambda \frac{\partial f}{\partial p'}$ yields a negative *dλ,* hence an inadmissible response. However, when the internal friction angle (and hence coefficient *M*) are temperature dependent $\frac{\partial f}{\partial M} \neq 0$ indeed

$\frac{\partial f}{\partial M} < 0$ and $\frac{\partial M}{\partial T} > 0$ the expression in the parenthesis in eq.(14) may become negative, and hence $d\lambda > 0$. That imposes a restriction on the two thermal hardening functions

$$\frac{\partial f}{\partial p'_c}\frac{\partial p'_c}{\partial T} < -\frac{\partial f}{\partial M}\frac{\partial M}{\partial T} \text{ for } H < 0 \text{ and } \Delta T > 0 \tag{16}$$

Note, that both sides of the inequality depend on the stress state and ΔT.

For example for the modified Cam Clay shape of the yield surface (eq.1), and thermal softening described by eq. (3) (Hueckel & Borsetto, 1990), assuming $a_0 = 0$, and a linear thermal dependence of coefficient M (eq. 6) (Laloui, 1993) excluding cases of $g \leq 0$, substituting (1), the inequality is expressed in terms of effective isotropic stress, as follows

$$\frac{g\ \Delta T}{M} > -\left(\frac{a_1\Delta T + 2a_2\Delta T^2}{p'_c - p'}\right) \tag{17}$$

The left hand side of inequality (17) represents a relative change in coefficient M. The right hand side is related to the effect of plastic strain softening. The denominator is always positive, comprised between $p'_c/2$ and p'_c. Its lowest value equals $p'_c/2$. The numerator is dominated by coefficient $a_1 < 0$. In other words, there must be enough thermal increase of the internal friction to compensate for the thermal plastic strain softening to ensure the stress states at $p' < p'_c/2$ to be sustainable during drained heating. For a given temperature, a highest chance for inadmissibility is near $p' = p'_c/2$. Note that p'_c depends on ΔT and ε_v^p.

It is easy to see that similarly, in the hardening range, $H > 0$, an opposite inequality applies. However the right hand side denominator ranges now between $p'_c/2$ and 0, and the right hand side may easily reach very high values, near $p' = p'_c$. For a given temperature, a highest chance for inadmissibility is again, near $p' = p'_c/2$.

It is important to be aware of the above limitations to avoid misjudging the response in numerical simulations.

Analogously, different *stress rate* admissibility conditions arise in the strain-softening range ($H < 0$). In fact, the inequality in (11) admits outward stress rate excursions at cooling, at yielding. This takes place when the rate of yield locus growth at cooling is such that

$$\left(\frac{\partial f}{\partial p'_c}\frac{\partial p'_c}{\partial T} + \frac{\partial f}{\partial M}\frac{\partial M}{\partial T}\right)dT < -\frac{\delta f}{\delta \underset{\sim}{\sigma}'}d\underset{\sim}{\sigma}' \ ; \ \frac{\delta f}{\delta \underset{\sim}{\sigma}'}d\underset{\sim}{\sigma}' > 0 \ ; \ H < 0 \tag{18}$$

Vice versa, in the hardening range, both inward and outward stress rates are admissible if

$$\left(\frac{\partial f}{\partial p'_c}\frac{\partial p'_c}{\partial T} + \frac{\partial f}{\partial M}\frac{\partial M}{\partial T}\right)dT > -\frac{\partial f}{\partial \sigma'_{kl}}d\sigma'_{kl} \quad ; \ H > 0 \tag{19}$$

There is no experimental evidence to corroborate or reject that assertion.

Again, the above conclusions affect the way stress probing is conducted in the non-linear analysis in the strain softening range.

It needs to be noted that in isothermal plasticity an outward stress rate is ruled out at softening, as statically inadmissible (see e.g. Maier & Hueckel, 1979).

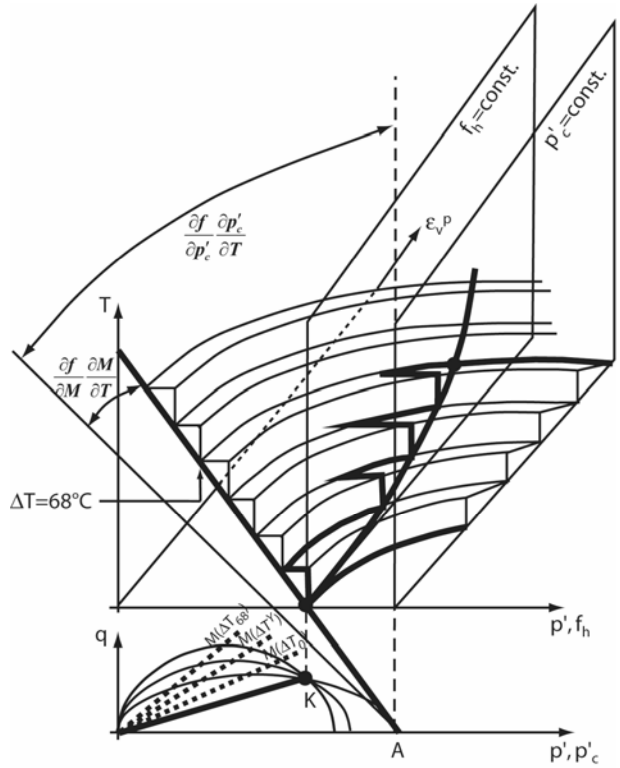

Fig. 4:. Schematic of the yield locus evolution and hardening compensatory mechanisms during heating at constant, non-isotropic stress (point K) for a temperature sensitive critical state material, preconsolidated to p'= p'c0 = 600 kPa.

4 IMPLICATIONS TO POST-HEATING STIFFNESS

An important factor in the evaluation of the experimental results of triaxial loading of drained-heated specimens for the *T*-variability of M is the post heating plastic stiffness modulus. Indeed, as such modulus depends on the amount of accumulated, prior ε_v^p, it is noted that generation of such strain during heating, via the hardening compensation mechanism is affected by the need to compensate for the changes in *M*, as well, as seen in eq.(9). Rewriting the consistency equation for the constant effective stress conditions

$$df_h = \frac{\partial f}{\partial p'_c}\frac{\partial p'_c}{\partial \varepsilon_v^p} d\varepsilon_v^p + \left(\frac{\partial f}{\partial p'_c}\frac{\partial p'_c}{\partial T} + \frac{\partial f}{\partial M}\frac{\partial M}{\partial T}\right) dT = 0 \qquad (20)$$

one can elegantly depict the compensatory mechanism in a diagram, Fig. 4 , in which the traditional variable of apparent precosolidation pressure p'_c is replaced by a complete (i.e. thermal plus strain-hardening) hardening part of the consistency expression (20), which is function $f_h = \int df_h$. In this Figure a surface $f_h = f_h\left[p'_c\left(\Delta T, \varepsilon_v^p\right), M\left(\Delta T\right)\right]$ is presented as a function of ΔT and ε_v^p. Compared to *M=cons.* case (Hueckel et al., 2009), the ΔT dependence of *M* mitigates the change in f_h. A constant stress condition implies a f_h = cons. requirement, which is shown as a plane on which the process trajectory is displayed. It is visible that after the heating process, which produces some thermo-plastic strain, the subsequent stress loading at T=*cons.* meets a visibly stiffer response, as being part of the same exponential relationship, but at more advanced values of ε_v^p .

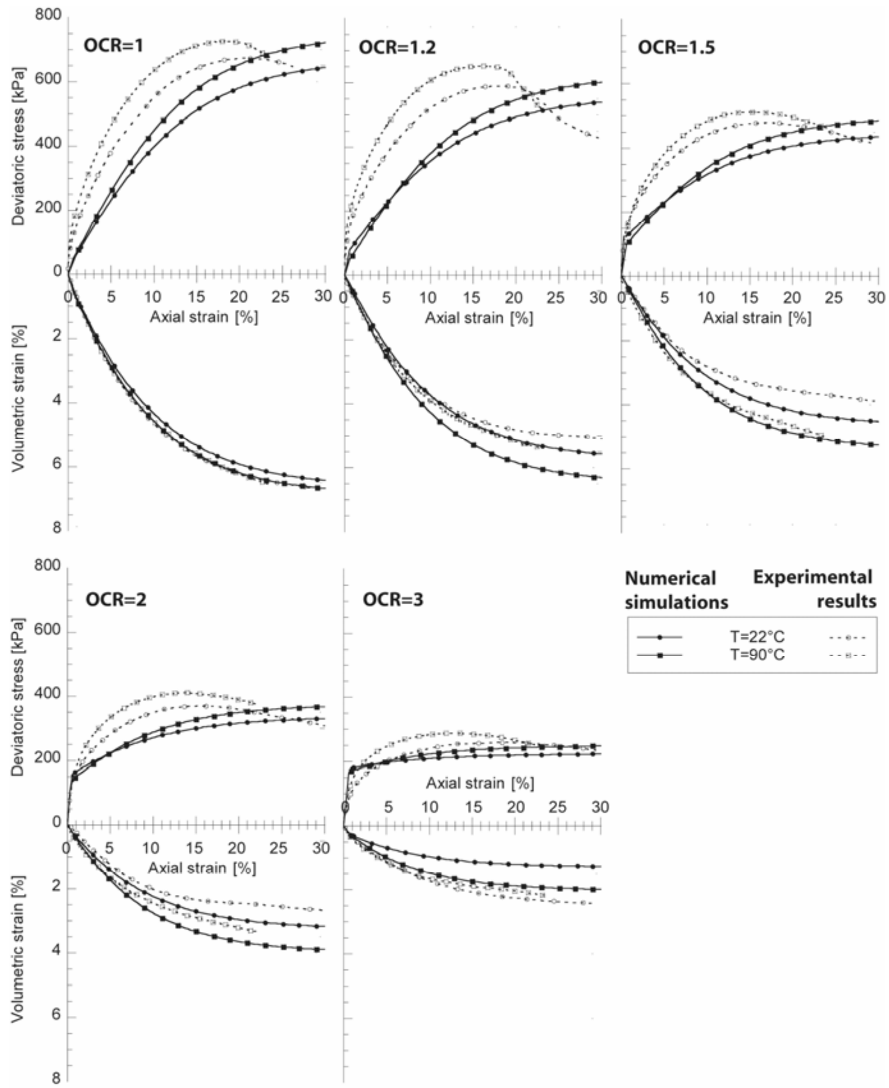

Fig 5: Results of numerical simulations of triaxial tests on Kaolin CH-1 by Cekerevac & Laloui, 2004, performed taking into account linear variability of coefficient of critical state M(ΔT): from M=0.80 to M(90°C)=0.88 (after Hueckel et a., 2009)

Simulations of the triaxial failure of Kaolin samples subjected to a prior drained heating is shown in Fig. 5 (Hueckel et al., 2009).

The main features captured by these simulations are the ultimate strength, which is higher at higher temperatures, as well as stiffness of the triaxial curves, which is increasing with temperature. For detailed comments see (Hueckel et al., 2009).

5 CONCLUSIONS

Temperature dependence of the static internal friction angle, or critical state coefficient *M*, may not necessarily be numerically impressive (e.g. *M* changing from 0.8 to 1.1 during heating from 20°C to 100°C). For some materials, it has not been seen at all. Still, it describes a practically very important fact, whether the ultimate failure load increases or not with temperature. This paper discusses a number of formal consequences of the thermal dependence of M. They concern (i) the compensatory hardening mechanism, (ii) different ranges of admissibility of both thermal and static loadings, and finally (iii) an altered effect on the post-heating elasto-plastic stiffness. These questions are important in evaluating numerical prediction of the response of materials in non-isothermal conditions. The study also reveals the areas where theoretical queries still await an experimental confirmation, or refutation.

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