TA SESSION # 7
ECON 341: ECONOMETRICS

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Problem 1. Consider the model,

$$y_i = \mathbf{x}_{1i}'\beta_1 + \mathbf{x}_{2i}'\beta_2 + \varepsilon_i$$

where $\mathbf{x}_{1i}$ is $k_1 \times 1$ and $\mathbf{x}_{2i}$ is $k_2 \times 1$.

Your colleague obtains the following regression:

$$y_i = \mathbf{x}_{1i}'\hat{\beta}_1 + \hat{\varepsilon}_i$$

and defines the error variance estimator as

$$s^2 = \frac{1}{n - k_1}\sum_{i=1}^{n}\hat{\varepsilon}_i^2$$

Is the estimator of the error variance unbiased? i.e., find the $E(s^2|X)$.

Problem 2. Assume you have the following model:

$$\mathbf{Y} = \mathbf{X}\beta + \varepsilon$$

$$E(\varepsilon|\mathbf{X}) = \mathbf{0} \text{ and } E(\varepsilon\varepsilon'|\mathbf{X}) = \Omega$$

Assume Ω in known.

(a) Find the GLS estimator of β, denote it $\tilde{\beta}$.
(b) Let $\hat{\beta}$ be OLS estimator of β. Find the conditional covariance between $\hat{\beta}$ and $\tilde{\beta}$
(c) Find the conditional covariance of the matrix $\hat{\beta} - \tilde{\beta}$.

Problem 3. Suppose you ran an OLS regression of the form

$$\hat{y} = \hat{\beta}_1\mathbf{x}_1 + \hat{\beta}_2\mathbf{x}_2$$

where y are executive salaries, $\mathbf{x}_1$ are sales, and $\mathbf{x}_2$ are profits, across a sample of 102 firms. The results are:

$$\hat{y} = 0.5\mathbf{x}_1 + 0.4\mathbf{x}_2$$

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$$\mathbf{X}'\mathbf{X} = \begin{pmatrix} 10 & 8 \\ 8 & 10 \end{pmatrix}$$

$$V(\hat{\beta}) = \begin{pmatrix} 0.7 & -0.5 \\ -0.5 & 0.7 \end{pmatrix}$$

(a) Someone suggested that high collinearity between sales and profits has prevented precise estimation of $\hat{\beta}$. Do you agree?

(b) Someone suggests you a method to eliminate this problem. Step 1, regress profits on sales and obtain residuals $\mathbf{x}_2^*$. Then, regress y on $\mathbf{x}_1$ and $\mathbf{x}_2^*$ to obtain the salary function. Find an expression for $\mathbf{x}_2^*$.

(c) Let the last regression be $\tilde{y} = \tilde{\beta}_1\mathbf{x}_1 + \tilde{\beta}_2\mathbf{x}_2^*$. Calculate the coefficients of this regression.

(d) Evaluate if this proposal has eliminated collinearity.

Problem 4. Suppose your classmate wants to know which of a set of 20 regressors has an effect on mid-term test scores. He regresses test scores on 20 regressors and reports the results. One of the 20 regressors (study time) has a large t-test (larger than 2.0), while other t-tests are insignificant (smaller than 2 in absolute value). He argues that the data show that study time is the key predictor for test scores.