TA SESSION # 8
ECON 341: ECONOMETRICS

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Problem 1. Assume the following model:

$$y_i = \mathbf{x}_i'\beta + e_i$$
$$E(e_i|\mathbf{x}_i) = 0$$
$$z_i = (\mathbf{x}_i'\beta)\gamma + u_i$$
$$E(u_i|\mathbf{x}_i) = 0$$

You goal is to estimate γ. You use a the following two-step estimator:

(i) Estimate $\hat{\beta}$ by OLS using y_i and $\mathbf{x}_i$.
(ii) Estimate $\hat{\gamma}$ by least squares of z_i on $\mathbf{x}_i'\hat{\beta}$.

Is $\hat{\gamma}$ consistent? What is the asymptotic distribution of $\hat{\gamma}$.

Problem 2. You have a sample (y_i, x_{1i}, x_{2i}) for $i = 1, \ldots, n$. You estimate two LS regressions:

$$y_i = x_{1i}'\tilde{\beta}_1 + \tilde{\varepsilon}_i$$
$$y_i = x_{1i}'\hat{\beta}_1 + x_{2i}'\hat{\beta}_2 + \hat{\varepsilon}_i$$

and calculate the residual variances estimates:

$$\tilde{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} \tilde{\varepsilon}_i^2$$
$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} \hat{\varepsilon}_i^2$$

Show that for any $w \in (0, 1)$, there is a constant $a \in (0, 1)$ such that:

$$\frac{1}{n}\sum_{i=1}^{n}[w\hat{\varepsilon}_i + (1-w)\tilde{\varepsilon}_i]^2 = (1-a)\hat{\sigma}^2 + a\tilde{\sigma}^2$$

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Problem 3. Assume you have the following model:

$$y = \mathbf{X}\beta + \varepsilon$$
$$E(\varepsilon|X) = 0$$
$$E(\varepsilon\varepsilon'|X) = \Omega = \begin{pmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_1^2 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \sigma_1^n \end{pmatrix}$$

then,

$$E(\hat{\sigma}^2|\mathbf{X}) = \frac{1}{n} tr(M\Omega)$$

where $\hat{\sigma}^2$ is the error variance estimator, and $M = I - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$. Show that:

$$E(\hat{\sigma}^2|\mathbf{X}) = \frac{1}{n}\sum_{i=1}^{n}\sigma_i^2 - \frac{1}{n}b_n$$

with

$$b_n \xrightarrow{p} tr(Q^{-1}D)$$

Find Q and D.