TA SESSION # 9
ECON 341: ECONOMETRICS

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Problem 1. Let y_i and z_i be random scalars and let $\mathbf{x}_i'$ be a $1 \times k$ random vector, where one element of $\mathbf{x}_i$ can be unity. Consider the population model,

$$y_i = \mathbf{x}_i'\beta + \gamma z_i + \varepsilon_i$$
$$V(e_i|\mathbf{x}_i, z_i) = \sigma^2$$

We want to estimate β. Assume $\gamma \neq 0$, $E(e|\mathbf{x}, z) = 0$ and, $\mathbf{x}_i$ and z_i are orthogonal, i.e., $E(\mathbf{x}_i z_i) = \mathbf{0}$. Consider two estimators based on N independent and identically distributed observations:

Procedure 1: $\hat{\beta}$ which is obtained from the estimation of y on $\mathbf{x}$ and z.
Procedure 2: $\tilde{\beta}$ that is obtained from the regression of y on x.

Are both estimators consistent? If so, which is asymptotically more efficient?

Problem 2. Assume that the pair (y_i, x_i) are iid, and we observe a random sample $i = 1, \ldots, n$. We want to estimate $\mu = E(y_i)$, and we know that $E(x_i) = 0$, where $x_i \in \mathbb{R}$. Find an efficient GMM estimator of μ.

Problem 3. Consider the model,

$$y_i = \mathbf{x}_i'\beta + \varepsilon_i$$

with

$$E(\mathbf{x}_i\varepsilon_i) = \mathbf{0}_{k\times 1}$$
$$E(q_i\varepsilon_i) = \mathbf{0}_{m\times 1}$$

and assume you have observations $(y_i, \mathbf{x}_i, q_i)$ for $i = 1, \ldots, n$ where x_i is $k \times 1$ and q_i is $m \times 1$. Find the efficient GMM estimator of β.

Problem 4. Assume you have y that is a $T \times 1$ vector with the price of gasoline, and $\mathbf{X}$ is a $T \times k$ matrix of determinants of gasoline. Assume that you believe that there was a change

Date: November 20, 2008.

in the behavior of consumer demand such that de demand for gasoline can be expressed as,

$$y_1 = \mathbf{X}_1\beta_1 + \varepsilon_1$$
$$y_2 = \mathbf{X}_2\beta_2 + \varepsilon_2$$

where,

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad \mathbf{X} = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix}$$

and we have,

$$E \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \begin{pmatrix} \varepsilon_1 & \varepsilon_2 \end{pmatrix} = \sigma^2 \begin{pmatrix} I_{T_1} & 0 \\ 0 & I_{T_2} \end{pmatrix}$$

Show that the F-test of the null hypothesis $H_0 : \beta_1 = \beta_2$ can be written as:

$$\left(\frac{T_1 + T_2 + 2K}{K}\right) \frac{(\hat{\beta}_1 - \hat{\beta}_2)'[(\mathbf{X}_1'\mathbf{X}_1)^{-1} + (\mathbf{X}_2'\mathbf{X}_2)^{-1}](\hat{\beta}_1 - \hat{\beta}_2)}{y_1'M_{X_1}y_1 + y_2'M_{X_2}y_2}$$

where $M_{X_1} = \mathbf{I} - \mathbf{X}_1(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1$ and $M_{X_2} = \mathbf{I} - \mathbf{X}_2(\mathbf{X}_2'\mathbf{X}_2)^{-1}\mathbf{X}_2$.