Optimizing Airport Accessibility

Team 781
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1 Introduction

Air travel is an important means of transportation in this modern era. It opens up greater horizons for exploration for many people, permitting modern citizens to become significantly more well-traveled than their predecessors. However, air travel is not without its hardships: it requires effort and often assumes physical mobility. Passengers have to move through sometimes labyrinthine airline terminals and negotiate crowded hallways during peak hours. They must drag along heavy baggage and perhaps travel up and down stairs to reach their gate or board their plane. All this hardship is magnified by the inherent time constraint of group travel: they must reach their desired gate by a certain deadline or miss their flight, resulting in a loss of time and money. This situation is compounded by connecting flights: travelers must reach another gate in an airport, often in a shorter time frame than they would like and often in an airport that is totally unfamiliar to them.

Travelers of limited mobility are particularly challenged by the difficulties of commercial air travel. However, federal statute stipulates that airlines must provide certain forms of assistance to passengers that request it.[1] It also declares that this includes wheelchairs and service personnel for arriving at the gate and for timely transportation between the gates in a connection. The airlines must comply by making these resources available, lest they face regulatory penalties or lowered profit by lost fares.

1.1 Restatement of Problem

Airlines must provide assistance to customers of limited mobility in going to, from, and between gates at the airport. However, in an effort to maximize profits by minimizing expenses,[2] airlines seek to determine an optimal strategy for this general task: allocating service personnel, wheelchairs, and other transportation aids in an airport to help their passengers move between gates. A successful model will produce an optimal strategy under varying conditions and take into account the many relevant factors that exist in the situation. Our consulting firm aims to explore the issues surrounding passenger assistance and produce a strategy for minimizing all costs to maximize profits for Epsilon Airlines.

[1] 14 CFR §382.39 [17]
[2] One might say Epsilon seeks to bring expenses arbitrarily close to zero...

1.2 Approach

An airport is a large and complex system that undergoes many changes. Furthermore, each airport is different from all others in some respect. In the United States alone there are almost 15,000 airports.[3] In designing a strategy that can be applied to many different types of airports in different types of situations, flexibility is key. An optimal algorithm must be able to integrate many sources of data and configurations and still produce good results.

So in devising a strategy to optimize passenger movement in airports, we started by analyzing the various ways customers and their wheelchairs could be transported. The main ways we studied were by individual escort, by airline-provided electric vehicle, and by airport-provided mass transport service. We model these three means by analyzing specific airports as case studies. Finally, we synthesize these models into a larger strategy that can accommodate an airport with any mixture of transportation means. This last algorithm balances all the costs involved and produces an optimal arrangement of resources.

1.3 Reactions to Previous Work and Ideas

Several ideas struck us with how to optimize the assistance of passengers in an airport. However, we decided to make our decision on which ways to pursue based on the importance of larger airports and airlines. One approach would involve finding and dictating a detailed scheduled for each escort over the course of a day, perhaps factoring in exact plane and passenger arrivals obtained daily. However, this model would not scale well at all: large airports have so many flights and therefore so many passengers that a schedule for a single escort would be painfully long, even for one single day. As well, any deviation over the course of the day (a late plane or unexpected passenger requiring assistance) would render the schedule meaningless. Scaling this effect to airports with literally dozens of terminals is not practical. Perhaps this "personal scheduling" idea would work for airports of limited passenger flow, but as we will see, airports of limited passenger flow are almost trivially taken care of. Also, smaller airports typically do not have connecting flights passing through them: large airports serve as hubs and therefore large airports are where quick gate-to-gate transportation is key. All this combines to show that optimizing passenger flow for large airports is where the real work needs to be done.

1.4 Terms

- **Customers**: An abbreviated reference to passengers of limited mobility that have requested help from an airline representative for moving around the airport complex. Since we do not need to discuss the other passengers as much, we can ignore them with this notation. One side point is that passengers of limited mobility with their own wheelchairs that travel with

[3] See [6]

an accompanying assistant are not included in this designation, since they require no special action by the airline (in the terminal itself).

- **Wheelchair**: A mechanical device for transporting an individual without the need for the individual to walk. Unless specially mentioned, wheelchairs are unpowered and require another person to push them.
- **Escort**: An abbreviated reference to airline customer service personnel that are tasked with assisting passengers of limited mobility, typically by pushing them in wheelchairs.
- **Airline**: A corporate entity that makes its money by transporting paying passengers between locations. We define it here to distinguish its employees from the employees of the airport. Since we are optimizing cost from the standpoint of an airline, any work by airport employees is free to us.
- **Concourse**: A section of an airport that contains several gates. It consists of a single building or contiguous section of a building with at least one airline operating planes out of its gates. It also can be referred to as a terminal and can be a useful abstraction for describing airport design.
- **Gate**: The place where passengers embark or disembark from a plane. It is found in terminals in airports.
- **Tram**: A motorized electric vehicle designed to transport people between locations in an airport. In our model we assign trams to help move passengers of limited mobility and their wheelchairs between terminals. Trams are capable of carrying between three and seven people excluding the driver.

1.5 Assumptions and Hypotheses

- **Customers only require assistance to and from the gate**. Once customers arrive at the gate, the responsibility for getting them on or off the plane belongs to the crew of the plane they will be boarding or exiting, as opposed to the staff of the airport. All moderately-sized commercial planes are required to have a wheelchair permanently aboard and flight attendants are trained in passenger assistance.[4]
- **Customers can arrive at and depart from the airport without airline assistance**. The responsibility for transporting customers to the airport does not belong to the airline. This joins with the previous assumption and generalizes to the fact that we only consider customers in the airport itself.
- **In general, connecting flights are the same as final flights or initial flights**. The security checkpoint can be thought of as an arrival gate and the baggage claim can be thought of as a departure gate.

[4] See [17]

- **Planes arrive and depart on time**. It rarely makes economic sense to delay plane departure in any significant amount for one passenger. Therefore, planes are assumed to follow their pre-determined schedule. This also means that connection times are "reasonable" and not unintentionally shortened by delay.

- **The distribution of people on planes is uniform**. That is, all planes are the same size and carry the same amount of people. This means we treat all planes as "average planes" and is only required when we use plane statistics (which is rarely).

- **All days are equal in an airport**. That is, the distribution of plane and passenger arrivals is uniform across separate days We present our results as optimal and applicable in a per-day snapshot. We do not consider seasonal effects, holidays, or other special cases.

- **As long as passengers reach the gate before the flight leaves, we are successful**. We ignore changing customer dissatisfaction based on how long they wait for assistance. We assume that the unhappiness based on missing a flight far overshadows it.

- **One out of ninety passengers have limited mobility**. We determined this based on an American Airlines statement that one out of the ninety million passengers they have annually is "disabled" in some form.

- **Customers and airplanes are conserved**. The number of customers that enter our system must also leave our system. They cannot be left inside or disappear. Tom Hanks will not live for months in our terminal.

1.6 Variables

Notation	Meaning
h	number of escorts
t	time
$r(t)$	rate at which customers come in (at time t)
d	time needed for a customer to get to his/her destination gate
$n(t')$	number of customers per day whose d is equal to t'
$cost_{miss}$	cost due to customers missing their flights (per day)
$cost_{system}$	cost of operating the system, including wages and maintenance (per day)
$cost_{day}$	daily cost: $cost_{day} = cost_{miss} + cost_{system}$
$cost_{setup}$	the cost of setting up the system

1.7 Constants

Constant	Value
Escort speed (with customer on wheelchair)	1 m/s
Escort speed (with empty wheelchair)	1.3 m/s
Electrical-powered tram	\$2500 + \$10/day (Maintenance)
Electrical-Powered wheelchair	\$1200 + \$6/day (Maintenance)
Wheelchair	\$200 + \$1/day (Maintenance)
Wage	\$10/hour (For escorts and tram drivers)
Penalty if a customer missed the next flight	\$200
Proportion of customers among all travelers	1/90

Values were approximated based on online product searches. For products not generally available to the public, prices were estimated.

2 Cost Modeling

Our ultimate goal is to minimize the total cost, which consists of two parts: the cost of setting up the system ($cost_{setup}$) and the daily cost ($cost_{day} = cost_{miss} + cost_{system}$).

$cost_{setup}$ is easy to calculate: \$2500 times the number of trams needed + \$1200 times the number of powered wheelchairs needed + \$200 times the number of normal wheelchairs needed.

$cost_{system}$ varies depending on how many hours escorts/drivers work per day.

To calculate $cost_{miss}$, we need to first model the probability $P(d)$ that a customer is going to miss his or her next flight. We assume that the customer is ensured to catch the flight if d is less than 1 hour, and he/she is ensured to miss the flight if d is more than 3 hours. In addition, we assume $P(d)$ is linear when $1 < d < 3$.

$$P(d) = \begin{cases} 0 & : \text{ if } d \leq 1 \\ (d-1)/2 & : \text{ if } 1 < d < 3 \\ 1 & : \text{ if } d \geq 3 \end{cases} \tag{2.1}$$

Therefore, we obtained the formulas of $cost_{miss}$ for discrete models and continuous models, respectively.

For discrete models:

$$Cost_{miss} = 200 \sum_{t'} P(t')n(t') \tag{2.2}$$

For continuous models:

$$Cost_{miss} = 200 \int P(t')n(t')dt' \tag{2.3}$$

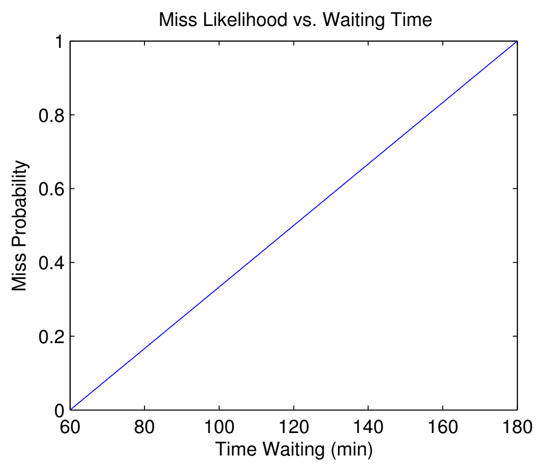

Figure 1: Probability of Missing Next Flight

We meld several factors in our cost modeling in order to simplify later analysis. The "maintenance" rate we charge for materials includes the cost of repairing or replacing the item after a certain time, as well as the "storage cost" for the item in question. To simplify matters, Epsilon Airlines could think of that money as going to a special-purpose fund to deal with all continued non-human costs associated with the system. Additionally, we allow ourselves to ignore the distinction between customers with and without wheelchairs by shifting the cost of wheelchairs. Lowering it would mean we, on average, need to provide less wheelchairs at the airport since more passengers arrive with their own.

Additionally, in later sections we will concern ourselves only with minimizing $cost_{miss}$ and $cost_{system}$. That is, we will in effect ignore $cost_{setup}$. This is not a huge stretch because after a few months the daily cost of paying wages and maintenance costs overshadows the initial costs of equipment. Also, the fact that we view equipment as "expiring" (needing to be replaced every so often) supports this simplification.

3 Plane Arrival Analysis

Although we treat all days as equal, we do not treat all hours within the day as equal. As experienced air travelers know, there are predictable busy and light times of day in airports. This is due to many factors which will not be explored here: we simply wish to approximate this phenomenon's effect. To accomplish this we collected flight arrival data from Chicago O'Hare International Airport.[5] The slight morning rush of flights can be seen, as well as a much larger afternoon and early evening rush (probably contributed to by connecting flights arriving

[5] See [5]

from multiple locations). To allow this data to be used in our later continuous models, we approximated it with a quartic polynomial in terms of two hour chunks (Equation 3.1).

$$-0.16x^4 + 1.92x^3 - 0.20x^2 - 17.37x + 19.49 \tag{3.1}$$

In order to generalize this behavior to airports of different size we can multiply by a constant scaling factor to change the daily flow of the airport.

However, our final model is designed to cope with a steady-state flow. An airline cannot adjust to changing flow in a very finely-tuned pattern. For example, they cannot add more workers once every hour and make an employee only work a few hours: the complexity of having intricate work schedules would make it too complex. So for later models we would obtain two solutions for the most optimal arrangement: one for heavy use, typically from 10 AM to 10 PM and one for lighter use, the remaining hours.

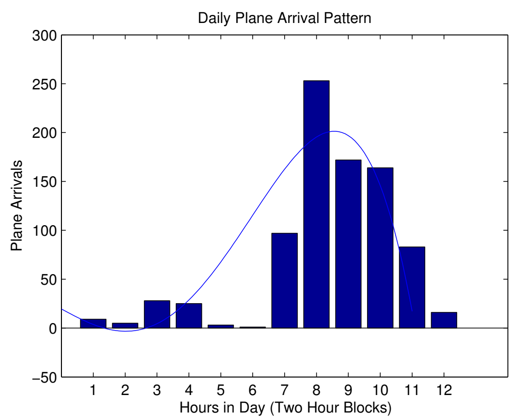

Figure 2: Plane Arrival Patterns of Chicago O'Hare

4 Single Concourse Analysis

Quite often, an airline only operates flights out of one terminal.[6] So naturally our first area of inquiry is the intra-concourse behavior of customers and escorts. In general we analyze the ability of a certain number of escorts to transport passengers from gate to gate within a local area (one concourse).

[6]Empirical analysis from several airports, see references.

4.1 Simulation: Raleigh-Durham International Airport Terminal C

For this task, we simulated a relatively small concourse, Terminal C from the Raleigh-Durham International Airport. In RDU, no airline services both its terminals (A and C), so we can simulate only one while ignoring the other's effects. Customers arrive in a certain pattern (obtained from the analysis in section 3, scaled to RDU's size) at random gates and are assigned the closest available escort. If no helper is available, they will wait morosely by their arrival gate until the first escort becomes available. The escort then accompanies the passenger to their desired destination. We found the commercial traffic of RDU to be approximately 12,000 passengers per day.[7] From our assumptions we estimate there to be 65 customers (requiring assistance) each day in Terminal C.

4.2 Simulation Parameters

Our single-terminal simulation also requires calibration for the distance between gates. There are 26 gates in Terminal C, and using FAA aerial maps of RDU [8] we determined this distance to be 40 meters between adjacent terminals. Terminal C is in a linear configuration with several gates clustered at each end so we set the distances between gates accordingly. We estimated from timing ourselves walking around that an escort pushing a customer in a wheelchair could go 1 meter per second and about 1.3 meters per second without a customer. We then started the simulation with the minimum number of escorts, one, and viewed the results of several days. The histogram of wait times for individual passengers over the course of a week[9] with one helper is shown in Figure 3.

A surprising result emerges from this model: only one escort can successfully assist all passengers in reaching their gates in at most forty minutes, without sophisticated schedules or movement patterns. But this effect is not too surprising: customers arrive at the busiest time only once per ten minutes and it takes about ten minutes to traverse the entire terminal. Since the flow slackens off in some hours and some passengers do not need to traverse the entire length of the terminal, the single escort can easily keep up. Since in section 2 we declared that passengers cannot be late if they arrive within an hour, we see that this situation is (luckily) optimal. Clearly, employing only one escort is the most cost-effective way of helping passengers if they can manage the entire task since they must employ some escorts. The cost of employing one escort with one wheelchair for a day comes out to $241 per day.

[7] See [6] and [13]
[8] See [6]
[9] Obtained by simulating one day seven times in a row

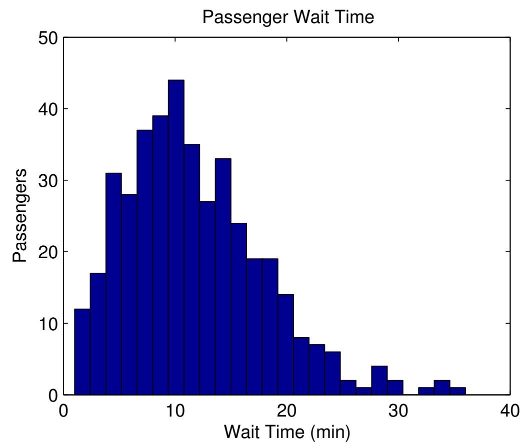

Figure 3: Wait Times in RDU Terminal C

4.3 Determining Intra-Terminal Flow

We also wish to use our simulation to determine the ability of a certain number of escorts to manage the movement of a certain number of passengers for use in a later model. All the physical parameters of the simulation can be customized to a given airport's specifications (gate arrangement, distances, escort speed) in order to determine the flow rate or throughput of a certain number of escorts. Then, an infinite number of passengers are simulated as waiting at each gate. The helpers escort passengers in order to their respective gates and at the end of the day the number of passengers who were helped is calculated. This gives us an approximate relation between escorts and flow rate within a terminal, which will prove to be useful. We will show the results of this simulation for another airport in a later section.

5 Publicly-Connected Terminals

Most airports constructed today utilize several terminals that are all joined by some series of walkways or concourses. Although most airlines are confined to a single terminal, several of the larger airlines occupy two or more terminals. There then exists a need to transport customers between terminals to reach their connecting flights. We consider this situation in two different contexts. The first case involves the nature of this system when a large-scale public transportation system is already in place. The other investigation is a strategy for inter-terminal customer transportation for large airports without a major transportation system. Both of our models with take a similar attack on the situation.

The motivation for our strategy comes from a rather unlikely source. The model that FedEx uses for overnight packages has proven to be an extremely successful business strategy [10]. The idea is that all the packages that need to processed are flown to a central location, sorted, and then flown back out to their destinations. The initial site of this central system for FedEx was Memphis, Tennessee as it provided a large airport with temperate weather and centralized location. Although we don't like bluegrass music as much as Fredrick W. Smith, we felt as though his approach had many applications to the problem posed by Epsilon Airlines. By creating a centralized inter-terminal transportation system, it would be possible to bring all customers to the entrances to the terminals, relocate them to the necessary terminals, and then allow individual helpers to distribute them to the necessary gates. This is also an effective strategy due to current security concerns. Most security check points are at the entrances to terminals and it would be extremely difficult to allow a large tram or train to pass through a security check point. From these considerations we recommend a centralized inter-terminal transportation system for moving customers between terminals.

5.1 For an airport with public transportation between concourses: A simple continuous model

Many of the major airports in the world, such as Chicago O'Hare International Airport, have public transportation systems to transport passengers from one concourse to another. By "public" we mean that it is owned and operated by the airport itself and therefore does not cost the airline to have its passengers use it. O'Hare calls its public transportation the Airport Transit System (ATS), and we will analyze it in this section. Our strategy is to make use of the ATS so that our escorts can remain localized in their concourses instead of transporting customers throughout the airport. The algorithm that we will utilize is as follows: We assign a certain number of escorts at each concourses. Once a customer, who needs help to go to concourse B to catch his next flight, arrives at concourse A, one of our escorts at concourse A will take him or her to the nearest ATS station and help him get on the next train coming in. At the same time, the escort will call concourse B so that they will send an escort to the ATS station at concourse B to pick up the customer. The nature of this model is continuous as customers come in and leave as flows. The walkways between concourses and their corresponding ATS stations are considered to be pipes with a certain capacity, which depends on the number of escorts on duty at a give time. It is the airport's responsibility, not the airline's, to set up ATS and make sure it works. We assume that the ATS has infinite capacity and works under any conditions.

Variables/Functions:

1. h – number of escorts.

[10] See [7]

2. $w(t)$ – the amount to time a customer has to wait if he or she lands at time t, before being sent to the nearest ATS station.

3. t_{max} – the maximum value for $w(t)$.

4. t_1 – time (in hours) needed for an escort to go from an concourse to the nearest ATS station and come back. The maximum capacity of our escorts is taking $\frac{h}{t_1}$ customers from concourses to stations, and picking up $\frac{h}{t_1}$ customers at stations and sending them to their destination concourses per hour.

5. t_2–time (in hours) needed for an ATS train to run one circle.

6. $r(t)$–rate (in number of customers per hour) at which customers requiring assistance, who need to go to another concourse to catch the next flight, arrive. It is a function of time t.

7. d –time needed for a customer to get to his/her destination gate

8. $n(t')$–number of customers per day whose d is equal to t' (defined in our variables section). Since our model is continuous here, $n(t')$ will be the distribution function.

Assumptions:

1. It takes virtually no time to move between gates within a concourse, but it takes a long time to move from a concourse to its nearest ATS station.

2. There are 4 concourses in the airport, all the customers will need to go to another gate to catch the next flight. In addition, it is equally likely for a customer to go to any of the 4 concourses to catch his or her next flight. That is to say that 1/4 of the customers will stay in the same concourse to catch the next flight, 3/4 of the customers will need to go to another concourse.

3. Escorts and wheelchairs are distributed in the most efficient way possible, which means escorts and their corresponding wheelchairs will move to other concourses if they have nothing to do and another concourse needs help because of high customer volume in that concourse. Escorts can move very fast if they are not taking any customers with them, so it takes virtually no time for them to spread.

4. Our rule in helping our customers is "first comes, first served."

5. $t_1 = 1/6$ is equivalent to 10 minutes (5 minutes to go one-way, 10 minutes to go and come back); $t_2 = 1/6$ is equivalent to 10 minutes. This is reasonable for a modern airport.

6. Customers always arrive at ATS stations when a train is supposed to come, and the trains are always on time. So customers do not have to wait at the stations. This is not hard to achieve because if $t_1 = ct_2$ where c is a positive integer, and if an escort arrives at the station with a customer when a train is at the station, then it takes the escort t_1 to bring his/her next customer to the station; since t_1 is a multiple of t_2, there should be another train at the station. In our case $t_1 = t_2$, so $c = 1$, the assumption can be achieved.

7. Customers do not have to wait once they get off the train. Since customers are sent to ATS at a rate of at most $\frac{h}{t_1}$ per hour, we assume that they get off uniformly with respect to time (they do not get off at the same time), the rate at which they get off should not exceed $\frac{h}{t_1}$ per hour. So there are always escorts available at the station to pick them up.

8. Assuming here that we have h escorts working all day. Although no one can work for 24 hours a day, it costs the same amount of money to hire an escort to work for 24 hours a day as to hire three escorts to work for 8 hours a day, therefore our total cost will not be affected.

9. All the flights through the airport are Epsilon Airlines flights.

Our goal is to compute the distribution of waiting time for all our customers for each fixed h number of workers on duty, thereby determining the cost of the system.

We use the data of Chicago O'Hare Airport in 2004, they have approximately $3.61 * 10^7$ passengers per year[11]. So there are approximately

$3.61 * 10^7/365/90 \approx 1100$ customers requiring assistance per day. Using Equation 3.1, the distribution of the flights over a day is $-0.16x^4 + 1.92x^3 - 0.20x^2 - 17.37x + 19.49$, we obtained $r(t) = 3/4 * 1.09 * (-0.01t^4 + 0.24t^3 - 0.05t^2 - 8.685t + 19.49)$, the rate of incoming customers.[12]

If we have enough escorts (h is at least 28) at any time of the day, then no customer will have to wait. This can be seen from the fact that the total capacity of 28 workers is larger than maximum inflow rate of customers. We then noticed that the maximum value of $r(t)$ is at about $t = 15$ or 16, which means that the busiest hours of the day will be around 3 or 4 PM. If we do not have enough escorts for certain periods of the day, assuming that we have enough escorts for the whole day except the busiest hours(See Figure 4), then we only need to consider the situation at and after the busiest hours. During and after these time periods there will be people waiting and additional cost must be added to account for this waiting time and any missed flights, but prior to them the only cost is for paying the salaries of the helpers.

If $r(t)$ crosses the maximum system capacity (the horizontal line $\frac{h}{t_1}$) for the first time at $t = t_a$ (See Figure 7.1), then people arriving at a time t between t_a

[11] See [6]

[12] We used this equation in MATLAB, which carried more digits in this expression. Thus, points evaluated with the truncated form might differ from our later numbers

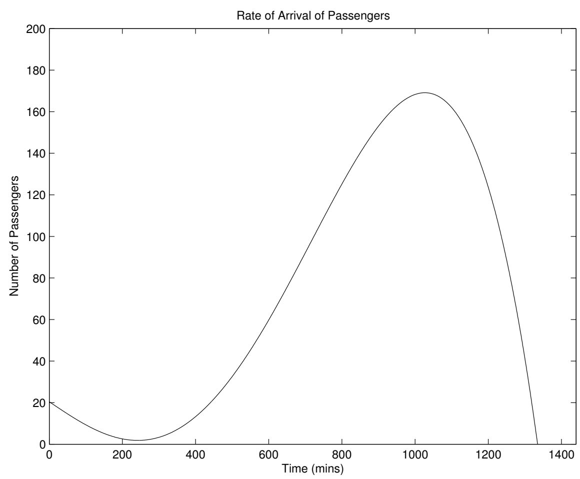

Figure 4: Flow rate as function of time

and t_b will have to wait at the concourse before there is escort available to take them to the ATS station, where t_b satisfy the condition that $\int_{t_a}^{t_b} r(y) - \frac{h}{t_1} dy = 0$. Note that $t = t_b$ is not the second intersection of $r(t)$ and $\frac{h}{t_1}$ on the graph, it is to the right of the intersection. In addition, customers arriving at time $t > t_a$ or $t < t_b$ will not have to wait. Suppose a customer who arrives at time t ($t_a < t < t_b$) will have to wait for $w(t)$ hours.

Now let us calculate $w(t)$. First, we calculate the maximum capacity of escorts: $\frac{h}{t_1} = 6h$, which means that they can help $6h$ customers per hour. When a customer A arrives at time t ($t_a < t < t_b$), the escorts are still busy helping customers who arrive at time between t_a and t. Starting from t_a, it takes the escorts $\frac{\int_{t_a}^{t} r(y)dy}{6h}$ (hours) to send those customers to their destination gate.[13] They will finish their work at time $t_a + \frac{\int_{t_a}^{t} r(y)dy}{6h}$ and come to help customer A. So A will have to wait for $\frac{\int_{t_a}^{t} r(y)dy}{6h} - (t - t_a) = \frac{\int_{t_a}^{t} (r(y)-6h)dy}{6h}$ hours. That is, $w(t)$ ($t_a < t < t_b$) we need (See Figure 6):

$$w(t) = \frac{\int_{t_a}^{t} (r(y) - 6h)dy}{6h} \tag{5.1}$$

The next step is to determine the distribution of customers' waiting time $num(t')$: number of customers who will have to wait for time t'. More strictly speaking, the number of customers who will have to wait for time t', where $T_1 < t' < T_2$, is equal to $\int_{T_1}^{T_2} num(t')dt'$. Suppose the maximum value for $w(t)$

[13]In fact, y here is a dummy variable for time, we choose y instead of t so that it has no confusion with the limit of integration t

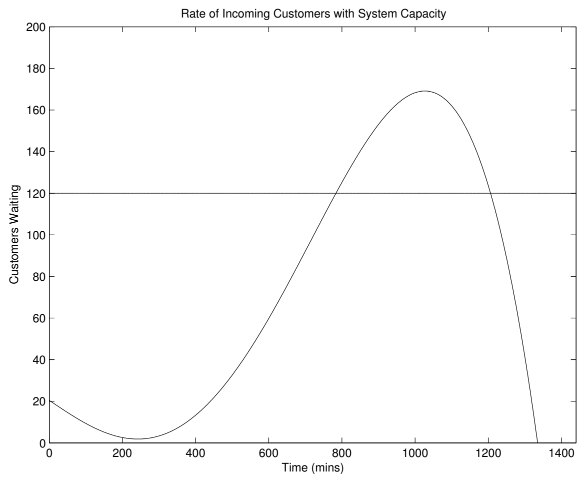

Figure 5: Flow rate with maximum system capacity

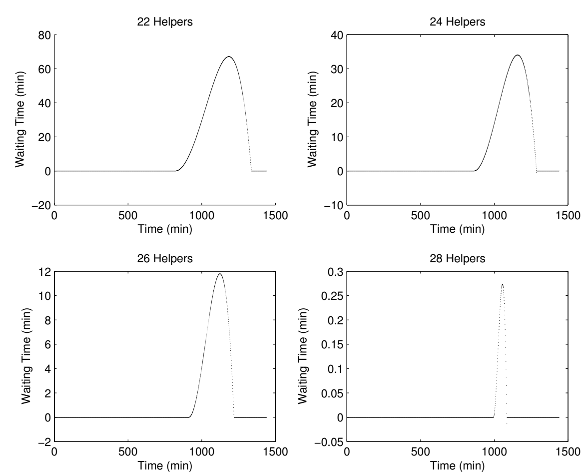

Figure 6: Plot of waiting time for h= 22, 24, 26, 28 helpers

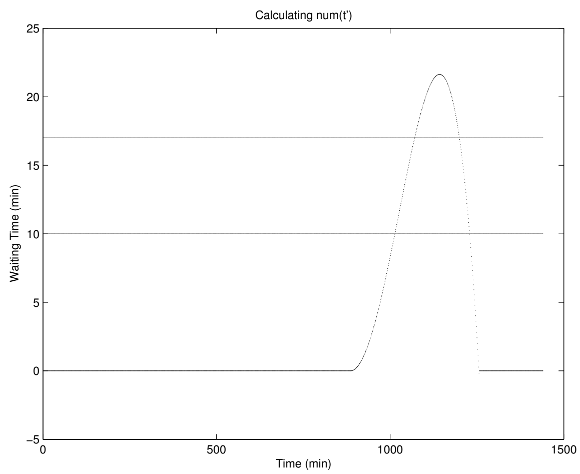

Figure 7:

is t_{max}. Obviously, for $t' > t_{max}$, $num(t') = 0$; for t' such that $0 < t' < t_{max}$, we need to calculate $num(t')$.

Consider $\int_{t'}^{t'+\Delta t} num(y)dy$. Suppose $w(t) = t'$ at $t' = t_\alpha$ and $t' = t_\beta$, $w(t) = t' + \Delta t$ at $t' = t'_\alpha$ and $t' = t'_\beta$ (See Figure 7). Then $\int_{t'}^{t'+\Delta t} num(y)dy$ will be equal to the number of customers coming in between time $t' = t_\alpha$ and $t' = t'_\alpha$, and between time $t' = t'_\beta$ and $t' = t_\beta$. For positive Δt small enough, we get $t'_\alpha - t_\alpha \approx \frac{\Delta t}{w'(t_\alpha)}$ and $t'_\beta - t_\beta \approx \frac{\Delta t}{w'(t_\beta)}$ (which means $t_\beta - t'_\beta \approx -\frac{\Delta t}{w'(t_\beta)}$, remember that $t_\alpha < t'_\alpha$ and $t_\beta > t'_\beta$). So $num(t') \approx \frac{\int_{t'}^{t'+\Delta t} num(y)dy}{\Delta t} \approx (\frac{\Delta t}{w'(t_\alpha)} r(t_\alpha) - \frac{\Delta t}{w'(t_\beta) r(t_\beta)})/\Delta t \approx \frac{r(t_\alpha)}{w'(t_\alpha)} - \frac{r(t_\beta)}{w'(t_\beta)}$ (See Figure 8).

If a customer waited for t' hours at the concourse he landed at, it is $t' + t_1/2 + t_2 + t_1/2 = t' + 1/3$ hours after he landed when he get to his destination gate. Therefore $n(t' + 1/3) = num(t')$. According to the global assumption we made about the cost due to customers missing flights (See Equation 5.2), the **expected value** for the cost is $cost_{miss} = 200 * (\int_0^{t_{max}+1/3} (P(t')n(t'))dt') = 200 * (\int_1^3 n(t') * \frac{1}{2}(t' - 1)dt' + \int_3^{max(3,t_{max}+1/3)} n(t')dt') = 200 * (\int_{2/3}^{8/3} num(t')(\frac{1}{2}t' - \frac{1}{3})dt' + \int_{8/3}^{max(8/3,t_{max})} num(t')dt')$. Ideally, we do not make our customers wait for more than 2 hours before we take them to their gates ($t_{max} < 2$), hence we get (5.2)

$$cost_{miss} = 200 * \int_{2/3}^{8/3} num(t')(\frac{1}{2}t' - \frac{1}{3})dt' = 200 * \int_{2/3}^{t_{max}} num(t')(\frac{1}{2}t' - \frac{1}{3})dt'$$

Notice that if t_max is less than $2/3$, $cost_{miss}$ will be 0.

We do not start to calculate the cost using our data at this point because we realized that the strategy we used here is not optimal: The busiest time of

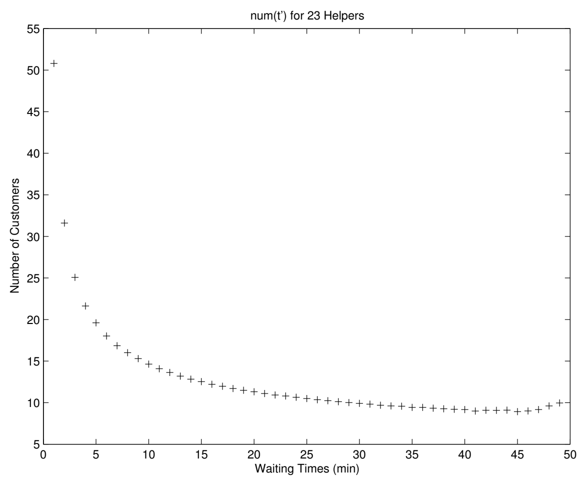

Figure 8: Number of customers waiting for given time period

the day is in the afternoon and evening, for the rest of the day we have too many escorts. Although our strategy should be able to solve the problem in the airport, we spend too much money on hiring escorts. So we decide to introduce an improved strategy in the following section.

5.2 An improved and more economical strategy

Having realized that it is not efficient to have the same number of escorts for any time of the day, we decided to divide every day into two parts: busy hours and idle hours. Busy hours are the several hours in the afternoon and evening when we has maximum number of customers. The rest are the idle hours when we do not have many customers.

Variables:
Same as above, in addition,

1. h – number of escorts during busy hours
2. h' – number of escorts during idle hours

Assumptions: same as above, in addition,

1. 10 AM - 10 PM (a twelve-hour interval) are the busy hours, 10 PM - 10 AM the next day (a twelve-hour interval) are the idle hours.
2. Unlike the situation during busy hours, we will not under-staff during idle hours, which means that we always keep enough escorts to send customer

to their destination gates as soon as they arrive. The reason is that with less escorts during idle hours, our system is more vulnerable to fluctuations in the number of customers.

We use the same data, hence the same $r(t)$ function as the previous subsection (See Figure 4). During the busy hours, assume that we are not going to make our customers to wait for more than two hours, so h is at least 20, according to function $w(t)$ and t_{max}. Hence the possible values for h are between 20 and 28 (we do not go over 28 because 28 is enough for any time of the day and it is a waste to hire more than 28 escorts). In addition, we found that if h is greater than or equal to 24, t_{max} is less than 2/3, which means $cost_{miss} = 0$. So it is wasteful to go beyond 24 escorts. h is between 20 and 24 (inclusive).

The maximum value of $r(t)$ during idle hours is $r(10) = 59.5$. $h' > \frac{max\{r(t), t\ belongs\ to\ idle\ hours\}}{6}$, the best h' will be 10. We notice that the daily cost of the system is not much less than the setup cost because hiring an escort for 24 hours will cost more than buying a wheelchair. In this case, it will be more important for us to minimize the daily cost rather than the setup cost. Now we calculate the daily cost for different possible h values to get an optimized plan.

The last equation in the previous subsection gives us a formula to calculate $cost_{miss}$ (See Equation 5.2).

(5.3)
$$cost_{system} = \$1 * h(forwheelchairs) + \$10 * (12h + 12h')(forwage) = \$(121h + 120h')$$

(5.4)
$$cost_{day} = cost_{miss} + cost_{system}.$$

(5.5)
$$cost_{setup} = \$200 * h.$$

value of h	$cost_{miss}$	$cost_{system}$	$cost_{day}$	$cost_{setup}$
20	465	3620	4085	4000
21	222	3741	3963	4200
22	76	3862	3938	4400
23	5	3983	3988	4600
24	0	4104	4104	4800

We achieve lowest daily cost at $h = 22$ (\$ 25 lower than $h = 21$). Although the setup cost for $h = 22$ is \$ 200 higher than that for $h = 21$, the lower daily cost brings $h = 22$ into advantage after 8 days. Obviously, we would prefer to choose h to be 22.

Conclusion:

The optimal plan will be to keep 22 escorts from 10 AM to 10 PM, and 10 escorts from 10 PM to 10 AM the next day. The expected value of daily cost will be \$3938, and the setup cost will be \$4400, which is a reasonable amount of money for an airline company.

Future work:

After looking at the table above, we realized that wage costs much more than maintenance of wheelchairs. So it makes sense to use electrical-powered wheelchairs so that customers with good mental health should be able to go to their destinations by themselves. Therefore, we do not have to hire as many escorts. Although the initial setup cost will increase by a large amount ($ 2000 - $ 200 = $ 1800 per powered-wheelchair bought), the daily cost will decrease (by $ 10/hour * 24 hours - ($ 6 - $ 1(the increase in maintenance))= $ 235 per day per powered-wheelchair bought) and it is beneficial in the long run. We will need to look up data about proportion of mental disabilities among people of limitd mobility in order to determine how many electrical-powered wheelchairs we can use.

5.3 Attempting to include the impact of customers on the ATS system

Description of the model:

Since it is hard for a handicapped customer to get on an ATS train (i.e. it takes longer), the train will have to wait for the customer, hence this may delay the train. We try to take this factor into account.

Variables:

1. h –number of escorts we have (global variable)
2. $w(t)$ –The amount to time a customer has to wait if he/she lands at time t, before he/she is sent to the nearest ATS station.
3. t_1–time (in hours) needed for an escort to go from an concourse to the nearest ATS station and come back. The maximum capacity of our escorts is taking $\frac{h}{t_1}$ customers from concourses to stations, and picking up $\frac{h}{t_1}$ customers at stations and sending them to their destination concourses per hour.
4. t_2–time (in hours) needed for an ATS train to run one circle.
5. v_0–average speed of the ATS trains.
6. $r_0(t)$–rate (in number of customers per hour) at which handicapped customers, who need to go to another concourse to catch the next flight, arrive. It is a function of time t.
7. r(t)–rate (in number of customers per hour) at which handicapped customers leave the concourse (for the nearest ATS station). Ideally, $r(t)$ will be equal to $r_0(t)$. However, if a lot of handicapped customers arrive in a short period of time and we do not have enough escorts to bring them to their destinations, then $r(t)$ will be less than $r_0(t)$ temporarily. In fact, the maximum value of $r(t)$ will be $\frac{h}{t_1}$. To make up for this, if $r_0(t)$ is to drop below $\frac{h}{t_1}$ later on, $r(0)$ will have to stay at $\frac{h}{t_1}$ until all the customers

are sent to the ATS station. The integral of $r(t)$ and $r_0(t)$ are equal in the long run.

Assumptions:

1. It takes virtually no time to move between gates within a concourse, but it takes a long time to move from a concourse to its nearest ATS station.

2. Escorts and wheelchairs spread out in the most efficient way, which means, they will move to other concourses if they have nothing to do and another concourse needs help because too many customers are coming in through that concourse. And escorts can move very fast if they are not taking any customers with them, so it takes virtually no time for them to spread.

3. Our rule in helping our customers is "first comes, first served."

4. $t_1 = 1/6$, which is equivalent to 10 minutes (5 minutes to go one-way, 10 minutes to go and come back); $t_2 = 1/6$, which is equivalent to 10 minutes. This is reasonable for a modern airport.

5. Customers always arrive at ATS stations when a train is supposed to come, which means that if the train is on time, they will not need to wait at the station. However, if the train is delayed, they will have to wait the amount of time the train is delayed.

6. Customers do not have to wait once they get off the train. Since customers are sent to ATS at a rate of at most $\frac{h}{t_1}$ per hour, we assume that they get off uniformly with respect to time (they do not get off at the same time), the rate at which they get off should not exceed $\frac{h}{t_1}$ per hour. So there are always escorts available at the station to pick them up.

7. It takes a while for a handicapped person to get on a ATS train, hence the train can be delayed.

8. The following idea comes from the differential equation describing the motion of an oscillator under a known force, and we use the corresponding equation to model the movement of trains. The movement of ATS trains can be described by a second order linear differential equation, with oscillation frequency being the frequency at which the trains operate at: $\omega_0 = \frac{2\pi}{t_2}$. In addition, we assume that whenever a train is delayed, the driver will try to catch up. However, he/she will not get ahead, which means that our equation describes an under-damped oscillation.

According to Assumption 8, here is the equation we use to model the movement of trains:

$$x'' + 2\gamma x' + \omega_0^2 x = r(t - t_1/2) \tag{5.6}$$

Initial condition: $x(0) = 0$; $x'(0) = 0$. x is the distance by which the train is behind. Notice that the rate at which (handicapped) customers get on the train at time t is equal to the rate at which (handicapped) customers leave the concourse they come from at time $t - t_1/2$ because it takes the escorts $t_1/2$ to bring customers from concourses to ATS stations. That explains why it is $r(t - t_1/2)$ instead of $r(t)$ on the right hand side of the equation. Now we solve for $x(t)$.

We apply the method of variation of parameters to solve the equation. First, we solve for the associated homogeneous equation:

$$x'' + 2\gamma x' + \omega_0^2 x = 0 \tag{5.7}$$

The characteristic equation is $r^2 + 2\gamma r + \omega_0^2 = 0$, whose roots are $r_1 = -\gamma - \sqrt{\gamma^2 - \omega_0^2}$ and $r_2 = -\gamma + \sqrt{\gamma^2 - \omega_0^2}$

According to assumption 8, our model is under-damped, which means that both roots are real ($\gamma > \omega_0$). Two solutions of the associated homogeneous equations are $x_1(t) = e^{r_1} = e^{(-\gamma-\sqrt{\gamma^2-\omega_0^2})t}$ and $x_2(t) = e^{r_2} = e^{(-\gamma+\sqrt{\gamma^2-\omega_0^2})t}$. The solution to the original equation will have the form $x(t) = u_1(t)x_1(t) + u_2(t)x_2(t)$

Now we impose the following conditions on $u_1(t)$ and $u_2(t)$:

$$u_1'(t)x_1(t) + u_2'(t)x_2(t) = 0, u_1'(t)x_1'(t) + u_2'(t)x_2'(t) = r(t - t_1/2) \tag{5.8}$$

Under the conditions above, we have $x' = u_1'x_1 + u_1x_1' + u_2'x_2 + u_2x_2' = u_1x_1' + u_2x_2'$ and $x'' = (u_1x_1'' + u_2x_2'') + (u_1'x_1' + u_2'x_2')$. So $x'' + 2\gamma x' + \omega_0^2 x = (u_1x_1'' + u_2x_2'') + (u_1'x_1' + u_2'x_2') + 2\gamma(u_1x_1' + u_2x_2') + \omega_0^2(u_1x_1 + u_2x_2) = u_1(x_1'' + 2\gamma x_1' + \omega_0^2 x_1) + u_2(x_2'' + 2\gamma x_2' + \omega_0^2 x_2) + (u_1'x_1' + u_2'x_2') = u_1'x_1' + u_2'x_2' = r(t - t_1/2)$. We can solve for $u_1'(t)$ and $u_2'(t)$ using the system of equations above, the original equation is then satisfied.

We get:

$u_1'(t) = -\frac{r(t-t_1/2)}{2\sqrt{\gamma^2-\omega_0^2}} e^{(\gamma+\sqrt{\gamma^2-\omega_0^2})t}$ and $u_2'(t) = \frac{r(t-t_1/2)}{2\sqrt{\gamma^2-\omega_0^2}} e^{(\gamma-\sqrt{\gamma^2-\omega_0^2})t}$

We still need to choose $u_1(0)$ and $u_2(0)$ such that the initial condition is satisfied.

$x(0) = u_1(0)x_1(0) + u_2(0)x_2(0) = u_1(0) + u_2(0)$. Since $x(0) = 0$, we get $u_1(0) + u_2(0) = 0$.

$x'(0) = u_1(0)x_1'(0) + u_2(0)x_2'(0) = (-\gamma - \sqrt{\gamma^2 - \omega_0^2})u_1(0) + (-\gamma + \sqrt{\gamma^2 - \omega_0^2})u_2(0) = -\gamma(u_1(0) + u_2(0)) - \sqrt{\gamma^2 - \omega_0^2}(u_1(0) - u_2(0)) = -\sqrt{\gamma^2 - \omega_0^2}(u_1(0) - u_2(0))$. Since $x'(0) = 0$, we get $u_1(0) - u_2(0) = 0$.

Now it is easy to see that $u_1(0) = 0$ and $u_2(0) = 0$. Hence $u_1(t) = \int_0^t u_1'(y)dy$, $u_2(t) = \int_0^t u_2'(y)dy$.

So here is our solution:

(5.9)

$$x(t) = e^{(-\gamma-\sqrt{\gamma^2-\omega_0^2})t}\int_0^t(-\frac{r(y-t_1/2)}{2\sqrt{\gamma^2-\omega_0^2}}e^{(\gamma+\sqrt{\gamma^2-\omega_0^2})y})dy+e^{(-\gamma+\sqrt{\gamma^2-\omega_0^2})t}\int_0^t(\frac{r(y-t_1/2)}{2\sqrt{\gamma^2-\omega_0^2}}e^{(\gamma-\sqrt{\gamma^2-\omega_0^2})y})dy$$

According to assumption 6, the amount of time $w(t)$ that a customer who arrives at time t will have to wait consists of two parts:

$w_1(t)$–the amount of time he/she has to wait at the concourse he/she landed at

$w_2(t)$–the amount of time he/she has to wait at the ATS station

$w(t) = w_1(t) + w_2(t)$

$w_1(t)$ is a function that depends on $r_0(t)$. We will determine $w_1(t)$ later. Assume we have $w_1(t)$ for the moment, the customer who arrives at the airport at time t will get to the ATS station at time $t' = t + w_1(t) + t_1/2$. By the time he/she gets to the ATS station, the ATS train is behind by a distance of $x(t')$, which means the train is delayed by approximately $\frac{x(t')}{v_0}$. We can say that $w_2(t) = \frac{x(t')}{v_0}$.

$$w(t) = w_1(t) + \frac{x(t + w_1(t) + t_1/2)}{v_0} \tag{5.10}$$

However, when we start to test the model, we found that it does not work very well. We started with enough escorts so that $w_1(t) = 0$, but x(t) seems to be always 0, the train is not delayed even at t = 15 or 16 (3 PM or 4 PM) when we have the largest number of customers. It is possible that we may have picked some incorrect parameters for the simulation.

5.4 Summary of Public Transportation Model

By making use of the pre-existing public transportation system our model incorporates an effective strategy for minimizing the total cost of the system from the standpoint of both cost of employees and cost of wait times for customers. By using flow analysis it is possible to arrive at the number employees to hire for both day and night shift to maximize the cost effectiveness of the system. By modifying the parameters utilized in this analysis it is possible to use our model to perform the same flow analysis on any hospital with a means of public transportation. This then provides a process for optimizing airline efficiency at any airport. This makes the flow analysis developed here extremely practical from a business standpoint. The next consideration is the necessary steps the airline must take for airports without a public transportation system.

6 Privately-Connected Terminals

In the situation in which a major airport does not possess any form of public transportation between terminals, it will be necessary for Epsilon Airlines to implement its own form of inter-terminal transportation for passengers of limited mobility. This system is especially important for large airlines that often occupy more than one terminal in major airports. The system must be capable of transporting customers between terminals efficiently without breaking down due to excessive volume. Since this system is being developed in a larger airport, special consideration must be taken to include the influences of the sizeable number of people processing through the airport. Although this provides additional complications to our analysis, it also enables us to make several assumptions that will provide us with a continuous model for the description of our system.

The first part of our systerm utilizes the concept that it is significantly easier to transport customers through the airport by utilizing a centralized system of mass transportation. This is the same concept that was developed in the previous section for airports that already possessed a pre-existing form of public transportation. Our proposal is that all of the customers requiring additional assistance will be brought to the entrance of the terminal via the system developed in the one terminal simulation. In essence the entrance to the terminal can be viewed as a gate and the model developed for one terminal can again be applied successfully. A series of trams will then be utilized to transport the customers between the different terminals. The trams will be outfitted with a communication system to enable them en route to communicate to the next terminal's substation, the number of customers that will require helpers. These helpers will then assist in unloading the customers and their wheelchairs and transporting them to the necessary gates within the terminal. Again the entrance to the terminal can be thought of as a gate with incoming passengers to provide coherence with the single terminal model. Since the workings of the single terminal have already been investigated, it is necessary to understand the workings and efficiency what we will call the "Tram Model."

6.1 Tram Model Analysis

In order to perform an analysis on the Tram Model we assumed that the volume of the number of people processed by a major airport was large enough to allow for a continuous analysis of the system. In terms of a physical system, the Tram Model has several unique properties. First, the trams run in a cyclic patern. This implies that there can be a period associated with their movement. Second, should there be no customers to process and no additional passengers in the airport, the Tram Model would oscillate stably without any fluctuations. Lastly, if the tram stops in order to pick up a customer, it would fall behind and there would be a function to describe the the magnitude of the deviation between the tram's actual position and its ideal posititon. These properties will enable us to make a very unique analysis of the Tram Model.

In nature there are many phenomenon that demonstrate very conflicting forms of behavior. In one situation they may present unique patterns, but in the next they may react chaotically. In mathematics these systems are oftern described as being non-linear. We assume in this analysis that the Tram Model can be described as being non-linear. This assumption arises from the nature of major airports. The number of people being processed and the choatic delays that can occur in aircraft arrival and departure times contribute significantly to this behavior. Therefore our assumption derives from the fact that like most other large airport systems, the Tram Model will also be subject to these influences. In addition to this assumption, the properties pointed out in the previous paragraph enable us to do a very specific analysis on the Tram Model. Since the Tram Model would oscillate stably without forcing, it can be described as a harmonic oscilator. Thus the nature of the Tram Model lends itself naturally to the same analysis that can be performed on natural non-linear harmonic oscillators.

6.2 The Mathieu Equation and Parametric Resonance

The equation that is utilized for describing non-linear harmonic oscillator systems is called the Mathieu equation.

$$x'' + 2\gamma x' + \omega_0^2[1 + f(t)]x = 0 \tag{6.1}$$

In the above equation γ refers to the damping coefficient of the system, ω_0 refers to the natural angular frequency of the system, and $f(t)$ represents the forcing function of the system. It is this forcing function that makes the system non-linear since it represents a different form of forcing. The forcing that is represented in the Mathieu equation is referred to as parametric forcing rather than direct forcing since the forcing is a product of the actual displacement x itself rather than the forcing associated with a non-homogeneous differential equation. In a physical sense this implies that force exerted on the system is dependent upon the actual state of the system, a property the Tram Model possesses.

The next step for us in the analysis of the Tram Model is to determine the nature of the forcing that the system will undergo. We know that planes arrive at regular intervals due to national aviation guidelines that set minimums for the distance between planes. Therefore it is safe to assume that our customers will be entering the system in discrete groups. This type of forcing can be modeled by delta functions in which there arrives a pulse on a periodic schedule and the pulse is of some magnitude. The Mathieu equation can then be rewritten with all coefficients normalized by ω_0^2.

$$x'' - 2\gamma x' + [1 + \epsilon \sum_{n=-\infty}^{\infty} \delta(t - n\Delta t)]x = 0 \tag{6.2}$$

Since the coefficient ϵ describes the number of people entering the system, the function $x(t)$ can be thought of as describing the number of people by which the system is either ahead or behind. Since it takes an average time to process each person, this value can be converted to time if necessary. In order to perform the analysis on this equation we will follow the analysis done by Bechhoefer and Johnson [14] on the Mathieu equation with positive amplitude delta function forcing.

The first step in the analysis is to recognize that in between the delta function forcing, the system is nothing but a second order ordinary differential equation that can be solved by initial conditions. Since the equation is homogeneous, the eigenvalues are found to be $-\gamma \pm \sqrt{1-\gamma^2}$ and the resulting equation is:

$$x_n(t) = A_n e^{\lambda_1 (t-t_n)} + \text{complex conjugate} \tag{6.3}$$

where λ_1 is one of the eigenvalues and the complex conjugate is represented by the other eigenvalue. From here we make two assumptions concerning the nature of the tram. First that its difference between its actual and its ideal point is continuous and second that its speed is discontinuous. These two assumptions make sense in the context of the Tram Model since the Tram can't suddenly fall behind or get ahead of itself in jumps. Second, the speed of the Tram can be discontinuous when it stops to pick up or drop off customers. Since the tram is most likely not going to be moving very fast (if this isn't the case, we have bigger problems) the tram's time of acceleration and deacceleration can be considered negligible. These conditions can be summarized in the following equations for two consecutive periods of time.

$$x_{n+1}(t_{n+1}) = x_n(t_{n+1}) \tag{6.4}$$

$$x'_{n+1}(t_{n+1}) = x'_n(t_{n+1}) - \epsilon x_n(t_{n+1}) \tag{6.5}$$

By solving the ordinary differential equation for its inital conditions and separating it into its real and imaginary parts, the following matrix system of equations can be created.

(6.6)

$$\begin{pmatrix} A^r_{n+1} \\ A^i_{n+1} \end{pmatrix} = e^{-\gamma \Delta t} \begin{pmatrix} cos(\omega' \Delta t) & -sin(\omega' \Delta t) \\ sin(\omega' \Delta t) + \frac{\epsilon}{\omega'} cos(\omega' \Delta t) & cos(\omega' \Delta t) - \frac{\epsilon}{\omega'} sin(\omega' \Delta t) \end{pmatrix} \begin{pmatrix} A^r_n \\ A^i_n \end{pmatrix}$$

(6.7)

$$\omega' = \sqrt{1-\gamma^2}$$

The 2x2 matrix above coupled with the exponential term provide us with a matrix that describes the change in amplitude of the tram's status from one time period to the next. In order to account for the different number of trains in the system we divide ϵ by the number of trains currently deployed to better account for the average number of people the system is behind by. The magnitude will then enable us to determine the number of trams to be opperating based on the current volume of people being processed by the system.

[14] See [1]

6.3 Determining the Number of Trams: Operating Procedure

In order to determine the number of trams to operate at a given period of time, the current lag of the system is carefully monitored. The amplitude of the lag determines the strain on the system and whether or not additional trams should be committed to the system, thereby lessening the load. However, it is not economical to be modulating the number of trams operating every minute. Therefore we determined that the number of trams should be allowed to be modified only if it has been at least fifteen minutes since the last modulation occurred. By allotting this waiting period between modifications, it enables us to determine potential patterns in the volume of customers and thereby optimize the number of trams operating. Ideally we would like to have no customers waiting at any time. However, since this is not practical due to the high number of trams it would require, a different standard must be set. Thus we determined that a normalized standard would be most efficient. If the number of people that the system is behind by ever reaches more than the capacity of all the trams operating, then a new tram would be sent out. This determination was made based on the idea that if there are more people waiting than can be processed in one cycle, we will have wait times longer than one cycle of the system, which would prove to be unacceptable for getting customers to their connecting flights successfully. A similar algorithm is utilized for determining when trams should be pulled from the system. If the number of people the system is behind by is ever less than the capacity of one fewer than the number of trams, then a tram is pulled. This allows the system to operate most efficiently by attempting to fill each tram as close to its capacity without exceeding it. In addition to the algorithm being utilized for determining the number of trams operating, careful supervision should be paid to the state of the system by the managers at each of the substations closest to the beginning of the terminals. There exist conditions in which the system could potentially brake down chaotically with devestating results. Several of these cases and the reasons behind them will be explored in the sensitivity analysis.

Another consideration to be made when determining the operation of the tram is the issue of conservation of wheel chairs. When a customer is removed from the tram and no customer is to board the tram, then an empty wheel chair should be placed on the tram in the customer's place. Similarly in the case that a customer is to board a tram with an empty space, an empty wheelchair will be removed from the tram and left at the current terminal. In essence this procedure will keep the number of wheelchairs at any one terminal equal. If this procedure were not maintained, it is possible to conceive how wheelchairs would tend to congregate at the terminals most often utilized by Epsilon Airways, thereby leaving some terminals with a dearth of wheelchairs. It might be possible to delay the implementation of this procedure during start-up of the system, thereby better distributing the wheelchairs to the terminals most utilized by Epsilon Airways, but its practice should cease relatively soon after initialization or the above effects could occur. In order to determine the effectiveness of this operating procedure we utilize the Tram Model to perform a simulation in a large scale airport.

6.4 Example: Los Angeles International Airport (LAX)

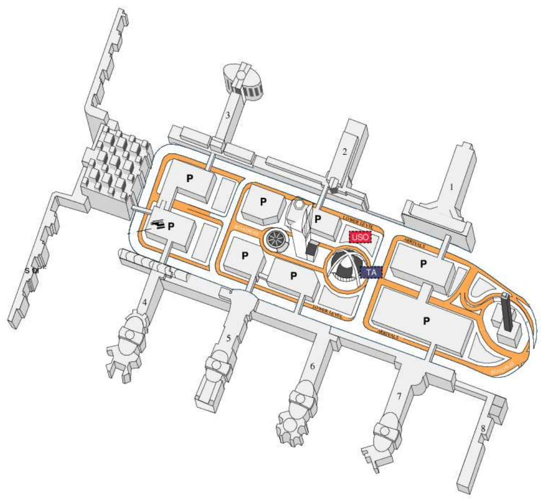

Figure 9: Los Angeles International Airport

One possible example of an airport in which the Tram Model could be implemented would be Los Angleles International Airport (LAX). LAX has eight terminals that are connected by walkways, but has no major public transportation means within the airport (see Figure 9). In addition to there being no major forms of public transportation, several major airlines are housed in more than one terminal. For example, United Airlines is housed in terminals six, seven, and eight. As a major carrier (assuming that its name has no correlation with its size) Epsilon Airlines would most likely have flights arriving in more than one terminal. Thus the Tram Model would need to be implemented in order to provide Epsilon Airlines with a means of moving impaired customers between the terminals.

To demonstrate the effectiveness of the Tram Model, we model the movement of customers through LAX during the period of one business day. We utilize the quartic model for the arrival of airplanes, but scale the magnitude, so as to match the average number of arrivals for LAX during a business day. This approximation should accurately reflect the number of incoming flights since most major airports follow the same set of air patterns and have a majority of their incoming flights landing in the afternoon. In addition to this we used the average number of people per flight and average number of customers on a flight to determined previously to develop an an equation for the magnitude ϵ of incoming customers and also for the average amount of time between flights. The average time between flights would be utilized as the step size for the simulation. In order for the simulation to work properly, the initial conditions must be such that at least one person is waiting somewhere in the system. Although it is early in the morning it is conceivable that such a situation exists. The last parameter that was necessary for us to fix was the damping coefficient γ. This value defines how quickly the system can recover from an input forcing, such as in this case, the arrival of the flight. For the purposes of this simulation we choose γ to be equal to zero. In terms of the physical significance of the system, this implies that the driver doesn't attempt to catch up if he falls behind by driving faster or rushing the loading and unloading of customers. This is a good approximation from a safety standpoint, but might not be realistic since most workers will attempt to make up for lost time by speeding up the rate at which they complete a task, but that is another model.

The simulation was performed by using MATLAB and stepping through the day by one minute at a time. The graph in Figure 10 shows the output of A_n or the number of people that the system was behind by at each minute of the day.

From this graph it is possible to see that the system responds well to the changing volume of passengers throughout the day. Although it appears that the graph is very chaotic, it is possible to understand it within the context of the problem. First, it must be understood that at every interval of time there are new planes landing and therefore more customers entering the system. However, the tram is only capable of picking up these people and every time it reaches a terminal and can do nothing for them during its transit between terminals. Similarly when a tram reaches a terminal and drops off a customer, the number of waiting customers will drop significantly. Thus, although the tram system is working properly, the number of apparent people behind will vary greatly as a function of time. By carefully examining the graph it is possible to see that the maximum number of people that the tram system is ever behind by is about thirteen people. This is a very reasonable number and can be considered a successful test of the Tram Model.

6.5 Example: Sensitivity Analysis of the Tram Model and Harmonic Tongues

Despite the successful simulation of the Tram Model on a regular business day at LAX, there still exists several conditions which could cause a catastrophic collapse of the Tram Model. These conditions arise out the instability of non-linear systems. In the case of this system, the rate of arriving flights and the number of people crowding the airport would put severe stress on the system and result in failure. In this sensitivity analysis we will analyze two situations in which could result in the failure of the Tram Model: modulation of the waiting time before the number of trams can be modified and changing the rate at which planes arrive at the airport.

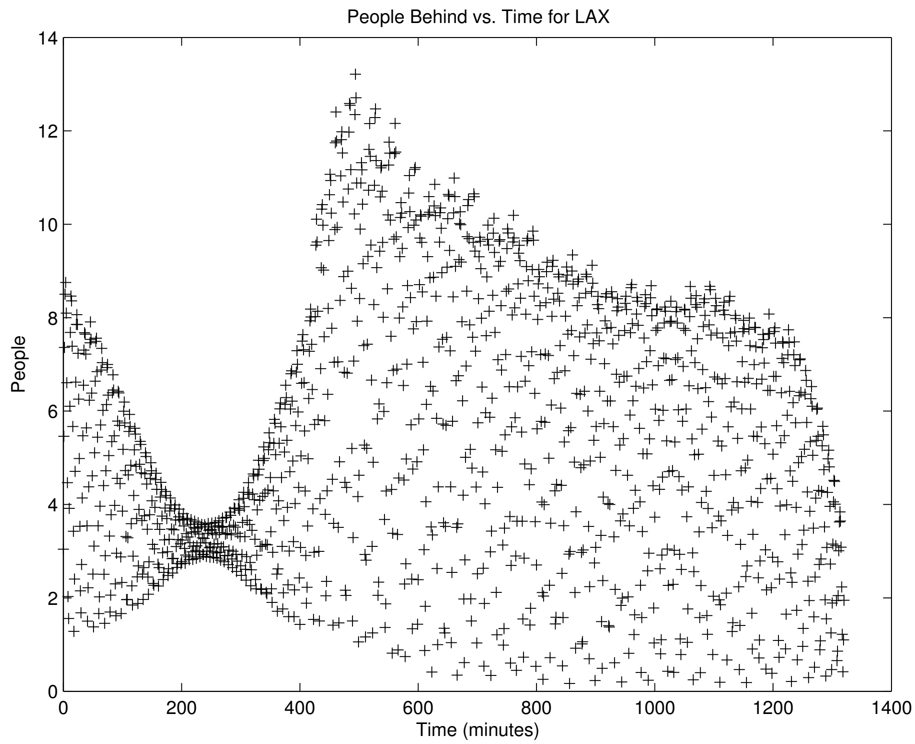

Figure 10: Response of Tram System During a Business Day at LAX

In the first case, if the amount of time between modification of the number of trams was to be inceased to a point where it was too large, it could have devastating effects on the Tram Model. In this model the same simulation was made except for the waiting time was increased by five minutes to twenty minutes. The results can be viewed in Figure 11.

Despite the fact that only a small time difference was made, there was an extraordinary change in the behavior of the system. This is due to the nature of the response of the system. The system has certain monitoring times which could result in an unstable system. For example, if the waiting time is again increased to thirty minutes the system stabilizes as can be seen in Figure 12. The nature and reason behind this phenomenon will be explained at the conclusion of this section.

The other instability that we will investigate is the nature of the rate of arrivals at the airport. In this simulation we again utilize the Tram Model presented in the simulation of LAX, but instead of using the regular distribution of flights for a large scale airport like LAX, we instead choose some constant rate within the daily range for arrival flights. We chose to use 100 flights per hour as the rate. Although this seems to be large, the number of flights arriving

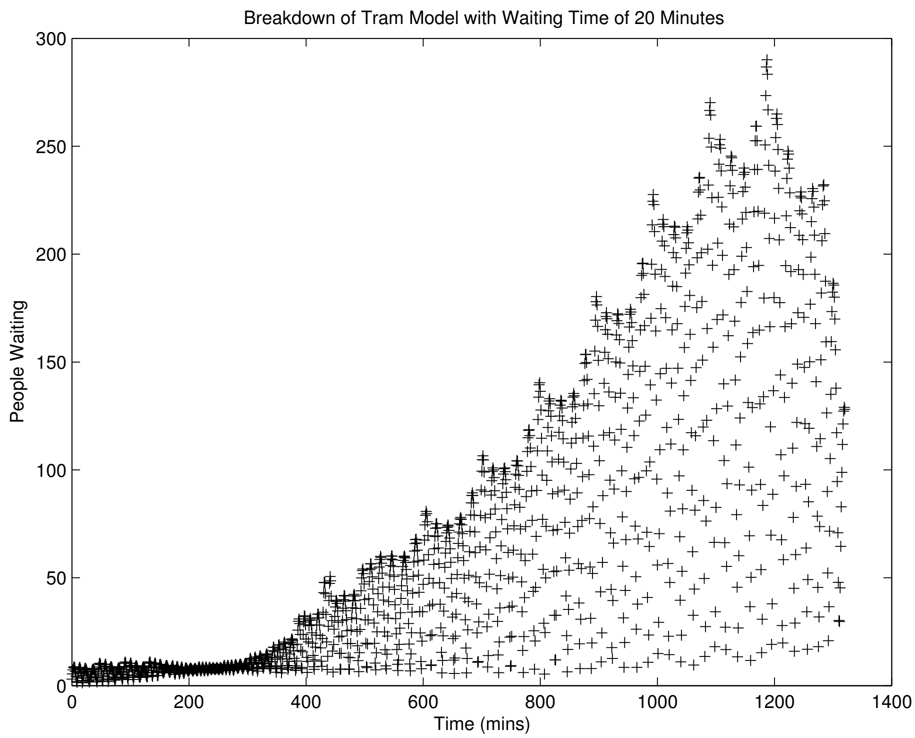

Figure 11: Response of Tram System During Break Down

at LAX during the peak of the afternoon is around 166 flights an hour. Thus 100 flights per hour is a rate that must occur throughout the day. However, when this rate is simulated, the system completely crashes and people are left waiting in numbers that would never actually occur. This can be observed in Figure 13.

Intuitively this result doesn't make sense, however, we will show that this is classic response of a non-linear system and should be considered in the when implementing the Tram System.

Although these behaviors seem like a total collapse of our system and the Tram Model, they are in fact realities of the real world with which we all must deal. The difference is that people are capable of recognizing these instabilities and changing the settings of the system so that the system is no longer unstable. This is the motivation behind the human supervision of the Tram Model. In order to understand the effect mathematically we perform stability anlysis on the solution to the Mathieu equation.

By knowing the eigenvalues of the equation, we can determine the change

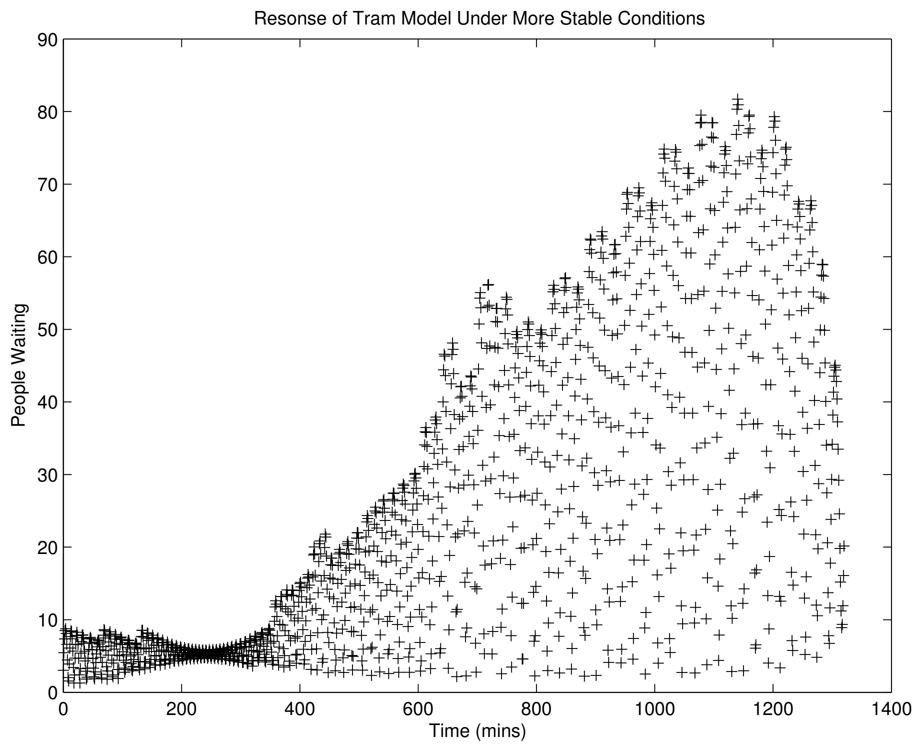

Figure 12: Resonse of Tram System in a More Stable Condition

of the amplitude for each time step. According to Bechhoefer and Johnson,[15] whenever the magnitude of the eigenvalues exceed the magnitude of the exponential term associated with the matrix, the system becomes unstable. By solving for this equation, they were able to derive a function of the time interval between the forcing function (arrivals) for the magnitude of the delta functions (number of customers) that would result in an unstable system. That equation for LAX is shown below.

$$\epsilon_c(\Delta t) = 2\omega'(\frac{cos(\omega'\Delta t) \pm cosh(\gamma\Delta t)}{sin(\omega'\Delta t)} \tag{6.8}$$

Here ϵ_c represents the threshold number of customers the system can handle for a given rate of arriving flights (Δt). Any number of incoming customers larger than ϵ_c for a given rate will result in an unstable system. This can be understood for the above to situations in the following manner: in the first example by changing the amount of time the system must wait before modulating the number of trams, we caused the system to change its relative view of the

[15] See [1]

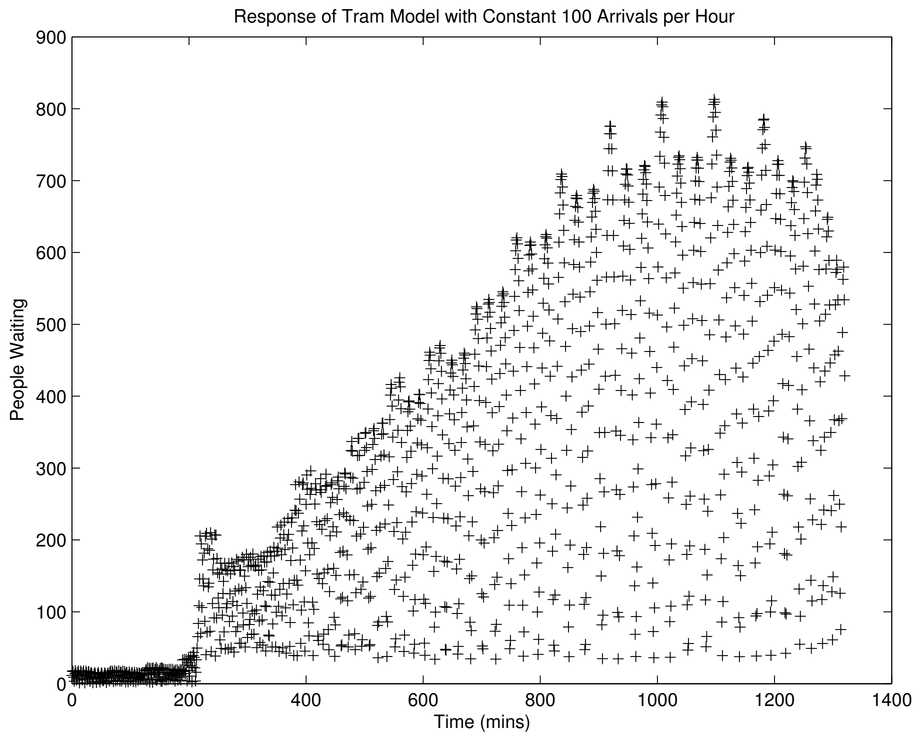

Figure 13: Response of Tram System at 100 Flights per Hour

incoming flights. Essentially, after the number of trams is change the system views a twenty minute block of incoming flights. Twenty minutes happens to be one of the time periods in which the system is unstable and it therefore crashes. However, at thirty minutes the system moves towards stability. Similarly when the arrival rate of flights is fixed at 100 flights per hour, the system becomes unstable despite the fact that at somepoint during the original simulation it was at 100 flights per hour and didn't crash. This implies that there exists certain values of the arrival time at which the system becomes unstable if left there for long periods of time. From these two results we can hypothesize that there exists instabilities within the system that occur periodically over the range of the arrival rate of flights and magnitude of incoming customers.

This result agrees exactly with the response of non-linear harmonic oscillators. The cyclic instabilities are referred to as harmonic tongues and can be seen in Figure 14.

This demonstrates the cyclic nature of the instabilities and agrees completely with the results found from the simulation of the Tram Model.

The Tram Model does have several means of controling these instabilities. Coincidentally, the number of trams has a direct effect on the uding additional

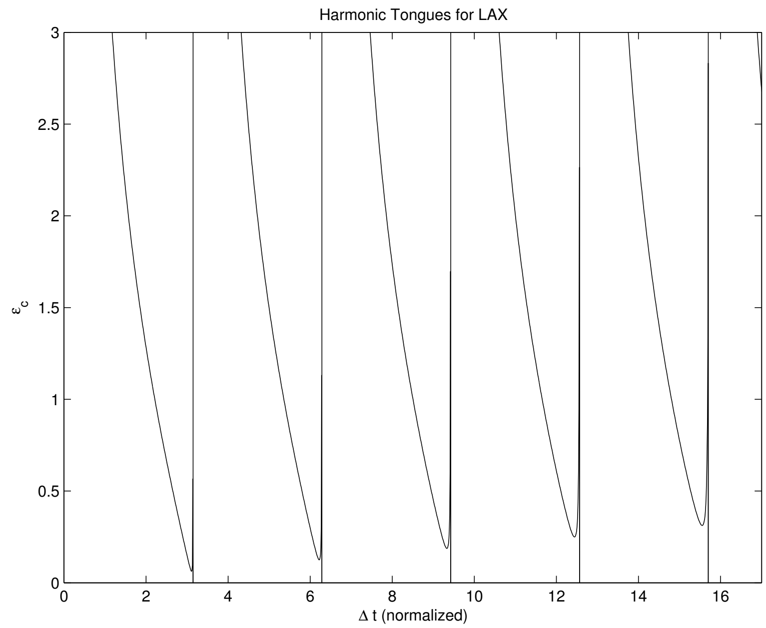

Figure 14: Normalized Harmonic Tongues for LAX

trams, the chance for the system to fail falls off drastically since more trams shifts the instabilities to points where Δt are extremely large. This effect can be observed in Figure 15.

Although the Tram Model can be unstable, the natural cycle of air traffic and human supervision can prevent the system from remaining in unstable states for extended periods. There are also additional measures that the operator of the system can take to manage these instabilities such as flight scheduling to avoid unstable arrival rates and incrementing the number of trams operating to reduce the probability of reaching an unstable state. The Tram Model can be unstable, but the instabilities are manageable.

6.6 Cost Analysis

To determine the cost of the Tram Model, we determined that it is first necessary to know the rate at which the Tram Model processed customers. The number of people that the system is capable of handling is directly dependent upon the number of trams that are operating. Therefore it is necessary to derive an equation for the number of people transported as a function of the number of trams operating. Since, we solved the Mathieu equation independent of the average angular frequency of the tram, it is possible to utilize the results from the LAX simulation to determine this relationship. By utilizing MATLAB we

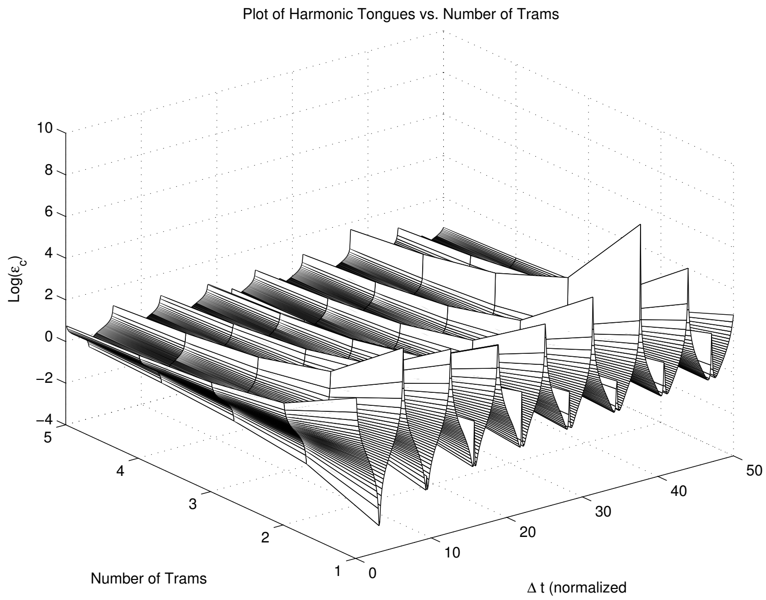

Figure 15: Location of Instabilities as a Function of Trams

integrated over the number of people waiting for different time periods in which different trams were running. Since we know that customers are conserved, we subtracted the total number of input customers from the number waiting for different periods and then divided by the number of trams operating during that time. We then averaged all of these rates for the different number of trams to come up with an average rate per number of trains. We again used MATLAB to fit a line to these points. The reason for utilizing a linear fit will become apparent later in the paper. The resultant flow equation is shown below.

$$f(m) = 12.458 * m - 6.621 \tag{6.9}$$

Here m is the number of trams and f is the flow or the number of people processed per unit time. We can then invert this equation to come up with a function of the the number of trams necessary as a function of flow. By applying the costs outlined in the chart on page 4, it is possible to then develop an equation for cost as a function of flow. This equation incorporates both the cost of the trams and the workers who operate them.

$$cost_{setup} = 2500 * \max(m) \tag{6.10}$$

$$cost_{system} = 10 * \max(m) + 10 * \sum_{t=0}^{24} \frac{10 * (r(t) - 6.621)}{12.458} \tag{6.11}$$

$$cost_{miss} = 200 * \int_{0}^{24} (r(t) - f(m(t)))dt \tag{6.12}$$

Using these cost equations it is possible to calculate the cost of both the system set up and the daily operating cost for the Tram Model. These equations provide a means of estimating the cost of the Tram Model for any airport and can greatly aid in allocating the necessary monetary resources in the annual budget.

6.7 Summary of the Tram Model

The Tram Model provides a strategy for customer movement in a large airport that doesn't possess a public transportation system within the airport. This model relies on the idea that it is more efficient to bring all customers to central system and then use this system to redistribute the customers to their necessary terminals. This central system is implemented in the form of a tram that cycles regularly between the terminals. The number of trams that are cycling at any given time is dependent upon the input flow of people into the system. Due to the nature of this system, it can become unstable and crash. However, with both computer and human monitoring and the use of several unique strategies for changing the system's state, the system can remain stable and operate effectively. The cost of the system can be modeled using linear equations which can provide a useful tool for estimating the total cost of the system at an airport over a fiscal year. The Tram Model provides a well-developed, controllable, and cost effective startegy for supporting customer travel through a large airport devoid of public transportation.

7 Multicommodity Network Flow

Until now we have been examining various pieces of the complex puzzle that is a modern airport with various mathematical and computational tools. Now we attempt to synthesize our results into a large model that incorporates many sources of data available to an airline. The tool we will use is network flow.

7.1 Modeling Airports as Graphs

We can view an airport as a graph where the vertices are the gates (or to simplify computation with another layer of abstraction, terminals) and the edges are the various connecting features between them. Air travelers flow between vertices along the edges, each trying to get to their destination. This is a typical network flow graph, familiar to those comfortable with graph theory. The next question is, where are the source and sink vertices? That is, where does the flow enter and leave the network? From our formulation of vertices representing gates we know that some gates should be sources and other should be sinks. Passengers appear in the airport system as planes arrive at gates and leave the system as they board other planes and depart. However, over the course of a day (or perhaps even an hour) a given gate may act as both source and a sink. Further, even if we assigned half the gates as sources and half the gates as sinks in some pattern, the maximum flow would probably be degenerate: each passenger arriving at a source would simply go to the nearest sink, mostly likely very close to the source (since we might assign source and sink labels randomly). Also, this does not fully approximate airport behavior: each passenger has one specific destination: simply reaching any departure gate is not the answer. A passenger cares *where* he or she is going, not simply that he or she is departing. To model the selective behavior of passengers in the network, we make the graph a *multicommodity* network flow graph. Instead of a single substance flowing from any source to any sink, we have multiple substances (commodities) each with a specific source and specific sink. Each group of passengers desiring to board a specific departing flight are one "commodity" in our network. This means the airline must support the movement of customers from many gates to many other (possibly far away) gates, in a specific pattern.

7.2 Obtaining Useful Results

While simple (single commodity) network flow problems have several provably fast algorithms for finding the maximum flow, we cannot use these in this case: we have multiple commodities. Additionally, Epsilon Airlines is not merely interested in the most people they can move through an airport. Our client is interested in the most cost-effective way of allocating resources to move people through the airport. Thus, we are actually computing a cost minimization flow network. Rather than finding the maximum amount of sustainable flow through the network, we will fix the overall flow of the network and then determine the most cost-effective arrangement of flows across various edges. We can do this by assigning a cost function that scales with the flow across each edge. To be able to handle more people in a terminal, for example, the airline needs to hire more helpers. The number of helpers required to sustain a particular flow determines the cost of the flow along that edge. The amount of customers we can help per helper or tram per unit of time which we determined earlier will come back into play here.

To determine the overall "fully optimal" arrangement for the airline, we suggest an iterative application of our algorithm: vary required flows and compute the minimum cost for supporting each. The overall flow determines the number of passengers who miss their planes, which imposes another cost. Finding the balance point between these two costs will lead to the overall best strategy for the airline.

7.3 Example: Pittsburgh International Airport

This strategy might grow clearer with an example. Here we will present Pittsburgh International Airport as our case study for multicommodity flow. For the moment we ignore the distinctions between airlines (perhaps our friends at Epsilon Airlines are so successful that they are the only carrier operating out of Pittsburgh). We also view the terminals as abstract, aggregate "gates" where passengers both arrive and depart from, in order to simplify our graph and to apply our flow information from Part 4.

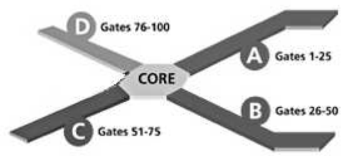

Figure 16: Pittsburgh International Airport

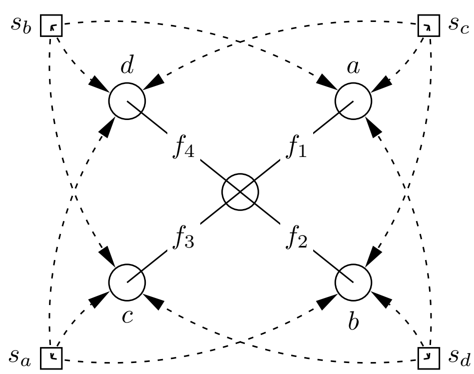

Figure 17: Abstraction of Pittsburgh Airport

The structure of the airport is clear: four terminals equally spread around a "core" that all inter-terminal traffic must pass through. Edges connect the terminals to the core to represent the corridors that permit human traffic. Round vertices and solid edges denote terminals and corridors, respectively, and in general actual physical features of the airport. Each type of passenger has one destination (their departure gate), but may come from several different sources (that is, other planes connecting to that flight or from the ticketing area). We need to make each commodity have a single source and a single sink, so we add "supersources" representing the source for all passengers going to the same destination. These supersources are depicted by rectangular nodes and the edges connecting them to terminals are dotted, to represent their abstract and conceptual nature.

7.4 Notation and Constraints

The supersource labeled s_i is the source for terminal i. We labeled the flow along the solid edges as f_i, but this is an abstraction to simplify understanding of the graph. In reality, the flow must be accounted for along each direction and for each type of commodity. We now introduce notation to make our life easier: we have the multicommodity flow network $[V, E, K]$ where V is the set of vertices, E is the set of edges, and K is the set of commodities. Say we have v vertices, e edges, and k commodities. For each drawn edge in our example, we actually have $2k$ edges: both directions and each commodity can have different flows. So we can refer to the directed edge from vertex i to j for commodity q E^q_{ij}. We have a flow along each edge of $f(E^q_{ij})$ and perhaps a capacity on the total flow through an edge for all commodities together, $c(E_{ij})$. Each commodity q has one source vertex from which it originates, s_q, and one sink vertex that it leaves from, n_q. The flow for each edge is also charged to the airline at a certain rate, $Cost(f(E^q_{ij}))$. Since we are minimizing the cost of the system, we prescribe the flow of each commodity r_q and then can minimize the cost of achieving that flow.

Now we can concisely state the constraints of the system that allow it to be solved[16]:

$$\sum_{q=1}^{k} f(E^q_{ij}) \leq c(E_{ij}) \quad \forall\ (i,j) \in (1...v) \times (1...v) \tag{7.1}$$

$$\sum_{j=1}^{v} f(E^q_{ij}) - f(E^q_{ji}) = \begin{cases} r_q & : \text{ if } i = s_q \\ -r_q & : \text{ if } i = n_q \\ 0 & : \text{ otherwise} \end{cases} \qquad \forall\ (i,q) \in (1...v) \times (1...k) \tag{7.2}$$

$$Cost_{System} = \sum_{i=1}^{v} \sum_{j=1}^{v} \sum_{q=1}^{k} Cost(f(E^q_{ij})) \tag{7.3}$$

Equation (7.1) states that the total flow through an edge for all its commodities must not exceed its capacity. Equation (7.2) is simply the conservation of flow, analogous to Kirchoff's law for electronics: any flow of one commodity in a vertex must equal the flow out of the vertex (except for the source and sink vertices). Equation (7.3) formalizes the cost of the system as the sum of the edge costs required to maintain the flows in each edge.

7.5 Application of Flow/Cost Relationships

Now we return to the previous sections of the paper where we determined the throughput of various numbers of escorts and trams. We use those relationships

[16]Here we adopt the notation of Tomlin. [15]

here, multiplying by the cost per helper or tram as the case may be. Figure 18 shows the throughput for one terminal in Pittsburgh, obtained after calibrating the simulation accordingly. Likewise Figure 19 shows the flow of a system of

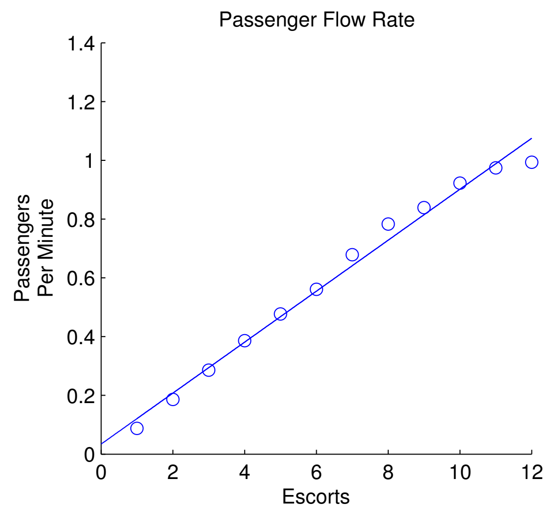

Figure 18: Flow Rate for Single Concourse

trams that go between each terminal in a cycle.

Now we can lay an edge between each vertex and assign cost functions based on what method of transport it uses. An astute reader will realize we now have quite a few linear equations and one large expression that we wish to minimize. We can apply linear programming with the capacity and conservation equations as our constraints and the $Cost_{System}$ expression as our objective function to minimize. The simplex or interior points methods will find the optimal answer since we have linear constraints on our system and our objective function is linear.

7.6 Actually Getting an Answer

So we have shown how to compute the optimal arrangement of resources to obtain certain flows between concourses. The final piece of the puzzle is determining $Cost_{miss}$ for a given flow rate between terminals. But this can be simply calculated from the average distances between gates (or concourses, depending on the level of abstraction) and applying a function representing how likely a passenger is to miss their flight based on the time it took to get to their gate (Equation 2.1). Once we can relate global flow to $Cost_{miss}$, we can solve the system. We iteratively vary total flow until $Cost_{System} - Cost_{miss}$ is minimized. The inner loop of this procedure is illustrated by Figure 20.

We use as input as much flight data as we can: either aggregate based on

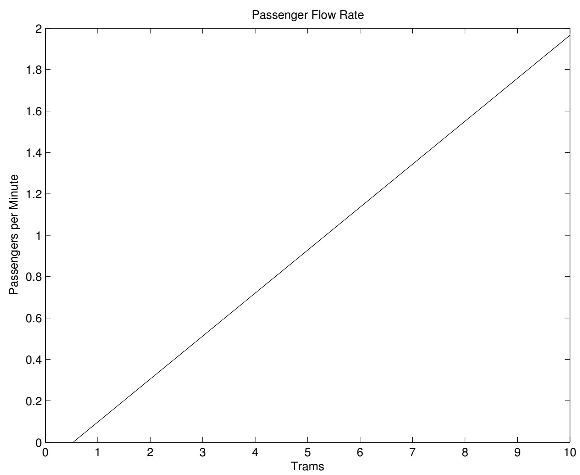

Figure 19: Flow Rate for Inter-Concourse Trams

average throughput from one terminal to another, or specific based on airline business knowledge. The flow data can be partitioned based into multiple periods of the day to determine different configurations for different worker shifts (for example, allocating more workers during busy times). However, the inter-terminal data is not immediately available to our consulting group and so we present this algorithm in purely strategy form, rather than computing answers based on rough estimates.

7.7 Flexibility of Strategy

The advantage of this strategy is its flexibility to many types of airport structures, states, and transportation means. All the transport methods we have discussed can be included in the flow model: escorts, trams, and public airport trains (trains simply have cost 0 and a fixed maximum flow since they are operated by the airport and cannot usually be varied). If flow data were obtained, this strategy could also handle new technologies like powered wheelchairs (higher flow and reduced escort cost) or people movers (constant flow and zero cost).

7.8 Strengths and Weaknesses

The algorithms developed for transporting customers developed in this paper reflect our best decisions as to a means of implementing a system capable of achieving the desired task. Howevever, even with the best ideas, problems can arise.

In our approach to this problem we initially assumed that the small effects would not have as large an effect on whatever system we decided to implement. We thereby took a more global view of the situation and attempted to attack the

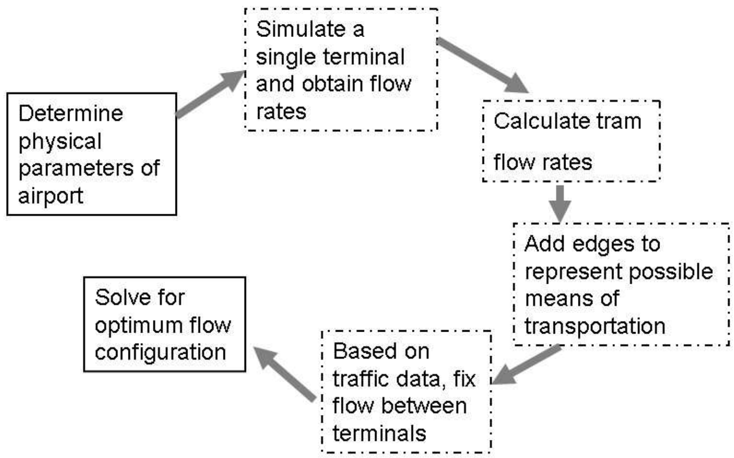

Figure 20: Strategy Overview

problem as a whole. Whether or not this assumption is valid is something that remains to be seen. Another of our model's potential flaws exists in the fact that we developed it by components. This would appear to be a sound approach, but in reality, might prove to be a weakness. The difficulties associated with a component development of a model are mainly related to difficulties dealing in the meshing of the different systems. It often occurs that the various components will operate at different rates or oscillate at different frequencies as it may. This can result in chaos and non-harmony existing between the systems. The other problem that could potentially arise from our solution to this problem is the level of abstraction utilized in developing it. Althought things may appear to work on paper, the real world can often produce drastically differing results. Thus although our system is abstract and complex and adaptable to many different circumstance, it is conceivable that a simpler and more effective model does exist.

The model that we outlined in this paper relies heavily upon abstractions that could carry some weaknesses. However, there are numerous strenghts to our model that cannot be ignored. By abstracting our work, it can be applied to numerous circumstances and can even transcend airline companies and all of the airports that they service. Our model could potentially be adapted to any circumstance where an item needs to be transfered from on location to another in the most efficient manner. Thus the potential source of weakness in our model is also the source of its greatest strenght.

Whether the model outlined in our paper would actually work in real life remains to be seen. The real world often differs greatly from that which occurs on paper. The success or failure of our algorithm can not be determined by any computer, but can only be determined by the actual implementation and testing. In this paper we can only hypothesize about the actual strengths and weaknesses of our model. The true nature of the model's properties will only become apparent with the realization of the model itself.

7.9 Extension Questions

In the coming years the "Baby Boom" generation will be entering into the ages where they will be eligible for Social Security and many other government-provided benefits [17]. In addition to putting a strain on the federal government, this changing population dynamic will also result in an increased stress on services such as hopsitals and healthcare providers. As the baby-boomers age and begin to retire they will begin to travel more. However, also as they age a large percentage of them will become impaired in some manner. Thus, this will produce an increasing stress on the airline industry to cater to aging baby-boomers still healthy enough to travel, but partially impaired. As the number of baby-boomers meeting this qualification increases, changes in the strategy developed above will have to be implemented.

There are several possible means with which the changing customer dynamic could be handled. One potential solution is to begin increasing the prices of airline tickets to account for the additional cost necessary to upgrade the current implemented system. Another potential solution is to modify the nature of airline flights to better accomodate the baby-boomers generation. This could include special flights scheduled only for senior-citizens. This would then allow the system to be specially designed for times of large flow of customers. For example, instead of having wheelchairs to pick people up at the gate, whole trams could be committed to one gate for a brief period of time. Lastly, flights could be scheduled to better reflect the travel patterns of baby-boomers. This would include direct or continuous flights from major cities to resort locations. This would negate the need for as many wheelchairs since customers would no longer need to change planes during the course of their travels. The implementation of any one of these strategies demands additional modeling before a decision and eventual implimentation could be performed. Modification of the system is essential in these circumstances as the system must adapt to reflect the changing nature of the customer base.

7.10 Conclusion and Parting Words

Through our mathematical analysis of the issue at hand, we believe that we have developed an optimal model of a solution. We feel as though our model has been developed in a manner that makes it capable of overcoming adverse situations both in the short and the long term. Four smaller models were developed as a means of moving customers throughout different sized airports. We then implemented a process of linear programming to optimizing all of these

[17] See [16]

strategies in order to develop an ideal approach for helping customers to meet their connecting flights. Although we cannot prove the success of the model, we know that it reflects our best decisions and common sense and would make an effective business strategy for Epsilon Airlines as it strives to improve its efficiency in a competitive airline industry.

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