Optical system for image rotation and magnification

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Flexibility is desirable in optical systems for transfer, processing, and display of pictorial information. Two important operations in this respect are image rotation and magnification. Simple means for their realization by one single system are discussed. Experimental verifications using diffractive and refractive elements are described.

Two optical transformations desirable in information processing situations are image rotation and scaling. They are both of predominant importance in manipulation of pictorial data, especially in matching the size and orientation of the light distributions to the parameters of systems and components. There exist numerous configurations to achieve these operations, e.g., prisms, mirrors, and lens assemblies for image rotation and lens combinations for zoom purposes. Here we want to present some thoughts on how these two functions can be handled by the same components in a system.

Passive reflective, refractive, and/or diffractive elements are commonly incorporated to realize optical systems capable of performing rotation and scaling operations. The functional influence of the device is then altered by changing the relative orientation and/or location of its members. Some practical considerations are on-axis operation, polarization insensitivity, and a minimum number of movable parts. In these respects refractive lens elements seem preferable. In this paper we will present some ideas toward this end. In order to explain the underlying concepts we first introduce and treat some combinations of diffractive elements before presenting an equivalent refractive system.

I. SPACE-VARIANT OPTICAL SYSTEM

Our starting point here is the space-variant system[1,2] of Fig. 1. The input plane O located in the front focal plane of the lens L is coherently illuminated with collimated light. In this plane the object $a(\mathbf{r})$, $\mathbf{r} = (x,y)$, together with a space-variant optical element $P(\mathbf{r})$ are introduced. The processed object appears in the display plane D located in the back focal plane of lens L. Thus, the optical system consists of the two parts P and L. To describe the image-forming properties we consider the light distribution in plane D.

Using the notation

$$A(\mathbf{r}) = a(\mathbf{r})P(\mathbf{r}) = a(\mathbf{r})\exp\{i\phi(\mathbf{r})\}, \tag{1}$$

we obtain in plane D as an amplitude distribution the Fourier transform

$$\begin{aligned}\tilde{A}(\mathbf{r}') &= \iint_{-\infty}^{+\infty} A(\mathbf{r})\exp\{-i(k/f_L)\mathbf{r}\mathbf{r}'\}\,d\mathbf{r} \\ &= \iint_{-\infty}^{+\infty} a(\mathbf{r})\exp\{i\psi(\mathbf{r})\}\,d\mathbf{r},\end{aligned} \tag{2}$$

where $k = 2\pi/\lambda$ and f_L is the focal length of L.

If the object distribution $a(\mathbf{r})$ changes only little during one oscillation of $\exp\{i\psi(\mathbf{r})\}$, then the Fourier integral can be evaluated using the method of stationary phase.[2] Then, $|\tilde{A}(\mathbf{r}')|^2$ is proportional to $|a(\mathbf{r})|^2$ and the geometrical transformation between both planes O and D is given by

$$\mathbf{r}' = (f_L/k)\nabla_{\mathbf{r}}\phi(\mathbf{r}), \tag{3}$$

where $\nabla_{\mathbf{r}}$ denotes the gradient in the input plane O.

II. PHASE ELEMENT SPECIFICATION

We required in our experiments that the phase elements fulfil two constraints:

(a) P is given by the superposition of M independent optical elements p_m, $m = 1, \ldots, M$, which results in

$$P(\mathbf{r}) = \prod_{m=1}^{M} p_m(\mathbf{r}). \tag{4}$$

(b) Optical elements are used that are quasiperiodic in $\mathbf{r}$, e.g., Fresnel zone plates described by

$$p_m(\mathbf{r}) = p[\phi_m(\mathbf{r})] = p[\phi_m(\mathbf{r}) + n2\pi] \qquad n = 0, \pm 1, \pm 2, \ldots. \tag{5}$$

Then, $p_m(\mathbf{r})$ can be represented by a Fourier series

$$p_m(\mathbf{r}) = \sum_{n_m=-\infty}^{+\infty} C_{n_m}\exp\{in_m\phi_m(\mathbf{r})\}, \tag{6}$$

where the coefficients C are defined as

$$C_{n_m} = \frac{1}{2\pi}\int_0^{2\pi} p_m(\mathbf{r})\exp\{-in_m\phi_m(\mathbf{r})\}\,d\phi_m. \tag{7}$$

Inserting Eq. (6) into Eq. (4) allows us to rewrite P as a sum of new "artificial" elements p_κ

$$P(\mathbf{r}) = \sum_{\kappa=0}^{\infty} B^\kappa \exp\{i\phi_\kappa\} = \sum_{\kappa=0}^{\infty} p_\kappa. \tag{8}$$

The amplitude and phase values of p_κ are given by

$$B^\kappa = \prod_{m=1}^{M} C_{n_{m,\kappa}} \tag{9}$$

and

$$\phi_\kappa = \sum_{m=1}^{M} n_{m,\kappa}\phi_m, \tag{10}$$

where $[n_{1,\kappa}, \ldots, n_{M,\kappa}]$ denote all possible combinations of the M integer numbers

$$[n_1, \ldots, n_M].$$

The imaging conditions (2) and (3) can now be applied independently to each "phase element" p_κ. Thus, different geometrical transformations

$$\mathbf{r}'_\kappa = (f_L/k)\nabla_{\mathbf{r}}\phi_\kappa(\mathbf{r}) \tag{11}$$

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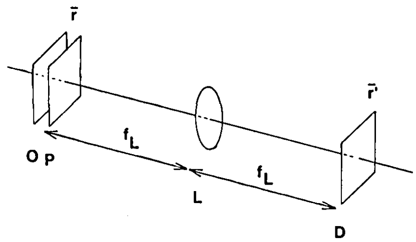

FIG. 1. System for performing geometrical transformations between planes O and D. Optical elements: L lens of focal length f_L and P combination of rotatable components.

can be performed simultaneously, as will be shown in the following.

III. ARRANGEMENT OF FRESNEL ZONE PLATES

A diffractive lens element is the Fresnel zone plate. Placing an off-axis portion of a zone plate in O of the setup in Fig. 1 results in several areas[2] of interest in plane D. These areas correspond to the different diffraction orders; a central spot forms the zeroth order and extended areas the first and higher orders. A one-to-one correspondence exists between these areas and the geometrical extension of the zone plate in O. A rotation of the zone plate does not influence the orientation of the pictorial information displayed in D.

If a second off-axis Fresnel zone plate is superposed onto the previous one in plane O, two sets of diffraction figures appear in plane D. In this case cross combination orders are also formed. The light corresponding to these locations has been diffracted by the first as well as the second zone plate. When the portions of the two zone plates used are identical, all the four areas corresponding to first (+1 and −1) orders are of the same size. In the locations corresponding to combination orders of the same kind, i.e., $(+n,+n)$ or $(-n,-n)$, areas appear with n times lateral magnification just as in the usual $\pm n$th order and in the locations belonging to combinations of the same orders but of opposite kind, i.e., $(+n,-n)$ or $(-n,+n)$, spots of light are formed just as on axis.

Nothing interesting toward our present aim seems to occur by a relative rotation of the two-zone plates.

IV. COMBINATION OF ONE-DIMENSIONAL ZONE PLATES

The situation is different with one-dimensional zone plates than with two-dimensional ones.[2] Here all cross combination diffraction figures are extended and the areas corresponding to orders of opposite kind are now, as we shall see, those with the useful properties.

The amplitude transmission $p(x)$ of a cosinusoidal one-dimensional Fresnel zone plate is

$$p(x) = (1/2) + (1/2)\cos\phi(x) \tag{12}$$

where $\phi = (k/2f_z)x^2$, $f_z = d^2/\lambda$ is the focal length, and d the half-width of the first zone. Replacing $\cos\phi$ by $(1/2)[\exp(i\phi) + \exp(-i\phi)]$ shows that light is diffracted into the +1st, 0th, and −1st order.

The phase term in Eq. (12) is now modified if the zone plate is rotated an angle φ around the optical axis and shifted the distance b from the origin $\mathbf{r} = \mathbf{0}$. Then, the coordinate x is replaced by

$$\xi = b + x\cos\varphi + y\sin\varphi, \tag{13}$$

where ξ is the coordinate in the rotated system. Inserting Eq. (13) into Eq. (12) leads to

$$\phi = (k/2f_z)(x^2\cos^2\varphi + y^2\sin^2\varphi + b^2 + 2xb\cos\varphi + 2yb\sin\varphi + 2xy\cos\varphi\sin\varphi). \tag{14}$$

Now we return to the formalism introduced in Sec. II, applied to the case of M superposed zone plates of different orientations φ_m, $m = 1 \ldots M$. Inserting the phase term ϕ_m according to Eq. (14) into Eq. (10) results in

$$\phi_\kappa = (k/2f_z)[x^2(\cos^2\varphi)_\kappa + y^2(\sin^2\varphi)_\kappa + (b^2)_\kappa + 2 \times (b\cos\varphi)_\kappa + 2y(b\sin\varphi)_\kappa + 2xy(\cos\varphi\sin\varphi)_\kappa], \tag{15}$$

where the lower index κ indicates the weighted summation as defined in Eq. (10).

The geometrical transformation between both planes O and D can now be performed according to Eq. (11)

$$\mathbf{r}'_\kappa = (f_L/f_z)(\mathbf{b}_\kappa + D^\kappa\,\mathbf{r}), \tag{16}$$

where the shift vector $\mathbf{b}_\kappa$ is given by

$$\mathbf{b}_\kappa = b\begin{pmatrix}(\cos\varphi)_\kappa\\ (\sin\varphi)_\kappa\end{pmatrix} \tag{17}$$

and the transformation matrix D^κ is defined as

$$D^\kappa = \begin{Bmatrix}(\cos^2\varphi)_\kappa & (\cos\varphi\sin\varphi)_\kappa\\ (\cos\varphi\sin\varphi)_\kappa & (\sin^2\varphi)_\kappa\end{Bmatrix}. \tag{18}$$

Equation (16) reveals that the origin $\mathbf{r} = \mathbf{0}$ of the input plane is displayed as a set of diffraction spots located at $\mathbf{r}' = \mathbf{b}_\kappa$. This pattern reflects the arrangement of zone plates in the input plane. Labeling each spot by $[n_{1,\kappa} \ldots n_{M,\kappa}]$ means that light is diffracted by the first zone plane in the $n_{1,\kappa}$th order, etc.

The geometry around each spot is determined by the matrix D^κ. With cosinusoidal zone plates only the orders $n = -1, 0, +1$ occur. Combining two plates, oriented at angles φ_1, and φ_2, yields nine diffraction spots $[0,0], \ldots, [-1,-1]$. Four classes of D^κ's occur in this case

$$D^{[0,0]},\ D^{[1,0]},\ D^{[1,1]},\ \text{and}\ D^{[1,-1]}.$$

The angles $\Omega = (\varphi_1 + \varphi_2)/2$ and $\omega = (\varphi_2 - \varphi_1)/2$ are introduced denoting the absolute and relative rotation of both zone plates (Fig. 2). Only the matrices of mixed first diffraction orders are of interest. The $[+1,-1]$ matrix can be written as

$$D^{[1,-1]} = -(f_L/f_z)\sin 2\omega\begin{Bmatrix}-\sin 2\Omega & \cos 2\Omega\\ \cos 2\Omega & \sin 2\Omega\end{Bmatrix}. \tag{19}$$

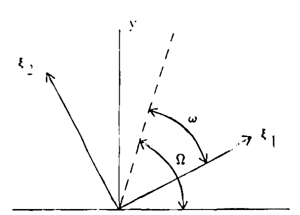

FIG. 2. Notations used to indicate orientations of optical elements in plane O of Fig. 1.

Thus, two factors determine the transformation. The scale factor depends on f_L/f_z and on the relative orientation 2ω of both zone plates and the rotation matrix on 2Ω. The [+1,+1] matrix is given by

$$D^{[1,1]} = \begin{Bmatrix} 2(\cos^2\Omega \cos^2\omega + \sin^2\Omega \sin^2\omega) & \sin 2\Omega \cos 2\omega \\ \sin 2\Omega \cos 2\omega & 2(\cos^2\Omega \sin^2\omega + \sin^2\Omega \cos^2\omega) \end{Bmatrix}. \tag{20}$$

This expression does not include a rotation matrix. Furthermore, the geometry obtained is distorted for $2\omega \neq \pi/2$; i.e., if both zone plates are not orthogonally oriented. For 3 combined cosinusoidal zone plates, 27 diffraction spots appear in plane D. These can be indexed by [0,0,0],..., [−1,−1,−1]. The arrangement of spots is shown in Fig. 3(a) for the case that the angle between two zone plates is 120°. If the third zone plate is reversed, then the arrangement of spots of Fig. 3(b) is obtained.

An analysis of all 27 transformation matrices D shows that only six matrices, represented by $D^{[0,0,0]}$, $D^{[1,0,0]}$, $D^{[1,1,0]}$, $D^{[1,-1,0]}$, $D^{[1,1,1]}$, and $D^{[1,1,-1]}$ are of interest. Furthermore, only in the neighborhood of the [1,−1,0] diffraction spot and its permutations is geometry preserved; i.e., $D^{[1,-1,0]}$ can be written as shown in Eq. (19).

V. EXPERIMENTS

The operation of the suggested method has been experimentally tested and verified. The optical arrangement of Fig. 1 was realized using collimated HeNe-laser light and a simple lens L of focal length $f_L = 10$ cm. Superposed portions of photographically produced binary Fresnel zone plates constituted the space-variant phase elements P. Off-axis portions of the zone plates were chosen so that the extension of the first and second diffraction orders were not overlapping, i.e., a spatial frequency range of 1 to 2 was used. Recordings were made on photographic material placed in the display plane D. A photographic transparency of 0.5 cm lateral extension served as the object. It was imaged with a telecentric system onto the zone plate assembly.

Figure 4 shows the case with two superposed two-dimensional zone plates. The frequencies of these plates were too low to allow processing of meaningful structured object information. Here only the square shape of the input plane appears. The different sizes of the cross combination diffraction orders are evident. In Fig. 4(a) a relative orientation of $\pi/2$ between the central lines in the two zone plates was adjusted and in Fig. 4(b) one of the plates was slightly rotated. The areas belonging to the combination orders remain in the same orientation.

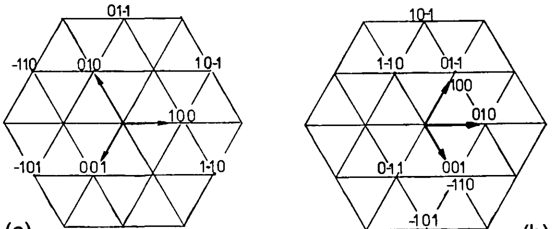

FIG. 3. Relative locations of frequency-spectra terms arising from a combination of three one-dimensional zone plates. Arrows indicate orientations of the zone plates.

Experimental results using two one-dimensional zone plates are illustrated in Figs. 5 and 6. In Fig. 5 the zone plates were crossed. The four recordings represent different orientations of the zone plate assembly relative to the optical setup. Here it is evident that only one specific pair of combination orders displayed is rotated and that the angle of rotation is twice that of the assembly. The results obtained when the zone plates were rotated in opposite sense relative to each other are shown in Fig. 6. Even in this case, only one pair of combination orders (plus and minus) is usable to obtain a scale change. The displayed configuration in the other pair is distorted when the zone plate setting is deviating from the crossed one.

Experiments were also performed using phase element assemblies consisting of three superposed one-dimensional Fresnel zone plates. Figures 7(a) and 7(b) show results corresponding to the arrangements illustrated in Fig. 3. The undistorted pictorial displays appear as expected in those combination orders that were formed by interaction between two orders of opposite kind. In each of these figures six displayed object distributions occur. Their rotations are determined by the orientations of the zone plate pairs in question. Because of the particular symmetrical angular positions of the plates in these cases, all six image configurations are of the same size.

VI. SYSTEM VERSION USING CYLINDER LENSES

The idea pursued so far to realize a system able to produce an image rotation as well as a magnification has been to combine a plus and a minus order from a set of superposed one-dimensional diffractive elements. Equation (12) clearly indicates that the phase variations corresponding to these orders can be achieved using a positive and a negative cylinder lens. This suggests the possibility of using a pair of cylinder lenses as an element assembly possessing the desired variable-phase properties in our optical system. The focal lengths of the cylinder lenses shall be equal but of opposite sign.

Such a cylinder lens combination has one peculiar property: it is equivalent to a system of crossed cylinder lenses, (positive and negative) the focal lengths of which have the same amount and vary with the angle 2ω. An attractive advantage using such a refractive element combination is its on-axis mode of operation.

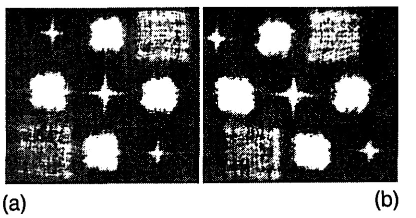

FIG. 4. Recordings of Fraunhofer diffraction configurations obtained from superposed two-dimensional Fresnel zone plates.

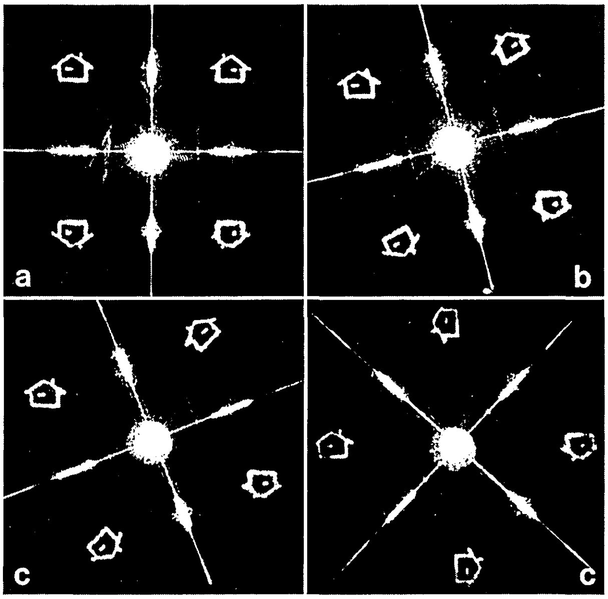

FIG. 5. Results using two crossed one-dimensional Fresnel zone plates as element P of Fig. 1. Rotations introduced: a, 0°; b, 30°; c, 45°; and d, 90°.

The total system version using cylinder lenses is illustrated in Fig. 8. The maximum magnification f_L/f_z occurs for crossed lenses as shown in Fig. 8(a). The focal length of the cylinder lenses is f_z. Zoom and rotation effects are introduced by turning the cylinder lenses in opposite directions and by rotating the whole cylinder lens assembly as indicated in Figs. 8(b) and 8(c).

Experimental verifications are shown in Fig. 9. A pair of cylinder lenses with $f_z = 10$ and -10 cm were mounted in ball bearings to allow individual as well as common rotation. In

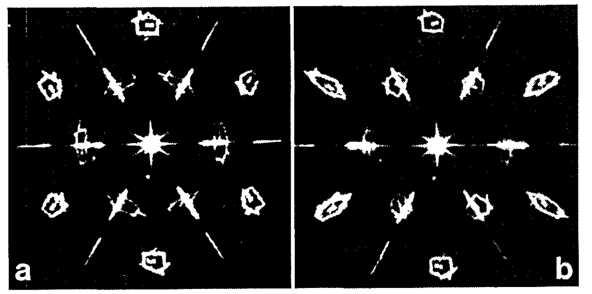

FIG. 7. Results using three superposed one-dimensional Fresnel zone plates as element P of Fig. 1 with orientations as indicated in Fig. 3.

Figs. 9(b), 9(c), and 9(d) multiple exposures were made to indicate how the image changes with variations in the orientations of the cylinder lenses.

VII. COMMENTS

One purpose of this work is to discuss ideas about composing optical systems so that several of its properties may be altered continuously. The thought is developed of how the combined function of a pair of one-dimensional diffractive elements can be incorporated for manipulation of pictorial information. In particular, it is shown how image rotation and magnification can be influenced separately. The adjustments are performed by rotation (relative and absolute) of the diffractive components.

The frequency spectrum of a one-dimensional Fresnel zone plate consists of sets of extended lines. The superposition of two plates results in lines inclined at corresponding angles. We have been using the combination orders which can be geometrically constructed by considering a specific order from each plate. The lines constitute the projections of the diffraction area in corresponding directions. It is evident that

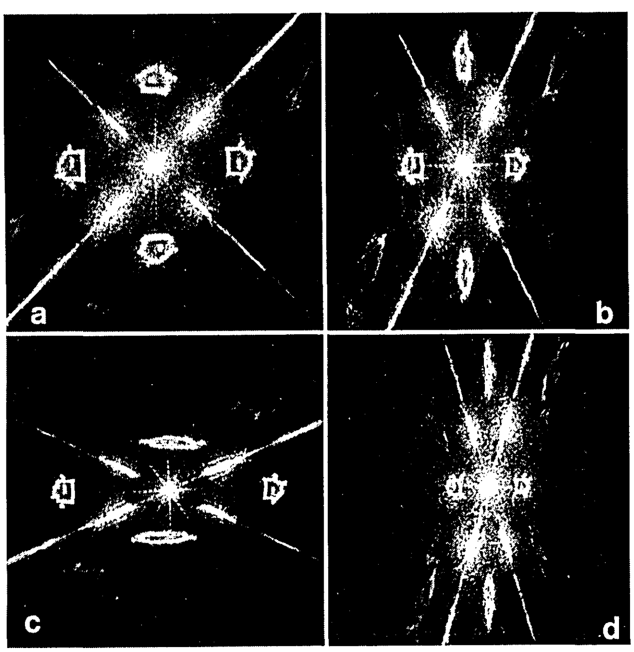

FIG. 6. Results using two one-dimensional Fresnel zone plates turned in opposite directions as element P of Fig. 1. Angle between the plates: a, 90°; b, 45°; c, 135°; and d, 30°.

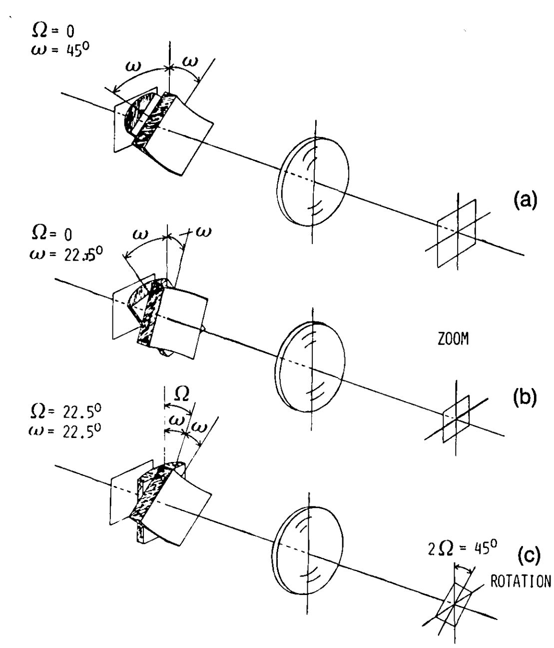

FIG. 8. System version using refractive elements. Indication of operation with different settings of the cylinder lens combination.

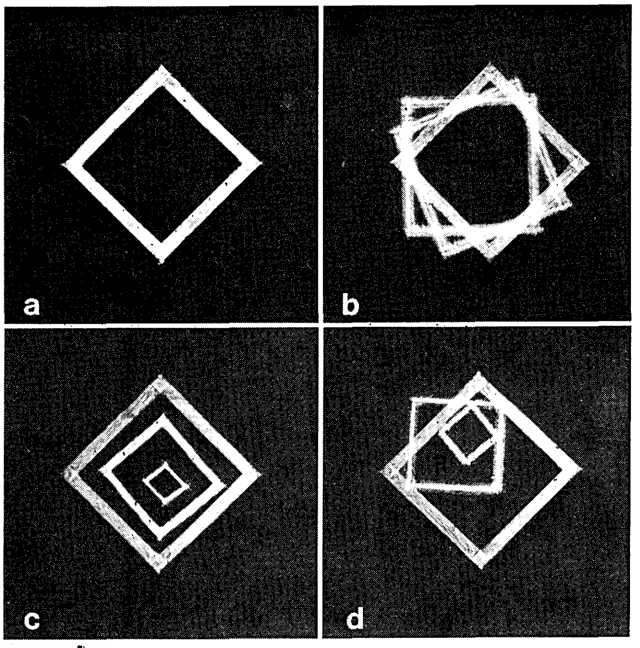

FIG. 9. Results using two cylinder lenses as element P of Fig. 1; a illustrates image configuration, b multiple exposures for different Ω, c for different ω, and d for different Ω and ω.

the area belonging to a particular line pair is maximum for crossed zone plates and decreases to zero for parallel or antiparallel plates. In our specific system only areas corresponding to a combination of orders of opposite sign result in undistorted displays. This is understandable if we consider the coresponding refractive systems. The results obtained equivalent to a combination of two plus to two minus first orders can be achieved by using a pair of positive or negative cylinder lenses. These systems are anamorphic for all angles 2ω except 0 and $\pi/2$. The situation is different for a combination of a positive and a negative cylinder lens. As indicated in Sec. VI such a system functions as a set of crossed lenses with focal lengths dependent on the relative orientations.

Inherent in our solution of Eq. (1) is the requirement that the frequencies of the object $a(\mathbf{r})$ have to be considerably lower than those of exp $\{i\psi(\mathbf{r})\}$. This restriction is also understandable from the set-up arrangement of Fig. 1. The distances are adjusted so that a power spectrum of the combined distribution in O occurs in plane D. However, the phase element P belongs to the optical system. For $f_z = f_L$ the Fourier transform of the object distribution is located close behind lens L. Another Fourier transform relationship is then necessary between this location and the display plane to form a perfect image in D. This determines the allowable aperture and the corresponding restricted frequency content of the object. In this respect, as well as concerning other practical factors, the refractive-system version possesses clear advantages. In other situations as those described here, where more complicated spatial phase variations are required, diffractive elements may be a possible realization.

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[2]O. Bryngdahl, "Geometrical transformations in optics," J. Opt. Soc. Am. **64**, 1092–1099 (1974).

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