

Preference for Flexibility and the Pricing of Assets

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Abstract

We consider agents who have a dynamic preference for flexibility, as in Krishna and Sadowski [2012] (KS). Such agents are uncertain about their future utilities. We first consider a version of the Lucas tree economy, where the representative agent behaves as in KS and is uncertain about his degree of future risk aversion. We show that in such an economy, the representative agent's uncertainty about his future risk aversion drives price volatility (in the sense of second order stochastic dominance). We then consider the Lucas tree economy and add an additional market stage, where the productive asset cannot be traded. We show that a representative agent with a higher degree of uncertainty about his future risk aversion implies less investment and more price volatility in this economy.

1. Introduction

This is a companion note to Krishna and Sadowski [2012] (henceforth KS). KS analyze choice on the recursive domain of Infinite Horizon Consumption Problems (IHCPs) first analyzed by Gul and Pesendorfer [2004]. They provide representations of preference for flexibility that are solutions to Bellman equations. In particular, they consider the case where there is no preference for flexibility with respect to continuation problems, but only with respect to consumption alternatives. Let $x \in Z$ be an IHCP, that is, a menu of lotteries p over present consumption $k \in K$ and continuation problems $z \in Z$. For this case, KS axiomatize a representation of

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Preference for Flexibility with Constant Beliefs (PFC),

$$V(x) = \int_{\mathcal{U}} \max_{p \in x} \left[\int_{K \times Z} [u(k) + \delta V(z)] dp(k, z) \right] d\mu(u)$$

where $\delta \in (0, 1)$ is the discount factor, \mathcal{U} is the space of consumption utilities (ie, the space of vN-M utilities over the set of consumption prizes K that are identified up to additive constant), and μ is a probability measure on \mathcal{U} . The measure μ is unique, up to scaling (see definition 2).

Following KS, we say that one DM has *greater preference for flexibility* than another DM, if the first prefers a nondegenerate menu over a singleton whenever the second does. KS characterize this property in terms of DM's beliefs in the context of the PFC representation: one DM has a greater preference for flexibility than another if, and only if, the corresponding beliefs are dominated in the increasing convex order. If the relevant part of the space of consumption utilities is one dimensional, this condition on beliefs amounts to second order stochastic dominance.

In this note, we first consider a version of the Lucas tree economy, where the representative agent behaves as in their model and is uncertain about his degree of future risk aversion. We show that in such an economy, the representative agent's uncertainty about his future risk aversion drives price volatility (in the sense of second order stochastic dominance).

In such an economy, if ownership of the productive asset constrains liquidity, we should expect correlation between underinvestment (a lower average price for the productive asset) and higher price volatility (in the sense of second order stochastic dominance), as both are driven by the representative agent's degree of uncertainty about future risk aversion. This leads us to reconsider the Lucas tree economy and add an additional market stage, where the productive asset cannot be traded. We affirm the intuition that a representative agent with a higher degree of uncertainty about his future risk aversion implies less investment and more price volatility in the economy.

2. PFC representation

Let K be a finite set of prizes with typical member k . We follow GP in defining an infinite horizon consumption problems (IHCP) as a collection of lotteries that yield a consumption prize in the present period and a new infinite horizon problem starting in the next period. Let Z be the collection of all IHCPs.¹ GP show that Z is a compact metric space, and that each $z \in Z$ can be identified with a compact set of probability measures over $K \times Z$. For the compact metric space $K \times Z$, let $\mathcal{P}(K \times Z)$ denote the space of probability measures endowed with the

(1) See GP for the recursive construction of Z .

topology of weak convergence, so that $\mathcal{P}(K \times Z)$ is compact and metrizable. Let $\mathcal{F}(\mathcal{P}(K \times Z))$ denote the space of closed subsets of $\mathcal{P}(K \times Z)$, endowed with the Hausdorff metric, such that $\mathcal{F}(\mathcal{P}(K \times Z))$ is a compact metric space. It can be shown that Z is linearly homeomorphic to $\mathcal{F}(\mathcal{P}(K \times Z))$. We shall denote this linear homeomorphism as $Z \simeq \mathcal{F}(\mathcal{P}(K \times Z))$. Typical elements $x, y, z \in Z$ are interpreted as *menus* of lotteries over consumption and continuation problems.

KS provide representations of choice over Z with the understanding that DM will choose from the IHCP he faces in every subsequent period. Take p, q to be typical lotteries in $\mathcal{P}(K \times Z)$. We will also consider the space of menus of consumption lotteries, $\mathcal{F}(\mathcal{P}(K))$, with typical members being a, b . By the recursive nature of Z , continuation problems are members of Z . Let A, B denote typical elements of the collection of menus of continuation lotteries, $\mathcal{F}(\mathcal{P}(Z))$. When there is no risk of confusion, we identify prizes and continuation problems with degenerate lotteries and lotteries with singleton menus.

As in KS, let $\mathcal{U} := \{u \in \mathbb{R}^K : \sum u_i = 0\}$ be the set of all vN-M utility functions over instantaneous consumption (ie, over K) that are identified up to a constant. The subjective state space relevant for the PFC representation is \mathcal{U} . To ensure that expected consumption utility under a measure μ is well defined, the measure μ must be *nice*, ie, must satisfy $\int_{\mathcal{U}} \mu u_k \, d\mu(u)$ is finite for each $k \in K$, which is to say the expected utility from each prize is finite.

Subjective states $u \in \mathcal{U}$ are naturally interpreted as consumption utilities, and the two terms are treated as synonyms. Similarly, in what follows, all probability measures are interpreted as subjective beliefs, and the two terms are used interchangeably. For $p \in \mathcal{P}(K \times Z)$, let p_k and p_z denote the marginal distributions on K and Z respectively.

Definition 1. Let \mathcal{U} be defined as above, μ a nice probability measure on (the Borel sigma-algebra of) \mathcal{U} , and $\delta \in (0, 1)$. We say that \succsim has a representation of *Preference for Flexibility with Constant Beliefs* (PFC), (μ, δ) , if there exists a continuous function $V : Z \rightarrow \mathbb{R}$, linear on Z , that satisfies

$$(2.1) \quad V(x) = \int_{\mathcal{U}} \max_{p \in x} [u(p_k) + \delta V(p_z)] \, d\mu(u)$$

and represents \succsim .

In the presentation above, $u(p_k) = \sum_{k' \in K} p_k(k')u(k')$, and $V(p_z)$ is the extension of V to $\mathcal{P}(Z)$ by linearity (and continuity), ie $V(p_z) = \int_Z V(z') \, dp_z(z')$. KS provide axioms on a preference relation, \succsim , that are equivalent to the existence of a PFC representation, (μ, δ) , of \succsim . Moreover, they establish that δ is unique and μ is unique up to scaling (Theorem 1 in KS).

Definition 2. Two probability measures μ and μ' on \mathcal{U} are *identical up to scaling*, if there is $\lambda > 0$ such that $\mu(E) = \mu'(\lambda E)$ for all measurable $E \subset \mathcal{U}$, where $\lambda E := \{\lambda u : u \in E\}$.

Preference for flexibility is the preference for non degenerate menus over singletons. Intuitively, one DM has more preference for flexibility than another if she has a stronger preference for menus over singletons. To make this precise requires us to consider the restricted domain $L \subset Z$ of *Infinite Horizon Consumption Streams (IHCSs)*. This domain consists of lotteries that deliver consumption for the present period and an IHCS for the next period. It is easy to show that L is a closed and convex subset of Z . In a manner analogous to the characterisation of risk aversion where lotteries are compared to certain amounts of money, characterizing preference for flexibility requires a comparison between IHCPs and IHCSs. This comparison is meaningful only if the preference restricted to L is non-trivial, ie, there exist $\ell, \ell' \in L$ such that $\ell \succ \ell'$. We refer to this property of \succsim as *Consumption non-triviality*. If \succsim has a PFC representation, KS show that \succsim satisfies Consumption non-triviality if, and only if, $\mu u \neq \mathbf{0}$.

Definition 3. \succsim^* has a *greater preference for flexibility* than \succsim if

$$x \succsim \ell \text{ implies } x \succsim^* \ell$$

for all $\ell \in L$ and $x \in Z$.

Note that the comparison in the definition requires that \succsim and \succsim^* rank IHCSs the same; that is, $\ell \succsim \ell'$ if, and only if, $\ell \succsim^* \ell'$.

Definition 4 (Dilation). Let $Q(u, D)$ be a Markov kernel from \mathcal{U} to itself. Then $Q(u, D)$ is a *dilation* if it is expectation preserving, ie, for each $u \in \mathcal{U}$, $\int_{\mathcal{U}} u' Q(u, du') = u$. If μ and μ^* are probability measures on \mathcal{U} , then μ^* is a *dilation* of μ if there exists a dilation Q , such that $\mu^* = Q\mu$, ie, $\mu^*(du') := \int Q(u, du') \mu(du)$.

The PFC representation (μ, δ) only identifies the measure μ up to scaling. In order to facilitate a comparison of measures, we shall say that a PFC representation (μ, δ) is *canonical* if $\|\mu u\|_2 = 1$. Obviously, \succsim admits a canonical PFC representation if, and only if, $\mu u \neq \mathbf{0}$ if, and only if, \succsim satisfies Consumption non-triviality.

THEOREM 1 (Theorem 4 in KS). *Let \succsim and \succsim^* have canonical PFC representations (μ, δ) and (μ^*, δ^*) , respectively. Then, the following are equivalent:*

- (a) \succsim^* has a greater preference for flexibility than \succsim .
- (b) $\delta = \delta^*$ and μ^* is a dilation of μ .

3. A Lucas Tree Economy

Consider a discretized version of the Lucas tree after Lucas [1978]. There is an economy with a representative agent and one productive asset. The asset produces $\omega \geq 0$ units of perishable output, or dividends, in each period. For simplicity, we assume that output is distributed identically and independently over time, according to the distribution $F(\omega)$ with finite support. In period 0 there is no production and no trade. Consider an agent whose period 0 preferences have a PFC representation of choice over IHCPs, where the support of F is in K .

The agent has $z \in [0, 1]$ shares in the asset, which gives him a proportional right to the output. Specifically, with probability z he gets all the output, and with complementary probability, he gets none of the output. Let \mathcal{U}^\dagger be the set of states he considers possible, ie, states in the support of μ , where we normalize $u(0) = 0$ for all $u \in \mathcal{U}^\dagger$.

There is a market where the agent can trade the probability q of getting all of the output and shares of the right to future output. The price of a unit of q is normalized to 1 in each state (ω, u) , while the price of a share is $p(\omega, u)$. Therefore, the agent's value function, when he owns z shares in the asset is

$$v(z, \omega, u) = \max_{q, x} \left[u([q; \omega]) + \delta \iint v(x, \omega', u') dF(\omega') d\mu(u') \right]$$

subject to

$$q + p(\omega, u)x \leq z + p(\omega, u)z$$

where $[q; \omega]$ is the lottery that gives ω with probability q and 0 with probability $1 - q$. Then, $u([q; \omega]) = qu(\omega) + (1 - q)u(0)$.

By following the arguments in Lucas, we can show that for each continuous $p(\omega, u)$, there exists a unique continuous, bounded, nonnegative function $v(z, \omega, u)$ that satisfies the Bellman equation above, and which is concave in z .

We know that in equilibrium, we must have $q = z + p(\omega, u)z - p(\omega, u)x$, so that $u([q; \omega]) = u(\omega)[z + p(\omega, u)z - p(\omega, u)x]$. Still following Lucas, we can show that the pricing function $p(\omega, u)$ takes the form

$$\begin{aligned} p(\omega, u)u(\omega) &= \frac{\delta}{1 - \delta} \iint u(\omega') dF(\omega') d\mu(u') \\ &= \frac{\delta}{1 - \delta} \mathbf{E}[u; F, \mu] =: \Lambda(F, \mu). \end{aligned}$$

Note that one can just as easily normalize the price of a share to be 1 in each state (ω, u) , in which case the price of a unit of q (the probability for immediate consumption) becomes $\psi(\omega, u) = 1/p(\omega, u)$. In what follows, it is more natural to work with the price $\psi(\omega, u)$. The solution to the pricing equation can thus be rewritten as

$$\psi(\omega, u) = \frac{u(\omega)}{\Lambda}$$

Suppose that there are only three levels of output, 0, 1/2 and 1. We are now in a position to relate the distribution of prices to the distribution of utilities. Consider two exchange economies, A and B , with representative agents A and B respectively. We assume that both agents have period 0 preferences with a PFC representation based on state spaces \mathcal{U}_i^\dagger , where $u(0) = 0 \leq u(1/2) \leq u(1) = 1$ for all $u \in \mathcal{U}_i^\dagger$, $i = A, B$. Thus, agents are uncertain about their risk aversion, which is captured by $u(1/2)$. We also assume that \succsim^A and \succsim^B agree on the intertemporal tradeoff for getting 1 instead of 0, which implies $\delta_A = \delta_B$.

Thus, for $\omega \in \{0, 1/2, 1\}$, prices are given by

$$\begin{aligned}\psi_i(0, u) &= 0 \\ \psi_i(1, u) &= \frac{1}{\Lambda_i} \\ \psi_i(1/2, u) &= \frac{u(1/2)}{\Lambda_i}\end{aligned}$$

where $u(1/2) \in [0, 1]$. In both economies the price of a unit (in probability) of consumption is constant across states if output is either 0 or 1. Let $\psi_i(0)$ and $\psi_i(1)$ denote these prices. Since utilities are stochastic, we can say something about the distribution of prices in the two economies for the case where output is $\omega = 1/2$. We let $H_i(\lambda) = \mathbf{P}(\psi(1/2, u) \leq \lambda)$ denote this distribution in economy i .

Proposition 5. In the two economies above, (a) and (b) are equivalent and imply (c).

- (a) Agent B has a greater preference for flexibility than agent A .
- (b) H_A second order stochastically dominates H_B .
- (c) $\psi_A(1) = \psi_B(1)$.

Proof. (a) implies (c): We must establish $\Lambda_A = \Lambda_B = \Lambda$. Let f_j be the probability that output is $j \in \{0, 1/2, 1\}$. Then we require that $\mathbf{E}[u; F, \mu_i] = \int [0 \cdot f_0 + u(1/2) \cdot f_{1/2} + 1 \cdot f_1] d\mu_i(u) = f_1 + f_{1/2} \int u(1/2) d\mu_i(u)$ is independent of i , which holds if and only if $\int u(1/2) d\mu_A(u) = \int u(1/2) d\mu_B(u)$ if, and only if, singletons are ranked the same by both agents, which follows from (i).

(a) iff (b): First note that

$$\mathbf{E}[\psi_i(1/2, u); F, \mu_i] = \frac{\int u(1/2) d\mu_i(u)}{f_1 + f_{1/2} \int u(1/2) d\mu_i(u)}$$

Hence, $\mathbf{E}[\psi_A(1/2, u); F, \mu_A] = \mathbf{E}[\psi_B(1/2, u); F, \mu_B]$ if, and only if, $\int u(1/2) d\mu_A(u) = \int u(1/2) d\mu_B(u)$ if, and only if, $\Lambda_A = \Lambda_B = \Lambda$. It is then immediate to see that H_A second order stochastically dominates H_B if, and only if, μ_A second order stochastically dominates μ_B . Applying Theorem 1 to the one dimensional case implies (a) iff (b), which concludes the proof. \square

The proposition says that in the context of our asset pricing model, where preference for flexibility stems from uncertainty about future risk aversion, a greater preference for flexibility corresponds to higher price volatility.

Now consider a setting where investment in the productive asset reduces liquidity. Since preference for flexibility manifests itself as preference for liquidity, preference for flexibility is associated with a tendency to underinvest in the productive asset. Proposition 5 suggests that this tendency to underinvest should correlate with price volatility. In order to capture this intuition in a simple asset pricing model like our Lucas tree economy, there must be periods where the productive asset, the tree, is not tradeable. We now consider this case.

4. A Lucas Tree Economy with Investment

Intuitively, preference for flexibility is associated with a tendency to invest less in the productive asset, as such investment reduces liquidity. In order to capture this intuition in a simple asset pricing model like a Lucas tree economy, there must be periods where the productive asset, the tree, is not tradeable. Consider the following environment: Odd periods ($t = 1, 3, 5, \dots$) are mornings, and even periods ($t = 2, 4, 6, \dots$) are evenings.

The tree produces a perishable good for every period according to the stationary distribution $F(\omega)$. Importantly, this production now happens with probability $1/2$ ‘just before’ and with probability $1/2$ ‘just after’ the period (and with it the ownership right) changes. Entering period t , the representative agent is endowed with shares z_t and proportional probabilistic rights to the tree’s output, ω_t . In the morning (t odd) domestic asset markets are open and the probabilistic right, q_t , to current output, ω_t , can be traded for future shares in the tree, z_{t+1} .²

The market structure of the economy in the evening is different: Because of the timing of production, in the evening, $t + 1$, ownership rights to the tree’s output, ω_{t+1} , are determined by q_t with probability $1/2$ and by z_{t+1} with probability $1/2$. This specification ensures that, in terms of expected received output in period $t + 1$, it is as good to own q_t as it is to own z_{t+1} . However, we assume that international goods markets are open only at the beginning of period $t + 1$. On those markets, domestic assets cannot be traded, but probabilistic rights to output of $\omega = 1/2$ can be traded at the fixed price $\kappa > 1$ for probabilistic rights to the output of $\omega = 1$. Thus, the tree’s output can only be traded on international markets if it is realized before the beginning of period $t + 1$. In expectation (from the morning’s perspective), therefore, ownership rights to output, q_t , provide the agent with more flexibility or ‘liquidity’ in the evening than shares of the productive asset, z_{t+1} , do.

(2) Whether or not output is realized just before or just after the beginning of period t will turn out to be irrelevant for odd t , as ownership of the tree is not permitted to change from period $t - 1$ to period t .

First we establish the pricing function $\psi(\omega, u)$ for a representative agent whose value function in odd periods, that is, at the time asset markets are open, takes the form

$$\begin{aligned} v(z, \omega, u) = \max_{q,x} & \left[qu(\omega) + \frac{1}{2}\delta x \int u'(\omega') \, dF(\omega') \, d\mu(u') \right. \\ & + \frac{1}{2}\delta q(f_{\frac{1}{2}} + \kappa f_1) \int \max[u'(\frac{1}{2}), \frac{1}{\kappa}u'(1)] \, d\mu(u') \\ & \left. + \delta^2 \int v(x, \omega', u') \, dF(\omega') \, d\mu(u') \right] \end{aligned}$$

subject to $q + p(\omega, u)x \leq z + p(\omega, u)z$.

Let us define $u^*(\mu) := \int \max[u(\frac{1}{2}), \frac{1}{\kappa}u(1)] \, d\mu(s) = \int \max[u(1/2), \frac{1}{\kappa}] \, d\mu(u)$ and $\mathbf{E}[u; F, \mu] := \int u'(y') \, dF(\omega') \, d\mu(u')$. Define

$$\phi(\omega, u) := p(\omega, u)(u(\omega, u) + \frac{1}{2}\delta(f_{\frac{1}{2}} + \kappa f_1)u^*(\mu))$$

and

$$\gamma := \frac{1}{2}\delta \mathbf{E}[u; F, \mu] + \delta^2 \iint (u'(\omega') + \frac{1}{2}\delta(f_{\frac{1}{2}} + \kappa f_1)u^*(\mu)) \, dF(\omega') \, d\mu(u')$$

Following the standard arguments in Lucas, we find

$$\phi(\omega, u) = \gamma + \delta^2 \iint f(\omega', u') \, dF(\omega') \, d\mu(u')$$

As in the Lucas model in KS, the unique solution is $\phi(\omega, u) = \gamma/(1 - \delta^2)$. Let $\Lambda := \gamma/(1 - \delta^2)$ and $\Gamma := \frac{1}{2}\delta(f_{\frac{1}{2}} + \kappa f_1)u^*(\mu)/\Lambda$, so that

$$\psi(\omega, u) = \frac{u(\omega)}{\Lambda} + \Gamma$$

We expect to verify the standard intuition that more preference for flexibility implies more demand for liquidity, or a lower willingness to invest in the productive asset, which will be reflected in a higher price for q_t in terms of z_{t+1} .

Note that Γ increases in the expected utility the agent derives from being able to access international markets.

As in KS, we now confine attention to the example where there are only three levels of output, 0, 1/2 and 1. Consider two exchange economies, A and B , with representative agents A and B respectively. We assume that both agents have period 0 preferences with a PFC representation based on state spaces \mathcal{U}_i^\dagger , where $u(0) = 0 \leq u(1/2) \leq u(1) = 1$ for all $u \in \mathcal{U}_i^\dagger$, $i = A, B$. Thus, agents are uncertain about their risk aversion, which is captured by $u(1/2)$. We also assume that \succsim^A and \succsim^B agree on the intertemporal tradeoff for getting 1 instead of 0, which implies $\delta_A = \delta_B$.

Thus, for $\omega \in \{0, 1/2, 1\}$, prices are given by

$$\begin{aligned}\psi_i(0, u) &= \Gamma_i \\ \psi_i(1, u) &= \frac{1}{\Lambda_i} + \Gamma_i \\ \psi_i(\frac{1}{2}, u) &= \frac{u(1/2)}{\Lambda_i} + \Gamma_i\end{aligned}$$

Let $H_i(\lambda) := \mathbf{P}(\frac{u(1/2)}{\Lambda_i} < \lambda)$ be the distribution of prices in the two economies for the case where output is $\omega = 1/2$, renormalized such that the two distributions can be compared in terms of second order stochastic dominance. Let $\bar{\psi}_i := \mathbf{E}[\psi_i(\frac{1}{2}, u), \mu_i]$ be the average domestic price in the case where output is $\omega = 1/2$.

Proposition 6. In the two economies above, (a) and (b) below are equivalent and imply (c).

- (a) Agent B has greater preference for flexibility than agent A .
- (b) H_A second order stochastically dominates H_B .
- (c) $\bar{\psi}_B \geq \bar{\psi}_A$ and $\psi_B(1) - \psi_A(1) = \psi_B(0) - \psi_A(0)$

Furthermore, if (c) holds for all prices $\kappa > 1$, then (a) and (b) are also implied.

The condition $\psi_B(1) - \psi_A(1) = \psi_B(0) - \psi_A(0)$ is the manifestation of the fact that singletons are ranked identically by both agents in terms of prices. Proposition 6 tells us to expect correlation between the price fluctuations and the degree of underinvestment in small economies that have open goods markets but closed asset markets: the average price for the productive asset is higher in economy A , or conversely the average price for current dividends is lower, $\bar{\psi}_B \geq \bar{\psi}_A$. The model suggests that the determinant of these two effects might be the level of uncertainty about future risk aversion in the economy.

Proof of Proposition 6. To investigate the effect of the liquidity provided by holding the right to output rather than shares, consider

$$\Gamma^{-1} = \frac{1}{1 - \delta^2} \left(\frac{(1 + 2\delta) \mathbf{E}[u; F, \mu]}{(f_{\frac{1}{2}} + \kappa f_1) u^*(\mu)} + \delta^2 \right)$$

Observe that $\frac{\partial \Gamma^{-1}}{\partial u^*(\mu)} < 0$ and therefore $\frac{\partial \Gamma}{\partial u^*(\mu)} > 0$. The proof rests on the following claim.

Claim: If μ_A SOSD μ_B , then $u^*(\mu_B) > u^*(\mu_A)$. Furthermore, if $u^*(\mu_B) > u^*(\mu_A)$ for all $\kappa > 1$, then μ_A SOSD μ_B .

Proof of Claim: Notice that $u^*(\mu) = \int \max[u(\frac{1}{2}), \frac{1}{\kappa}] d\mu(u(\frac{1}{2}))$. By definition μ_A SOSD μ_B if, and only if, $\int \max[u(\frac{1}{2}), \frac{1}{\kappa}] d\mu_A(u(\frac{1}{2})) > \int \max[u(\frac{1}{2}), \frac{1}{\kappa}] d\mu_B(u(\frac{1}{2}))$ for all $\kappa > 1$ (eg, page 33 in Laffont [1989]).

We can now establish the proposition. Equivalence of (i) and (ii) follows as in the case without investment. To establish that (ii) implies (iii), note $\psi_B(0) - \psi_A(0) = \psi_B(1) - \psi_A(1)$ if, and only if, $\Lambda_A = \Lambda_B = \Lambda$, if, and only if, $\mathbf{E}[u(\frac{1}{2}); F, \mu_A] = \mathbf{E}[u(\frac{1}{2}); F, \mu_B] = \bar{u}$. By the claim, $u^*(\mu_B) > u^*(\mu_A)$ for all $\kappa > 1$ if, and only if, μ_A SOSD μ_B , which is obviously the case if, and only if, H_A SOSD H_B . By the observation above $\Gamma_B > \Gamma_A$, if, and only if, $u^*(\mu_B) > u^*(\mu_A)$. Hence, $\bar{\psi}_B = \bar{u}/\Lambda + \Gamma_B \geq \bar{u}/\Lambda + \Gamma_A$.

Conversely, if (iii) holds for all $\kappa > 1$, then $\bar{u}_A = \bar{u}_B$, and $\Lambda_A = \Lambda_B$, and hence $\Gamma_B \geq \Gamma_A$, which implies $u^*(\mu_B) \geq u^*(\mu_A)$, and hence by the claim, μ_A SOSD μ_B . \square

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