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## Inter-Dealer Trading in Financial Markets\*

### I. Introduction

Trading between dealers who act as market makers is a common feature of most major financial markets. With the exception of equity markets that deal with relatively small order sizes, in most financial markets the customer order is filled by one dealer who then retrades with other dealers. Inter-dealer trading is an integral part of market design, particularly for institutional markets that deal with large orders. While the exact structure of inter-dealer trading is undergoing significant change (as we discuss below) two distinct kinds of inter-dealer trading, sequential trading and limit-order books are dominant in the marketplace.

In equity markets like the London Stock Exchange, inter-dealer trading constitutes some 40% of the total volume and occurs mainly via an anonymous limit-order book, although some

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We compare the following multi-stage inter-dealer trading mechanisms: a one-shot uniform-price auction, a sequence of unit auctions (sequential auctions), and a limit-order book. With uninformative customer orders, sequential auctions are revenue-preferred because winning dealers in earlier stages restrict quantity in subsequent auctions so as to raise the price. Since winning dealers make higher profits, dealers compete aggressively, thus yielding higher customer revenue. With informative customer orders, winning dealers use their private information in subsequent trading, reducing liquidity. Sequential trading breaks down when the customer order flow is too informative, while the limit-order book is robust and yields higher revenues.

inter-dealer trading occurs through direct negotiation between dealers. The evidence in Hansch, Naik, and Viswanathan (1998), Naik and Yadav (1997), and Reiss and Werner (1998) suggests that the layoff of large orders (risk sharing) is important reason why inter-dealer trading occurs in London. On the NASDAQ market, market makers can trade with each other on the SuperSoes system, the SelectNet system and on electronic crossing networks (ECNs) like Instinet. The SuperSoes systems allows market makers to place limit orders that are hit by other market makers, electronic crossing networks (ECNs) and day traders. Volume on SuperSoes is 20% of NASDAQ volume, a significant portion of this volume is inter-dealer trading. Market makers can also place quotes on SelectNet system. If these quotes are hit by other market makers (who must place orders that are a hundred shares more than the quoted size), the quoting dealer has the discretion to execute the order or withdraw the quote (this is called discretionary execution). Inter-dealer trading on SelectNet is around 10% of total NASDAQ volume. Finally market makers can trade with each other on ECNs like Instinet (which includes direct trades between institutions); Instinet has around 15% of total NASDAQ volume.<sup>1</sup>

In the market for U.S. Treasuries, two-thirds of the transactions are handled by inter-dealer brokerage firms such as Garban and Cantor-Fitzgerald, while the remaining one-third is done via direct interactions between the primary dealers. To improve market transparency in the U.S. Treasuries market, in 1990 the primary dealers and four inter-dealer brokers founded GovPX, which acts as a disseminator of transaction price and volume information. Hence, the trading between primary dealers can occur on a number of competing inter-dealer brokerage systems but is reported via GovPX to financial institutions.<sup>2</sup> More recently, the Bond Market Association responded to SEC pressure for more transparency in the bond market by setting up a single reporting system like GovPX for investment grade bonds.<sup>3</sup>

In the foreign exchange market, inter-dealer trading far exceeds public trades, accounting for about 85% of the volume.<sup>4</sup> Traditionally, inter-dealer trading on the foreign exchange market has been either by direct negotiation or brokered. Much of the inter-dealer trading via direct negotiation is sequential (an outside customer trades with dealer 1 who trades with dealer 2 who trades with dealer 3 and so on) and

1. See <http://www.marketdata.nasdaq.com> and <http://www.nasdaqtrader.com> for this information.

2. The GovPX web sit <http://www.govpx.com> provides useful information on the history of GovPX.

3. See the Bond Market Association web site <http://www.bondmarkets.com> or the related website <http://www.investinginbonds.com>. In an appearance before a House subcommittee on September 29, 1998, then SEC Chairman Arthur Levitt pushed for greater transparency in bond trading.

4. See Lyons (1995) for more details.

involves very quick interactions; hence, it is often referred to as “*hot potato*” trading. Today, 90% of this direct inter-dealer trading is done via the Reuters D2000-1 system which allows for bilateral electronic conversations in which one dealer asks another for a quote, which the originating dealer accepts or rejects within seconds.<sup>5</sup> Brokered trading is also executed via one of two electronic limit-order book systems, the Reuters Dealing 2002 system and the Electronic Brokerage System (EBS Spot Dealing System). The Reuters Dealing 2002 system was launched in 1992 and is discussed extensively by Goodhart, Ito, and Payne (1996). The EBS system was started to compete against the Reuters system and claims average volumes in excess of \$80 billion a day.<sup>6</sup>

The preceding discussion makes clear that two distinct kinds of inter-dealer trading, sequential trading (as in the Reuters D2000-1 system) and limit-order books (as in the EBS system, the SuperSoes system) are most prevalent. In our view, the literature on market microstructure does not explain how the structure of inter-dealer trading should differ according to the underlying trading environment.<sup>7</sup> While the seminal work of Ho and Stoll (1983) suggests that risk-sharing is a strong motive behind inter-dealer trading, it is not entirely clear why risk-sharing needs could not be met optimally by direct customer trading with several dealers. Even if inter-dealer trading is desirable, it is unclear which method of inter-dealer trading is more appropriate. Our view is echoed by Lyons (1996a) in his discussion of empirical results on the foreign exchange market where he states that “*a microstructural understanding of this market requires a much richer multiple-dealer theory than now exists (see e.g. Ho and Stoll 1983).*”

In this paper, we provide models of inter-dealer trading that include both single-price procedures with sequential trading (reflecting the traditional voice-brokering methods of inter-dealer trading) and multiple-price procedures like limit-order books (reflecting the recent move towards electronic books in the foreign exchange market). Also, we ask how the strategic behavior of dealers and the execution prices for customer orders differ with the exact structure of the inter-dealer market (single-price auction versus limit-order book) and with the motivation of customer trades (inventory versus information). We believe that

5. This information is provided in Evans and Lyons (2002) that uses the Reuters-2001 dataset.

6. The EBS partnership is a consortium of banks that are the leading market makers in the foreign exchange market. One key advantage of the EBS spot dealing system is that it links automatically with FXNET, a separate limited partnership of banks that provides for automated netting and hence reduces settlement risk. EBS’s web site <http://www.ebsp.com> provides more details. The Reuters web site <http://www.reuters.com> provides details on the Reuters Dealing 2002 system.

7. In the canonical models of Glosten and Milgrom (1985) and Kyle (1985), the outside order is taken completely by one dealer and no re trading occurs.

realistic modeling of inter-dealer trading is crucial to improve our understanding of dealership markets like the foreign exchange market, the U.S. Treasury market and the NASDAQ market.

Initially, we compare customer welfare between (1) two-stage trading that involve inter-dealer trading after a customer-dealer trade and (2) one-shot trading traditionally analyzed in the literature. We identify a key advantage of two-stage trading that is absent in one-shot trading environments.

Since Wilson (1979), it is known that single-price divisible good auctions are plagued with a “demand reduction” problem (see also Back and Zender (1993), Wang and Zender (2002), and Ausubel and Cramton (2002)). That is, uniform-price auctions have equilibria in which prices deviate substantially from economic values because bidders act strategically and steepen their demand curves. Two-stage trading alleviates this problem because customers do not split orders in the first stage, and in the second stage the dealer who wins in the first stage acts strategically. In particular, the dealer who wins in the first stage restricts the supply of the good in the second stage, i.e., engages in supply reduction. This raises the price in the second stage. Since the winner in the second stage has a higher utility in the first stage, there is an incentive to bid aggressively for the whole quantity in the first stage. This leads to higher revenue in the two-stage procedure. The ability of two-stage trading to elicit greater bidder competition gives the seller a potentially powerful weapon to cope with strategic bidding in single-price auction. This idea is worth emphasizing and is useful in understanding why multi-stage trading occurs in the real world.

To gain a deeper understanding of the nature of inter-dealer trading, we extend the analysis to the case when the dealers rely on a limit-order book at the second-stage. There are important differences between inter-dealer trading at one price (dealership) and inter-dealer trading at multiple prices (a limit-order book). In contrast to the dealership market, the benefit to a limit-order book market *decreases* with customer order size. In a single-price auction strategic bidding (i.e., a departure from bidding according to marginal valuations) is greatest at large quantities. In a limit-order book it is the greatest at small quantities.<sup>8</sup> Therefore, for a limit-order book, the revenue improvement of a two-stage versus a one-shot trading is mainly for small customer orders.

We generalize our two-stage, single price procedure to a sequential auction with multiple rounds of unit-auctions. In the sequential auction, the winning dealer (in any round) keeps some fraction of the object and

8. These differences in market makers' trading strategies across market structures are emphasized in Viswanathan and Wang (2002). The hybrid market structure considered there was not two-stage trading, but rather a concurrent setup which routes orders to different marketplaces using a size criterion.

sells the remaining via a unit-auction to one of the remaining dealers in that round. The procedure is in the spirit of the “voice-brokering” market where a customer sells to dealer 1 who then sells to dealer 2 and so on. We show that in such a sequential auction market liquidity falls as the auction progresses, and in each successive round the winning dealer in that round keeps a larger fraction of the order flow for himself. Further, we show that the seller is better off with more rounds of auctions. These results rationalize the use of sequential auctions and explain the phenomenon of “hot potato” trading.

We extend our analysis to environments where the customer order flow contains payoff-relevant information. Absent reporting of trades, the information asymmetry between the customer and the market makers imposes a cost on inter-dealer trading that adversely affects the attainment of efficient risk-sharing. More asymmetric information in order flow lowers market liquidity and yields uniformly lower price competition between dealers. With large information asymmetry, a (linear strategy) equilibrium does not exist in a sequential auction. In contrast, inter-dealer trading with a limit-order book is less sensitive to private customer information and does not break down. Thus limit-order books are more robust to market breakdown than sequential auctions.

The paper proceeds as follows. Section II discusses the prior literature and our relation to it. Section III presents a model of two-stage trading in a dealership setting. The basic intuitions of the model are laid out in this simple context by comparing customer welfare between two-stage trading and one-shot trading. Section IV analyzes a sequential auction procedure that extends two-stage trading. The important case of limit-order book trading is taken up in Section V. Section VI deals with informative customer trades; we also compare the customer’s expected revenues under different trading mechanisms. Section VII concludes.

## **II. Literature Review**

Naik, Neuberger, and Viswanathan (1999) and Lyons (1997) consider the value of disclosing customer trades and inter-dealer trades in dealership markets. The focus of our paper is not on disclosure of customer trades but rather on the comparison of differing inter-dealer systems. Vogler (1997) considers the special case of a dealership market when dealers have the same pretrading inventory and concludes that inter-dealer trading always dominates the one-shot dealership market. Potential costs to two-stage trading in our model are absent from Vogler’s model because his is not a model of private information and because he assumes homogeneous inventories across the dealers. None of the above papers considers the limit-order book

as a possible mode of inter-dealer trading or considers sequential auction procedures.<sup>9</sup> Our modeling of the limit-order book allows dealers to use limit-orders of arbitrary sizes in the inter-dealing stage and for optimal bidding for the customer trade in the first round.

This paper is also related to the recent literature on limit-order books due to Biais, Martimort, and Rochet (2000), Viswanathan and Wang (2002) and Röell (1998). Viswanathan and Wang (2002) characterize the equilibria of a dealership market (a single-price setup) and a limit-order book (a multi-price setup) when the risk-averse dealers compete for the customer order via downward sloping demand curves. Röell (1998) provides related results when the order flow is drawn from the exponential distribution. Bias, Martimort, and Rochet (2000) characterize the limit-order book when the marginal valuation curve is downward sloping due to information in the order flow. Viswanathan and Wang (2002) discusses the relationships between these papers and their relation to the earlier work of Glosten on the monopoly specialist (1989) and on the competitive limit-order book (1994). While these papers characterize limit-order books, they do not allow for inter-dealer trading.

Finally, our paper is related to the literature on retrading in auctions by Haile (2000) and Gupta and Lebrun (1999). In these models, bidders are risk-neutral and re-trading occurs either because the object was not allocated to the bidder with the highest valuation (Gupta and Lebrun) or because valuations changed and the bidder with the highest valuation in the second stage is not the highest bidder in the first stage (Haile). In contrast, retrading in our model occurs because dealers wish to share risk and not hold all of the object themselves.

### III. Inter-Dealer Trading With a Single-Price Mechanism

#### A. The Model

There are  $N > 2$  risk-averse dealers (market makers or liquidity providers) in the game. Each dealer can potentially fill a sell order from a risk neutral outside customer<sup>10</sup> of size  $\tilde{z}$ , which is distributed over the unit interval  $[0, 1]$ . The dealers act to maximize a mean-variance derived utility of profit with the risk aversion parameter  $\rho$ .<sup>11</sup> A typical

9. Werner (1997) presents a double auction model of inter-dealer trading with initially identical dealers. Following the extensive literature on double auctions with unit demands, all dealers in Werner (1997) submit limit orders to buy or sell a fixed amount in the second stage. Inter-dealer trading is also taken as given in Lyons (1996b), where the focus is on the *transparency* of inter-dealer trades.

10. Customer buys are analyzed analogously.

11. With single-price inter-dealer trading (modeled as a uniform-price auction), this quadratic objective function may be derived from the standard assumption of constant absolute risk-aversion utility functions and normally distributed payoffs. For the limit-order book (akin to a discriminatory auction), these assumptions do not imply a quadratic objective function in the dynamic optimization problem.

dealer, generically referred to as dealer  $k$  ( $k = 1, 2, \dots, N$ ), is endowed with an *ex ante* inventory of  $\tilde{I}_k$ , which is drawn from some commonly known distribution that has support,  $[L, \bar{I}]$ . We denote the average (per dealer) initial inventory as  $\bar{Q} \equiv (1/N) \sum_{k=1}^N \tilde{I}_k$ . The underlying asset value,  $\bar{v}$ , is normally distributed as  $N(\bar{v}, \tau_v^{-1})$ , and  $\bar{v}$  is independent of the *ex ante* inventories,  $\tilde{I}_k$ .

To isolate the difference between two-stage trading and one-shot trading, we initially assume that the *ex ante* dealer inventories are common knowledge among all dealers. The main conclusions are not qualitatively different when this assumption is relaxed (see Section III.D). Furthermore, we assume that the customer order does not contain information about the fundamental value of the traded asset. The consequences of relaxing this assumption will be evaluated in Section VI.

As a benchmark, we consider a one-shot trading setup in which the dealers submit demand schedules to compete for fractions of the customer order which is split among the  $N$  dealers. The focus of this section, however, is two-stage trading in which the outside order is first filled *in its entirety* by a single dealer before it is divided among all dealers via inter-dealer trading. Order splitting during customer-dealer trading is disallowed.<sup>12</sup> Partly because most dealership markets are not anonymous, the customer expects to sell to one dealer instead of splitting the order among several dealers. As such, competition among the market makers in the first round is similar to bidding for an indivisible good.

We refer to the dealer who wins the customer order as dealer  $W$ , and to any dealer who loses the customer order as dealer  $L$  ( $\forall L = 1, 2, \dots, N, L \neq W$ ).<sup>13</sup> After the customer-dealer trading, the customer order size and transaction price become known to dealer  $W$ , but *not* to the other dealers.<sup>14</sup> Consequently, during inter-dealer trading, dealer  $W$  can condition his trading strategy on  $z$ , while dealer  $L$  cannot. Inter-dealer trading among all  $N$  dealers starts shortly after the first round.<sup>15</sup> The

12. The “all-or-nothing” assumption is adopted in part to avoid the difficulty of choosing among the multiple equilibria that result from analyzing a share auction. The main conclusions of the paper regarding the superiority of two-stage trading over one-shot trading are not driven by this assumption. What is important is the observation that the winning dealer in the initial customer-dealer trading engages in “supply reduction” to raise the inter-dealer price.

13. Throughout the paper, the subscript  $k$  denotes quantities for a typical dealer  $k$ , whereas the superscript  $W$  (or  $L$ ) denotes quantities for any specific dealer identified as the winning (or losing) dealer in the first round.

14. The issue of trade disclosure is important if the second stage is run as a limit-order book, since a key aspect of a book is its inability to condition on quantity. In this paper, we do not pursue issues related to disclosure but focus on the comparison of various modes of inter-dealer trading. See Naik, Neuberger, and Viswanathan (1999) for a paper that focuses on disclosure and customer welfare in a dealership market.

15. In a dynamic setting, dealer  $W$  would face the tradeoff between waiting for the next customer buy to arrive and initiating trading with other dealers right away. By focusing on inter-dealer trading that occurs soon after the customer order is filled by one particular dealer, we are essentially studying markets where the need to lay off the risk associated with an unbalanced portfolio is very significant.

efficacy of such two-stage trading will be compared to one-shot trading in which the customer can directly trade with many market makers.

It is convenient to visualize inter-dealer trading as involving three steps. First, dealer  $W$  hands over his *entire* holding of the asset,  $I^W + z$ , to an auctioneer (the inter-dealer broker). Then, the auctioneer solicits bids from all dealers (including dealer  $W$ ) in the form of combinations of price and quantity. Further, we restrict the analysis to demand schedules that are continuously differentiable and downward sloping. A typical dealer  $k$ 's trading strategy, as a function of the equilibrium price and possibly his own pretrading net position, is the quantity that is awarded to him by the auctioneer,  $x_k$ .<sup>16</sup> The equilibrium price of the single-price inter-dealer trading,  $\tilde{p}_2$ , is determined by equating demand and supply. After the auctioneer collects payment from all winning bids (those with price levels at or above the equilibrium price), the total proceeds are then returned to dealer  $W$  and are his to keep. Note that, at the conclusion of the inter-dealer trading, dealer  $W$ 's net holding is  $x^W$ , while dealer  $L$ 's net position is  $I^L + x^L$ . In other words,  $x^W$  denotes dealer  $W$ 's final allocation, while  $x^L$  is dealer  $L$ 's trade quantity. Results in the paper need to be interpreted with this convention in mind.

In the two-stage game, the winning dealer submits a supply curve during inter-dealer trading. An alternative is for the winning dealer to use the quantity choice as a strategy. However, with two stages of trading and no adverse selection problem, we can show that this alternative is revenue inferior for the winning dealers. The case of winning dealers making *sequential* quantity choices is analyzed in Section IV.

A distinct feature of the dealership market is that most transactions take place at a single price. Therefore, dealer  $k$ 's profit at the conclusion of inter-dealer trading is:

$$\tilde{\pi}_k^W = \tilde{v}\tilde{x}_k^W + \tilde{p}_2(I_k + \tilde{z} - \tilde{x}_k^W), \quad \text{if } k \text{ previously won the customer order,}$$

$$\tilde{\pi}_k^L = \tilde{v}(I_k + \tilde{x}_k^L) - \tilde{p}_2\tilde{x}_k^L, \quad \text{if } k \text{ did not win the customer order.}$$

Differences between inter-dealer trading via a dealership setup (which involves trading at a single-price) and inter-dealer trading via a limit-order book (which involves trading at multiple prices) are discussed in Section V.

16. It is often convenient to express the bidding strategies as the inverse demand schedule, i.e., price as a function of quantity and inventory. To save notation, arguments to the demand schedules are often suppressed.

For ease of presentation,<sup>17</sup> we maintain the following restrictions throughout the paper:

$$I_k \ll \frac{\bar{v}}{\rho\tau_v^{-1}}, \forall k,$$

$$\frac{1}{N} \ll \frac{\bar{v}}{\rho\tau_v^{-1}}.$$

With tractability in mind, we restrict our analysis to equilibria of the model that are characterized by linear trading strategies. Given the symmetry of the problem, we will search for equilibria in which the strategies of the nonwinning dealers ( $\forall L \neq W$ ) take the same functional form.

### *B. Benchmark: A One-Shot Dealer Market*

A one-shot model where the customer can trade directly with  $N$  competing market makers serves as a benchmark for comparison with the two-stage trading model just introduced. The following is a necessary condition for the equilibrium strategies in a single-price dealership market:<sup>18</sup>

$$\sum_{j \neq i}^N \frac{\partial x_j(p, I_j)}{\partial p} = - \frac{x_i(p)}{\bar{v} - p - \rho\tau_v^{-1}[I_i + x_i(p, I_i)]}.$$

A unique symmetric, linear solution to the above equation is the following:<sup>19</sup>

$$x_k(p, I_k) = \gamma_d(\bar{v} - \rho\tau_v^{-1}I_k - p), \quad \forall k = 1, 2, \dots, N,$$

where

$$\gamma_d = \frac{N - 2}{(N - 1)\rho\tau_v^{-1}}. \quad (1)$$

The equilibrium price and allocations are:

$$\tilde{p}_d = \bar{v} - \rho\tau_v^{-1}Q - \frac{(N - 1)\rho\tau_v^{-1}}{N - 2} \frac{\tilde{z}}{N},$$

$$\tilde{x}_k = \frac{\tilde{z}}{N} + \frac{N - 2}{N - 1}(Q - I_k). \quad (2)$$

17. We make these assumptions in order to restrict the discussion to just one side of the market, i.e., a customer sells and the market makers buy. Relaxing these parameter restrictions does not affect the conclusions of the paper in any substantive way.

18. See Viswanathan and Wang (2002) for a derivation.

19. This is similar to Kyle (1989).

The customer's total revenue is

$$\tilde{R}_d = \tilde{z}\tilde{p}_d.$$

At the end of trading, the net inventory positions are as follows:

$$\begin{aligned} I_k + \tilde{x}_k &= \frac{\tilde{z}}{N} + \frac{N-2}{N-1}Q + \frac{I_k}{N-1} \\ &= \frac{\tilde{z}}{N} + \frac{2I_k}{N} + \frac{N-2}{N} \left( \frac{\sum_{j \neq k} I_j}{N-1} \right), \end{aligned} \quad (3)$$

where the term in parenthesis in Eq. (3) is the average inventory of the other  $N - 1$  dealers.

In the ending positions of the dealers (Eq. (3)), the customer order is equally shared. However, each individual dealers' inventory is overweighted by "one extra share" and the average inventory of the other dealers is underweighted correspondingly.<sup>20</sup> We will use this observation to provide intuition for the revenue superiority of two-stage trading.

In submitting bids for the given quantity, each dealer understands the impact of his trade on the price. Hence a dealer who wishes to buy finds it optimal to steepen his demand curve to reduce the price. This is referred to as "demand reduction" (see Ausubel and Cramton (2002) for a general discussion of this phenomenon in uniform-price auctions) and results in a price lower than the dealer's marginal valuation of the object (see Eq. (2)). Symmetrically, a dealer who wishes to sell has strategic incentives to engage in "supply reduction", i.e., to restrict the supply at every price. This strategic power leads to each dealer overweighting his own inventory by "one extra share" of inventory.<sup>21</sup> The two-stage trading model that we present exploits this strategic incentive.

### C. Equilibrium Analysis of the Two-Stage Trading Model

The two-stage trading model is solved using backward induction. Under a dealership structure, the second-stage (i.e., inter-dealer competition) can be modeled as a single-price divisible good auction. The first round bidding between the market makers for the customer order can be viewed as a unit demand auction with private bidder valuation. With publicly known dealer inventories, the bidders' valuation is common knowledge and the outcome of the first stage bidding is standard.

20. Without this overweight, the ending inventory would be  $\frac{\tilde{z}}{N} + \frac{I_k}{N} + \frac{1}{N} \sum_{j \neq k} I_j$ . The overweight of one's own inventory implies that inventory hedging is incomplete by the end of trading.

21. Note that dealers have equal strategic power with respect to the customer order which is equally shared.

### 1. The Second Round: Inter-Dealer Trading

When the inter-dealer trading results in all trades clearing at a single price, the dealers' equilibrium trading strategies are provided below. Note that the solution in Proposition 1 forms an *ex post* equilibrium strategy in that it is independent of the distribution of customer order size or the dealer inventories.

PROPOSITION 1. If inter-dealer trading occurs at a single price, it has a unique linear strategy equilibrium characterized by the following trading strategies:

$$x^W(p, z, I^W) = \gamma_2(\bar{v} - p) + \frac{I^W + z}{N - 1}, \quad (4)$$

$$x^L(p, I^L) = \gamma_2(\bar{v} - p) - \frac{N - 2}{N - 1}I^L, \quad \forall L \neq W, \quad (5)$$

where the price elasticity of demand,  $\gamma_2$ , is given by:

$$\gamma_2 = \frac{N - 2}{(N - 1)\rho\tau_v^1}. \quad (6)$$

*Proof.* See Appendix I.

Using the above equilibrium strategies, it is straightforward to show:

$$\begin{aligned} \tilde{p}_2 &= \bar{v} - \rho\tau_v^{-1} \left( \tilde{Q} + \frac{\tilde{z}}{N} \right), \\ \tilde{x}^W &= \frac{N - 2}{N - 1} \left( \tilde{Q} + \frac{\tilde{z}}{N} + \frac{I^W + \tilde{z}}{N - 2} \right), \\ \tilde{x}^L &= \frac{N - 2}{N - 1} \left( \tilde{Q} + \frac{\tilde{z}}{N} - I^L \right), \quad \forall L \neq W. \end{aligned} \quad (7)$$

Thus, the dealers' net positions at the end of two rounds of trading will be:

$$\tilde{x}^W = \frac{2\tilde{z}}{N} + \frac{N - 2}{N - 1}\tilde{Q} + \frac{I^W}{N - 1} = \frac{2(\tilde{z} + I^W)}{N} + \frac{N - 2}{N} \left( \frac{\sum_{j \neq W} \tilde{I}_j}{N - 1} \right), \quad (8)$$

$$I^L + \tilde{x}^L = \frac{N - 2}{N - 1} \frac{\tilde{z}}{N} + \frac{N - 2}{N - 1}\tilde{Q} + \frac{I^L}{N - 1} = \frac{2I^L}{N} + \frac{N - 2}{N} \left( \frac{\tilde{z} + \sum_{j \neq L} \tilde{I}_j}{N - 1} \right). \quad (9)$$

These two expressions are similar to the last two terms of Eq. (3). Because this second round of trading involves retrading among the dealers, no additional "external" supply is available, which explains the absence of a

term similar to the first term in Eq. (3). The original customer order,  $\tilde{z}$ , shows up as part of dealer  $W$ 's pre-inter-dealer-trading inventory. Thus, it receives "one share of overweight" in  $W$ 's ending inventory position, much like the one-shot trading model where each dealer's own pre-trading inventory is overweighted in the final quantity allocation. This is due to the "supply reduction" engaged in by dealer  $W$  so as to raise the price in the second stage. Comparing Eq. (7) with Eq. (2), we have  $\tilde{p}_2 > \tilde{p}_d$ . Hence, dealer  $W$ 's supply reduction raises the price, and consequently dealer  $L$  has an ending inventory that underweights the customer order.

At the conclusion of inter-dealer trading, the customer order is split unevenly among the dealers, with dealer  $W$  retaining an above average fraction,  $2/N$ , and every other dealer retaining a below average fraction,  $(N-2)/[N(N-1)]$ , of the original customer order  $\tilde{z}$ . In addition, the price is higher during inter-dealer trading than in the one-shot trading model ( $\tilde{p}_2 > \tilde{p}_d$ ). This is an important difference between two-stage trading and one-shot trading.

Next we write dealer  $k$ 's ( $\forall k = 1, 2, \dots, N$ ) second-stage (certainty equivalent) utility, depending on whether or not he gets to fill the customer order in the first round:

$$\begin{aligned}\tilde{U}_k^W &= \bar{v}\tilde{x}_k^W - \frac{\rho\tau_v^{-1}}{2}(\tilde{x}_k^W)^2 + \tilde{p}_2(I_k + \tilde{z} - \tilde{x}_k^W) \\ &= \bar{v}\left[\frac{N-2}{N-1}\left(\tilde{Q} + \frac{\tilde{z}}{N} + \frac{I_k + \tilde{z}}{N-2}\right)\right] - \frac{\rho\tau_v^{-1}}{2}\left[\frac{N-2}{N-1}\left(\tilde{Q} + \frac{\tilde{z}}{N} + \frac{I_k + \tilde{z}}{N-2}\right)\right]^2 \\ &\quad + \left[\bar{v} - \rho\tau_v^{-1}\left(\tilde{Q} + \frac{\tilde{z}}{N}\right)\right]\left[I_k + \tilde{z} - \frac{N-2}{N-1}\left(\tilde{Q} + \frac{\tilde{z}}{N} + \frac{I_k + \tilde{z}}{N-2}\right)\right], \\ \tilde{U}_k^L &= \bar{v}(I_k + \tilde{x}_k^L) - \frac{\rho\tau_v^{-1}}{2}(I_k + \tilde{x}_k^L)^2 - \tilde{p}_2\tilde{x}_k^L \\ &= \bar{v}\left[I_k + \frac{N-2}{N-1}\left(\tilde{Q} + \frac{\tilde{z}}{N} - I_k\right)\right] - \frac{\rho\tau_v^{-1}}{2}\left[I_k + \frac{N-2}{N-1}\left(\tilde{Q} + \frac{\tilde{z}}{N} - I_k\right)\right]^2 \\ &\quad - \left[\bar{v} - \rho\tau_v^{-1}\left(\tilde{Q} + \frac{\tilde{z}}{N}\right)\right]\left[\frac{N-2}{N-1}\left(\tilde{Q} + \frac{\tilde{z}}{N} - I_k\right)\right].\end{aligned}$$

From the above, it is straightforward to compute the utility difference between the winning dealer and the losing dealers, which is useful in analyzing the first stage of customer-dealer trading.

**COROLLARY 1.** If inter-dealer trading occurs at a single price, the difference in dealer  $k$ 's second-stage (certainty equivalent) utility between winning and losing the customer order,  $\tilde{z}$ , is:

$$\begin{aligned}\tilde{U}_k^W - \tilde{U}_k^L &= \tilde{z}\left\{\bar{v} - \frac{\rho\tau_v^{-1}}{(N-1)^2}\left[I_k + N(N-2)\tilde{Q} + \left(N - \frac{3}{2}\right)\tilde{z}\right]\right\}, \\ &\quad \forall k = 1, 2, \dots, N.\end{aligned}\tag{10}$$

## 2. The First Round: Customer-Dealer Trading

For any given customer order  $\tilde{z}$ , Corollary 1 suggests that dealer  $k$ 's incentives for filling the customer order are based on the following private value function:<sup>22</sup>

$$\tilde{V}_k = \tilde{U}_k^W - \tilde{U}_k^L, \quad (11)$$

which is strictly decreasing in his own inventory level,  $I_k$ . Thus, without loss of generality, we can index the dealers in ascending order of their inventory positions, i.e.,  $I_1 < I_2 < \dots < I_N$ .

The first stage of trading is an unit auction under complete information. The dealer with the lowest *ex ante* inventory, dealer 1, wins the customer order, and he pays an amount equal to the reservation price of the dealer with the second-lowest inventory, dealer 2.

**PROPOSITION 2.** Assume that dealer inventories are publicly known. If inter-dealer trading occurs at a single price, the customer receives the following revenue in the first round of trading:

$$\tilde{R}_1 = \tilde{z} \left\{ \bar{v} - \frac{\rho \tau_v^{-1}}{(N-1)^2} \left[ I_2 + N(N-2)Q + \left( N - \frac{3}{2} \right) \tilde{z} \right] \right\} \equiv \tilde{z} \tilde{p}_1, \quad (12)$$

where  $I_2$  is the second-smallest inventory among all the dealers.

*Proof.* From Eq.s (10) and (11), it is clear that, for any given  $\tilde{z}$ , we have  $\tilde{V}_1 > \tilde{V}_2 > \dots > \tilde{V}_N$ .

Although the dealers do not know in advance the size of the incoming customer order,  $\tilde{z}$ , they compete by submitting a series of quantity-payment pairs, i.e., a demand schedule that states what the total payment to the customer,  $B_k$ , will be at each potential customer order size. In particular, the following is a set of equilibrium bidding strategies (as a function of the customer order size,  $\tilde{z}$ ):

$$\begin{aligned} B_1(\tilde{z}) &= V_2(\tilde{z}), \\ B_k(\tilde{z}) &= V_k(\tilde{z}), \forall k \neq 1. \end{aligned}$$

The tie-breaking rule is that the lower-numbered dealer wins when the same bid is submitted by more than one dealer.

Regardless of the actual customer order sizes, the outcome is always that dealer 1 wins and pays the customer dealer 2's reservation price of  $\tilde{V}_2$ .

22. Due to the two-stage nature of the model it is *not* always true that one could directly work with the certainty equivalent utility in computing the dealers' optimal trading strategy in the first round. See Lemma 1 in Appendix II for a proof of the validity of the certainty equivalent approach in our model.

Customer revenue from two-stage trading versus one-shot trading can be evaluated by comparing Eq.s (12) and (2):

$$\tilde{p}_1 - \tilde{p}_d = \frac{\rho\tau_v^{-1}}{(N-1)^2} \left[ \frac{N^2-2}{2N(N-2)} \tilde{z} + (Q - I_2) \right]. \quad (13)$$

In Eq. (13), there are two effects that could potentially work in opposite directions. The first is a “strategic effect” that always works in favor of the two-stage trading game. This is captured by the term proportional to  $\tilde{z}$ . The second effect arises due to the surplus extracted by the winning dealer in the first round of a two-round trading model. This “bidding effect” is the term proportional to  $Q - I_2$ .

The intuition for the strategic effect is as follows. Previously we established that in equilibrium all dealers overweight their own inventory. This results in the winning dealer retaining an extra share of the customer order: dealer  $W$  restricts quantity during the inter-dealer trading stage. Restricting the quantity available (“supply reduction”) to other bidders raises the price in the second round and yields higher profits to the winning dealer. Since the winner of the customer order expects to receive higher profits in inter-dealer trading, all dealers compete more intensely for the customer order. This yields the strategic term,  $\frac{\rho\tau_v^{-1}}{(N-1)^2} \left[ \frac{N^2-2}{2N(N-2)} \tilde{z} \right]$ .<sup>23</sup>

The strategic effect favors two-stage trading. Also, as the customer order size becomes larger, the winning dealer’s potential profit in the second round becomes greater, which implies more intense competition in the first round. Therefore, the net benefit of this strategic effect to two-stage trading (as opposed to one-shot trading) increases as the customer order becomes larger.

The bidding effect is related to the surplus extracted by the winning bidder in the first stage. Since the winning bidder has the lowest inventory, it must be that  $Q - I_1 > 0$ , i.e., the winning bidder wishes to buy additional units. This works in favor of the two-stage trading game as the customer order is allocated to a trader who desires it the most. However, the transaction price is not set by the dealer with the lowest inventory, but rather by the dealer with the second lowest inventory. If  $Q - I_2 > 0$ , the dealer with the second lowest inventory wants to buy and hence the price favors the two-stage game. If  $Q - I_2 < 0$ , the dealer with the second lowest inventory wants to sell. In this situation, setting the transaction price based on a dealer who does not want to add to his inventory will have a negative impact on the first-stage price.

23. Note that the customer receives  $\tilde{p}_1$ , not  $\tilde{p}_2$ . The strategic effect is less than  $\tilde{p}_2 - \tilde{p}_d = \frac{\rho\tau_v^{-1}}{N(N-2)} \tilde{z}$  because the winning bidder has to be compensated for the risk of holding additional inventory of size  $\frac{\tilde{z}}{N}$ , thus lowering the benefit of running the two-stage trading game for the customer.

Notice that this bidding effect arises only because of the surplus obtained by the winning bidder. One way to see this is to rewrite Eq. (13) as

$$\tilde{p}_1 - \tilde{p}_d = \frac{\rho\tau_v^{-1}}{(N-1)^2} \left[ \frac{N^2 - 2}{2N(N-2)} \tilde{z} + (Q - I_1) \right] - \frac{\rho\tau_v^{-1}}{(N-1)^2} (I_2 - I_1), \quad (14)$$

where the last term is the only negative term in the equation and reflects the surplus extracted by the winning bidder. Note that the size of the bidding effect is affected by the distribution of dealer inventories and not by the customer order size.

**COROLLARY 2.** Assume that dealer inventories are publicly known. Relative to the equilibrium price in a one-shot game,  $\tilde{p}_d$ , the equilibrium price in the first round of the two-stage game,  $\tilde{p}_1$ , has two properties: (i)  $\tilde{p}_1$  as a function of the customer order size  $\tilde{z}$  is always flatter (i.e., more price-elastic) than  $\tilde{p}_d$ ; (ii)  $\tilde{p}_1$  has a higher intercept (i.e., small-quantity quote) than  $\tilde{p}_d$  if and only if  $Q > I_2$ .

The bidding effect does not change as customer order sizes change, while the strategic effect is proportional to the order size. Thus, the strategic effect dominates at large order sizes. Therefore, the two-stage dealership market generally provides better execution than its one-shot counterpart for large-sized order flows. See Figure 1 for an illustration.

In empirical work, Naik and Yadav (1997) and Reiss and Werner (1998) find that the bid-ask spread on inter-dealer trades is smaller than the spread on customer-dealer trades. The model here produces the following result, which is consistent with the empirical observation.

**COROLLARY 3.** If  $Q - I_2 < \frac{(N-2)\tilde{z}}{2N}$ , then the bid-ask spread on inter-dealer trades is smaller than the spread on customer-dealer trades.

From Eq.s (7) and (12), it is easy to see that

$$\tilde{p}_2 - \tilde{p}_1 = \frac{\rho\tau_v^{-1}(N-2)}{2N(N-1)^2} \tilde{z} - \frac{\rho\tau_v^{-1}}{(N-1)^2} (Q - I_2).$$

Thus, when the bidding effect (the second-term above) does not dominate (e.g., when the dealer inventories are relatively homogeneous), we have  $\tilde{p}_2 > \tilde{p}_1$ . In this case, the bid price in the second stage is higher than in the first stage of trading. An analogous analysis of buy orders reveals that ask prices in the second stage are lower than the prices in the first stage of trading. Thus, the bid-ask spread is smaller on inter-dealer trades.

#### *D. Privately Known Dealer Inventories*

Now, any other dealer's higher private valuation will manifest itself through a lower value of  $\tilde{Q}$  in dealer  $k$ 's private value (notice the appearance of  $\tilde{Q}$  in Eq. (10)). Thus, when the dealers do not know

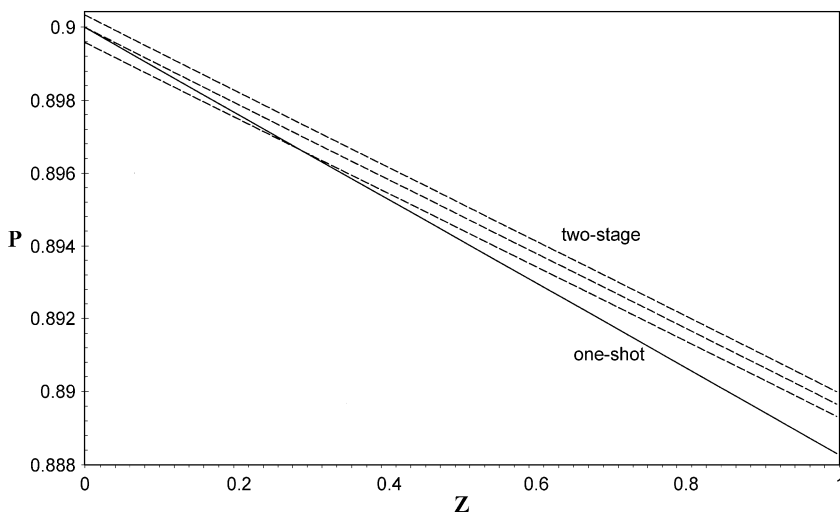


FIG. 1.—Equilibrium price vs customer order size in a dealership market. The solid line is for one-shot trading. The three dashed lines are for the initial stage of a two-stage trading model (the second lowest dealer inventory is 0.8, 1.0, 1.2 from top to bottom). The price for two-stage trading is generally higher than its one-shot counterpart at large customer order sizes, although at small customer order sizes the comparison is influenced by dealer inventory.

other dealers' inventory positions, the dealers' private values are "affiliated" in the sense of Milgrom and Weber (1982). Now the celebrated *revenue equivalence* theorem does not hold and our choice of auction form is relevant. For concreteness, we will assume the dealers have exponential preferences and model the customer-dealer trading as a second-price auction.

PROPOSITION 3. Suppose all dealers have exponential preferences and their inventories are exponentially distributed with a pdf of  $f(\eta) = \mu e^{-\mu\eta}$  and a cdf of  $F(\eta) = 1 - e^{-\mu\eta}$ , where the parameter  $\mu > 0$ .<sup>24</sup> If the customer-dealer trading is a second-price auction, then the customer's expected revenue is:

$$\tilde{R}_1 \equiv \tilde{z}\tilde{p}_1 = \tilde{z} \left\{ \bar{v} - \frac{\rho\tau_{\bar{v}}^{-1}}{(N-1)^2} \left( N - \frac{3}{2} \right) (2\bar{I} + \tilde{z}) \right\} + \frac{(N-2)}{\rho} \left[ (\mu - \tilde{\kappa})\bar{I}_2 - \ln \left( \frac{\mu}{\mu - \tilde{\kappa}} \right) \right],$$

24. For ease of computation, we do not impose the inventory restrictions listed in Section III.A. With the exponential distribution, inventories can be very large. Hence some dealers could be sellers instead of buyers at the single price that clears the second stage.

where,  $\bar{I}_2$  is the expected value of the second lowest inventory given by:

$$\bar{I}_2 = \frac{2N - 1}{\mu N(N - 1)},$$

and we require  $\tilde{\kappa} < \mu$ :

$$\tilde{\kappa} \equiv \frac{(N - 2)\rho^2\tau_v^{-1}}{(N - 1)^2}\tilde{z}.$$

*Proof.* See Appendix II.

We find that the price the customer expects to receive is usually higher with two-stage trading than with one-shot trading (for one-shot trading we find  $\tilde{p}_d$  by integrating Eq. (2) over  $Q$ ):

$$\begin{aligned} \tilde{p}_1 - \tilde{p}_d = & \frac{N^2\rho\tau_v^{-1}}{2N(N - 2)(N - 1)^2}\tilde{z} + \rho\tau_v^{-1}\left[\bar{Q} - \frac{(2N - 3)}{(N - 1)^2}\bar{I}_2\right] \\ & + \frac{(N - 2)}{\rho\tilde{z}}\left[(\mu - \tilde{\kappa})\bar{I}_2 - \ln\left(\frac{\mu}{\mu - \tilde{\kappa}}\right)\right]. \end{aligned} \quad (15)$$

As in the analysis preceding Eq. (14) in the known inventory case, we can decompose the price difference into two components. The first is the “strategic effect” which arises because of the supply reduction during inter-dealer trading. This effect is the first term in Eq. (15) which is strictly positive and proportional to the customer order size.

The second line in Eq. (15) is the “bidding effect”. As in the known inventory case, it is typically positive (so long as the log term does not dominate). This effect consists of two effects, the surplus effect and the winner’s curse (the winner’s curse did not exist with known inventories). As before, the surplus effect arises because the price is set by the second lowest inventory in Eq. (15). The winner’s curse arises because the winner does not know the inventories of the bidders below him and has to find their conditional expectation. This induces him to underbid and is reflected in the last term in Eq. (15).

Overall, the bidding effect is negative when  $\tilde{\kappa}$  approaches  $\mu$  from below, that is, when the customer order is large and/or when the dealer inventories are drawn from a distribution with high variance. For intuition, we recognize that the variance of the exponential distribution is  $1/\mu^2$ . So when  $\mu$  is small, the variance (and mean) of inventories is high, suggesting more surplus to the highest type (the dealer with the lowest inventory). Further, the winner’s curse is higher when the variance of inventories is higher. Under these circumstances, the bidding effect works against the two-stage auction.

The models we have analyzed in Section III all lead to similar conclusions. Two-stage trading enjoys a pricing advantage (from the customer's perspective) over one-shot trading. In two-stage trading, the winning dealer strategically restricts the amount sold in the inter-dealer stage in order to raise the resale price. This strategic effect favors two-stage trading and is linear in the customer order size. In addition, there is a bidding effect that favors two-stage trading unless the customer order is large and inventories are drawn from a distribution with a high variance.

#### IV. Sequential Auctions: Rationalizing “Hot Potato” Trading

Much of the inter-dealer trading in the foreign exchange markets is done via voice-brokering and has the following feature that was emphasized in the introduction: The customer trades with dealer 1 who trades with dealer 2 who trades with dealer 3, and so on. The quick sequence of bilateral inter-dealer trades following a customer trade is often referred to as “hot potato” trading. Given our finding in Section III that two-stage trading (an unit auction followed by single-price trading) is generally favored over one-shot, single-price trading, a logical question to ask is whether customer welfare is improved by having more trading rounds.

We construct a sequential trading model of a dealership market where the customer first sells his quantity  $\bar{z}$  to one of  $N$  dealers, who then resells a portion of the customer quantity to another dealer, and so on. This continues until there is a total of  $m \geq 4$  dealers left, at which point the dealer who has bought in the previous round resells a fraction of his quantity to the other  $m - 1$  dealers using a uniform-price auction. Hence the selling dealer in each round chooses a quantity to trade rather than a supply curve. Thus, the model in this section differs slightly from that in Section III in that a dealer submits a quantity rather than a supply curve. As we will see, the underlying intuition of restricting supply to raise the price will still hold.

In this section our focus is on the strategic bidding caused by sequential trading; thus we assume all dealers are symmetric in their initial inventory positions (set to zero). In other words, the “bidding effect” is absent here. Also, a dealer who has sold some quantity to another dealer cannot trade again. We will refer to the  $n$ -dealer stage of sequential trading, which eventually ends when it gets down to  $m$  dealers, as an  $(n, m)$  trading game, where  $n \geq m$ . When necessary, we use the superscript  $m$  and subscript  $n$  to denote quantities in the  $(n, m)$  trading game.

At the  $n$ -dealer stage, the dealer who purchased the quantity  $q_{n+1}$  from the previous round resells a portion of it,  $q_n$ , to the other  $n - 1$  dealers at the price of  $p_n(q_n)$ . We denote the selling dealer's expected utility  $U_n$  and the other dealers' expected utility  $V_n$ .

We conjecture that the inter-dealer trading price in an  $(n, m)$  game takes the following form:

$$p_n(q_n) = \bar{v} - \rho\tau_v^{-1}\lambda_n^m q_n. \quad (16)$$

The parameter  $\lambda_n^m$  is an inverse measure of the market liquidity: the lower is  $\lambda_n^m$ , the more liquid is the inter-dealer trading.

PROPOSITION 4. In the sequential trading model above, the liquidity parameter is determined by the following iteration formula:

$$\lambda_{n+1}^m = \frac{\lambda_n^m}{1 + 2\lambda_n^m} + \frac{\lambda_m^m - \frac{1}{2(m-1)}}{(m-1)(1 + 2\lambda_m^m)^2(1 + 2\lambda_{m+1}^m)^2 \dots (1 + 2\lambda_n^m)^2},$$

$$\forall n \geq m \geq 4, \quad (17)$$

with  $\lambda_m^m = (m-2)/[(m-1)(m-3)]$ .

*Proof.* See Appendix III.

The next result characterizes the evolution of market liquidity, the dealers' trading volume and the equilibrium price in the successive rounds of the sequential auction game.

COROLLARY 4. As inter-dealer trading progresses in the  $(n, m)$  trading game (i.e., as  $n$  becomes smaller), market liquidity decreases and the selling dealer retains a larger proportion of the quantity obtained in the previous round. Furthermore, the equilibrium price increases in later rounds.

*Proof.* An inverse measure of market liquidity is  $\lambda_n^m$ . We first prove by induction that  $\lambda_n^m$  is decreasing in  $n$ .

It is straightforward to verify that  $\lambda_{m+1}^m < \lambda_m^m$ . Now by assuming  $\lambda_n^m < \lambda_{n-1}^m$ , we have:

$$\lambda_{n+1}^m = \frac{\lambda_n^m}{1 + 2\lambda_n^m} + \frac{\lambda_m^m - \frac{1}{2(m-1)}}{(m-1)(1 + 2\lambda_m^m)^2(1 + 2\lambda_{m+1}^m)^2 \dots (1 + 2\lambda_n^m)^2}$$

$$< \frac{\lambda_{n-1}^m}{1 + 2\lambda_{n-1}^m} + \frac{\lambda_m^m - \frac{1}{2(m-1)}}{(m-1)(1 + 2\lambda_m^m)^2(1 + 2\lambda_{m+1}^m)^2 \dots (1 + 2\lambda_{n-1}^m)^2}$$

$$= \lambda_n^m.$$

From his optimization problem (see Eq. (A-15) in Appendix III), the selling dealer retains the following quantity:

$$q_{n+1} - q_n = \frac{2\lambda_n^m}{1 + 2\lambda_n^m} q_{n+1}. \quad (18)$$

Thus, the proportion he retains is increasing in  $\lambda_n^m$ . As  $n$  becomes smaller,  $\lambda_n^m$  becomes greater, and therefore, the selling dealer sells less and chooses to retain a greater share.

As for the last statement, multiplying (17) by  $q_{n+1}$  and using Eq. (18), it is easy to show that  $\lambda_n^m q_n$  is increasing in  $n$ . Thus, according to Eq. (16), price increases as trading progresses (i.e., as  $n$  decreases).

Corollary 4 demonstrates that the inter-dealer market becomes more illiquid as trading progresses. This deterioration of liquidity occurs for the following reasons. Mechanically, as sequential trading evolves, fewer prospective buyers remain. Thus, the potential for risk-sharing with the remaining participants diminishes. This has a negative impact on the liquidity of the market. Furthermore, the winning dealers in successive rounds engage in more quantity restriction by selling less and withholding a greater share from the inter-dealer market.

The fact that price increases over time is consistent with the two-stage model in Section III (see Corollary 3). As trading unfolds the sellers get higher prices at the expense of worsening liquidity and declining trading volume.

The effects of sequential inter-dealer trading on market liquidity, transactions volume, and dealer competition are illustrated in Figures 2 and 3.

Corollary 4 describes the evolution of liquidity, prices and volume along an auction sequence where the total number of trading rounds is fixed. To draw a closer parallel to the previous comparison of one-shot trading and two-stage trading, we study the customer revenue when the number of trading rounds is increased.

**COROLLARY 5.** A risk-neutral customer prefers a sequential dealership market with more stages of inter-dealer trading. For any given number of dealers, when there are more rounds of inter-dealer trading, the trading volume is higher.

*Proof.* The customer-dealer trade can be viewed as the first round of trading in the sequential trading game  $(N + 1, m)$ . Thus, the customer's expected revenue is:

$$E[\tilde{R}_{N+1}^m] = E[\tilde{z}p_{N+1}(\tilde{z})] = \int_0^1 (\bar{v} - \rho\tau_v^{-1}\lambda_{N+1}^m z)zg(z)dz. \quad (19)$$

Now consider a sequential auction with one fewer round of inter-dealer trading, i.e., sequential trading begins as an  $(N + 1, m + 1)$  game. In this case, the customer's expected revenue is:

$$E[\tilde{R}_{N+1}^{m+1}] = E[\tilde{z}p_{N+1}(\tilde{z})] = \int_0^1 (\bar{v} - \rho\tau_v^{-1}\lambda_{N+1}^{m+1} z)zg(z)dz.$$

To show that the risk-neutral customer always prefers more rounds of inter-dealer trading, it is sufficient to show that  $E[\tilde{R}_{N+1}^{m+1}] > E[\tilde{R}_{N+1}^m]$ ,

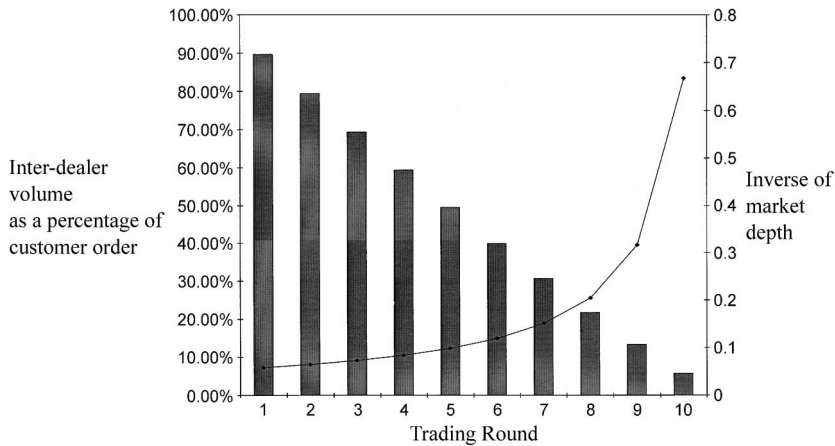


FIG. 2.—The inverse liquidity parameter,  $\lambda$ , and inter-dealer trading volume (indicated by the thick bars) vs the rounds of trading in a sequential inter-dealer auction market ( $N = 12$ ). As trading progresses, the inter-dealer transaction volume as a percentage of the customer order is decreasing, and market liquidity is also decreasing.

or equivalently,  $\lambda_{N+1}^m < \lambda_{N+1}^{m+1}$ . This is established by explicit calculation.

Lastly, from Eq. (A-15) in Appendix III, it is easy to see that  $q_n^m > q_n^{m+1}$ , since  $q_n^m$  is inversely related to  $\lambda_n^m$ .

Corollary 5 states that, starting with a given number of dealers, the more rounds of inter-dealer trading, the higher the customer’s expected revenue. The intuition is related to the “strategic effect” discussed in Section III. In the sequential trading game, dealers restrict quantities at each stage of inter-dealer trading. With more rounds of trading to go, the dealer who wins the customer order will have the incentive to withhold a smaller share. In equilibrium, a higher trading quantity implies higher liquidity, which ultimately benefits the customer. Therefore, extends our earlier result regarding the superiority of two-stage trading over one-shot trading to the multi-stage setting.<sup>25</sup>

Corollary 5 offers an explanation as to why sequential auctions may be beneficial as a trading institution. It demonstrates that more rounds

25. The sequential auction considered here must end when  $m = 4$ , with one dealer trading with three other dealers who share the good equally. An alternative would be to push the sequential auction analogy further and let the selling dealer sell via a unit auction to the three remaining dealers. The dealer who wins would then run a unit auction to sell to the two remaining dealers. In such an end-game, one dealer gets no quantity allocation at all. We find that, for a risk-neutral customer, this new sequential auction ending with a unit auction is preferred to a sequential auction ending with a share auction. This is consistent with our result that more rounds of inter-dealer trading improve customer welfare.

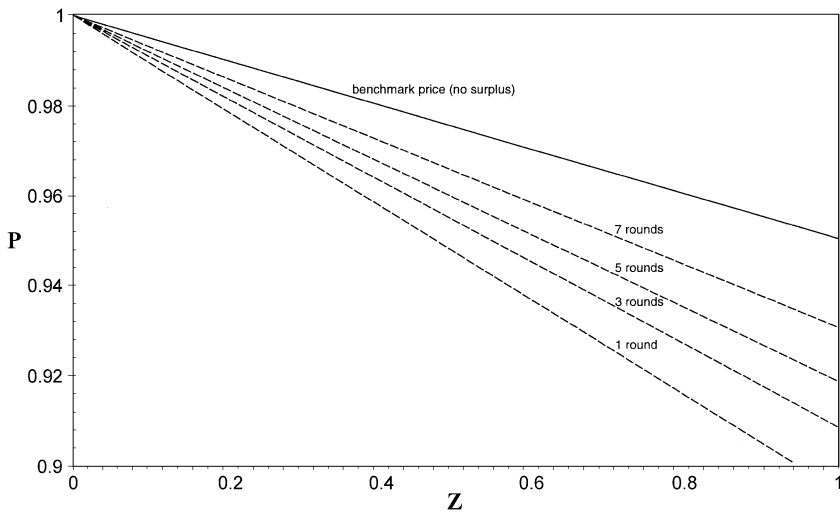


FIG. 3.—Equilibrium price vs customer order size in a sequential inter-dealer auction market ( $N = 10$ ). The solid line is the benchmark price for which there is no trading surplus for the dealers. The dashed lines are the dealers' demand curves with 1, 3, 5, and 7 rounds of trading remaining in the sequential game. With more rounds of inter-dealer trading remaining, dealer competition is more intense and the customer's expected revenue is higher.

of inter-dealer trading leads to higher expected revenue for the customer. Consequently, it provides a rationale for the “hot potato” phenomenon in the foreign exchange market. Numerical calculation shows that the model can generate trading volumes that rival the substantial inter-dealer trading in the foreign exchange market. For example, with 12 dealers and 10 rounds of trading, the total inter-dealer trading volume is 4.59 times the initial customer volume: inter-dealer trading is 82% of total volume. This is in line with the number reported by Lyons (1995) who states that 85% of trading in the foreign exchange market is attributable to inter-dealer trading.

#### V. Inter-Dealer Trading with a Limit-Order Book

While voice-brokering via sequential trading has traditionally been used in the foreign exchange markets, much volume has migrated to electronic limit-order book trading. In particular, as discussed in the introduction, both the EBS partnership and Reuters Dealing 2002 offer systems which have features of a limit-order book. Here we analyze a two-stage model where the inter-dealer competition occurs within a limit-order book which is akin to a discriminatory auction. In our analysis of the limit-order book, we assume that all dealer

inventories are identical,  $I_k = Q$ ,  $k = 1, 2, \dots, N$ . Then, dealer  $k$ 's trading profit is:

$$\begin{aligned}\tilde{\pi}_k^W &= \tilde{v}\tilde{x}_k^W + \tilde{p}'_2(Q + \tilde{z} - x_k^W) + \sum_{m \neq k}^N \int_{\tilde{p}'_2}^{\tilde{p}} \tilde{x}_m^L(\psi) d\psi, \\ \tilde{\pi}_k^L &= \tilde{v}(Q + \tilde{x}_k^L) - \tilde{p}'_2\tilde{x}_k^L - \int_{\tilde{p}'_2}^{\tilde{p}} \tilde{x}_k^L(\psi) d\psi,\end{aligned}$$

where  $\tilde{p}$  is the intercept of the demand schedule with the price axis, and  $\tilde{p}'_2$  is the market clearing price during the inter-dealer trading stage. We emphasize that, to run the inter-dealer market as an anonymous limit-order book, the customer order in the first stage cannot be disclosed to the dealers who do not receive the order in the first stage.

In contrast to the equilibrium in a single-price setting (Section III) which is independent of distributional assumptions, in this section we suppose that the distribution of customer order sizes has a linear hazard ratio. In particular, the pdf and cdf for  $\tilde{z} \in [0, 1]$  are the following:

$$\begin{aligned}g(z) &= \frac{1}{\theta} (1 - z)^{\frac{1}{\theta} - 1}, \\ G(z) &= 1 - (1 - z)^{\frac{1}{\theta}},\end{aligned}$$

where  $\theta$  is a positive parameter related to the first two moments of the distribution as follows:

$$\begin{aligned}E[\tilde{z}] &= \frac{\theta}{1 + \theta}, \\ \text{Var}[\tilde{z}] &= \frac{\theta^2}{(1 + \theta)(1 + 3\theta + 2\theta^2)}.\end{aligned}$$

Note that the case of  $\theta = 1$  corresponds to the uniform distribution.<sup>26</sup>

#### A. Benchmark: A One-Shot Limit-Order Book

For comparison purposes, a benchmark is presented in which the customer can trade directly with  $N$  competing market makers in a one-shot limit-order book. The following is a necessary condition for the equilibrium strategies in a limit-order book market:<sup>27</sup>

$$\sum_{j \neq i}^N \frac{\partial x_j(p)}{\partial p} = - \frac{\theta(1 - z)}{\bar{v} - p - \rho\tau\bar{v}^{-1}[x_i(p) + Q]}.$$

26. The only other distribution that yields a linear solution is the exponential distribution, which is a limiting case of the linear-hazard ratio class studied here. This can be seen by taking the limit of  $G(z) = 1 - (1 - \lambda z)^{\frac{1}{\theta}}$  as  $\theta$  approaches infinity, which is  $G(z) = 1 - e^{-\lambda z}$ .

27. See Viswanathan and Wang (2002) for a derivation.

The unique symmetric, linear solution to the above equation is given by:

$$x_k(p) = \gamma_b \left[ \bar{v} - \rho\tau_v^{-1}Q - \frac{\rho\tau_v^{-1}\theta}{N(1+\theta) - 1} - p \right], \quad \forall k = 1, 2, \dots, N.$$

where

$$\gamma_b = \frac{N(1+\theta) - 1}{(N-1)\rho\tau_v^{-1}}. \quad (20)$$

The equilibrium price and allocations are:

$$\tilde{p}_b = \bar{v} - \rho\tau_v^{-1}Q - \frac{\rho\tau_v^{-1}\theta}{N(1+\theta) - 1} - \frac{(N-1)\rho\tau_v^{-1}}{N(1+\theta) - 1} \frac{\tilde{z}}{N}, \quad (21)$$

$$\tilde{x}_k = \frac{\tilde{z}}{N}.$$

The customer's total revenue in this case is:

$$\tilde{R}_b = \tilde{p}_b \tilde{z} + \sum_{k=1}^N \int_{\tilde{p}_b}^{\tilde{p}} x_k(\psi) d\psi.$$

Note that dealer positions at the end of trading are as follows:

$$I_k + \tilde{x}_k = \frac{\tilde{z}}{N} + Q.$$

### B. The Second Round: Inter-Dealer Trading

With identical *ex ante* inventories, each dealer has an equal probability of winning the customer order in the first stage of trading. Without loss of generality, we designate the dealer who wins the customer order in the first round of trading as dealer *W*, and all other dealers are referred to as dealer *L*. It turns out that dealer *W*'s equilibrium strategy is independent of the distributional assumptions about inventory or customer order size.

PROPOSITION 5. If inter-dealer trading is run as a limit-order book, dealer *W*'s equilibrium strategy is:

$$x^W(p, z) = \begin{cases} \frac{\bar{v} - p}{\rho\tau_v^{-1}} & \text{if } z \in [\underline{z}, 1], \\ Q + z & \text{if } z \in [0, \underline{z}]. \end{cases} \quad (22)$$

That is, dealer  $W$  will sell a nonzero quantity in the inter-dealer market if and only if the customer order he fills in the first stage exceeds the following threshold size:

$$\underline{s} = \frac{2\theta}{(N-1)(1+\theta) + 2\theta + \sqrt{(N-1)^2(1+\theta)^2 - 4\theta}}. \quad (23)$$

*Proof.* See Appendix IV.

The analysis of the optimal strategy for a dealer who did not get to fill the customer order, dealer  $L$ , involves solving a dynamic optimization problem.

PROPOSITION 6. Assume that the dealer inventories all equal  $Q$ . If inter-dealer trading is run as a limit-order book, the trading strategy for dealer  $L$  is:

$$x^L(p) = \mu'_2 - \gamma'_2 p, \quad \forall L \neq W, \quad (24)$$

where

$$\gamma'_2 = \frac{(N-1)(1+\theta) - 2 + \sqrt{(N-1)^2(1+\theta)^2 - 4\theta}}{2(N-2)\rho\tau_v^{-1}}, \quad (25)$$

$$\mu'_2 = \gamma'_2 \left[ \bar{v} - \rho\tau_v^{-1}Q - \frac{2\rho\tau_v^{-1}\theta}{(N-1)(1+\theta) + 2\theta + \sqrt{(N-1)^2(1+\theta)^2 - 4\theta}} \right]. \quad (26)$$

Dealer  $L$  gets a nonzero quantity allocation if and only if the customer order size is greater than  $\underline{s}$ .

*Proof.* See Appendix V.

For  $z \in [\underline{s}, 1]$ , we can solve for the equilibrium price in the second stage as:

$$\tilde{p}'_2 = \bar{v} - \rho\tau_v^{-1}Q - \rho\tau_v^{-1} \frac{\frac{2(N-1)\theta}{(N-1)(1+\theta) + \sqrt{(N-1)^2(1+\theta)^2 - 4\theta}} + z}{1 + (N-1)\rho\tau_v^{-1}\gamma'_2}. \quad (27)$$

Therefore, we can express dealer  $W$ 's quantity allocation after the second stage trading,  $\tilde{x}^W$ , and dealer  $L$ 's acquired quantity from inter-dealer trading,  $\tilde{x}^L$ , as follows:

$$\tilde{x}^W = \frac{\bar{v} - \tilde{p}'_2}{\rho\tau_v^{-1}} \equiv \tilde{X}, \quad (28)$$

$$\tilde{x}^L = \mu'_2 - \gamma'_2 \tilde{p}'_2 - I^L \equiv \tilde{Y} - I^L, \quad \forall L \neq W. \quad (29)$$

Thus, the net positions for the dealers at the conclusion of inter-dealer trading are:

$$\begin{aligned}\tilde{x}^W &= \tilde{X}, \\ I^L + \tilde{x}^L &= \tilde{Y}.\end{aligned}$$

Since

$$\begin{aligned}\tilde{X} - \tilde{Y} &= \frac{4N(N-2)(1+\theta)(1-\tilde{z})}{[(N-1)^2(1+\theta) - 2 + (N-1)\sqrt{(N-1)^2(1+\theta)^2 - 4\theta}]} \\ &\times \frac{1}{[(N-1)(1-\theta) + \sqrt{(N-1)^2(1+\theta)^2 - 4\theta}]} \geq 0,\end{aligned}\quad (30)$$

the winner of the customer order in the first round retains a fraction of the customer order that is at least as large as the other dealers' fraction at the end of inter-dealer trading. Therefore, regardless of the nature of pricing rules (limit-order book vs dealership), the dealer who fills the customer order in the first round always retains a larger share and engages in quantity restriction. The winning dealer does this to raise the price and revenue in the second stage. In equilibrium, this leads to more dealer competition in two-stage trading than in one-shot trading.

For a typical dealer  $k$ , his (certainty equivalent) utility from inter-dealer trading is:

$$\tilde{U}_k^W = \bar{v}\tilde{X} - \frac{\rho\tau_v^{-1}}{2}\tilde{X}^2 + (Q + \tilde{z} - \tilde{X})\tilde{p}'_2 + \frac{1}{2\gamma'_2} \sum_{j \neq k}^N (\tilde{Y} - Q)^2,$$

if he wins the customer order in the first stage. If dealer  $k$  did not win the customer order during the first round of trading, the corresponding utility will be:

$$\tilde{U}_k^L = \bar{v}\tilde{Y} - \frac{\rho\tau_v^{-1}}{2}\tilde{Y}^2 - (\tilde{Y} - Q)\tilde{p}'_2 - \frac{1}{2\gamma'_2}(\tilde{Y} - Q)^2.$$

Therefore,

$$\begin{aligned}\tilde{U}_k^W - \tilde{U}_k^L &= (\bar{v} - \tilde{p}'_2)(\tilde{X} - \tilde{Y}) - \frac{\rho\tau_v^{-1}}{2}(\tilde{X}^2 - \tilde{Y}^2) \\ &\quad + \tilde{p}'_2\tilde{z} + \frac{1}{2\gamma'_2} \sum_{j=1}^N (\tilde{Y} - Q)^2.\end{aligned}\quad (31)$$

**COROLLARY 6.** When inter-dealer trading is run as a limit-order

book, the difference in dealer  $k$ 's second-stage (certainty equivalent) utility between winning and losing the customer order is given by:

$$\begin{aligned} \tilde{U}_k^W - \tilde{U}_k^L = & \\ & \begin{cases} (\bar{v} - \tilde{p}'_2)(\tilde{X} - \tilde{Y}) - \frac{\rho_{\tau_v}^{-1}}{2}(\tilde{X}^2 - \tilde{Y}^2) + \tilde{p}'_2 \tilde{z} + \frac{1}{2\gamma_2} \sum_{j=1}^N (\tilde{Y} - Q)^2 & \text{if } z \in [\underline{s}, 1], \\ \left\{ \bar{v}(Q + \tilde{z}) - \frac{\rho_{\tau_v}^{-1}}{2}(Q + \tilde{z})^2 \right\} - \left\{ \bar{v}Q - \frac{\rho_{\tau_v}^{-1}}{2}Q^2 \right\} & \text{if } z \in [0, \underline{s}]. \end{cases} \end{aligned} \quad (32)$$

### C. The First Round: Customer-Dealer Trading

For any given customer order  $\tilde{z}$ , Corollary 6 shows that all the dealers have the same reservation price,  $\tilde{V} \equiv \tilde{U}_k^W - \tilde{U}_k^L$ , ( $\forall k = 1, 2, \dots, N$ ).

Assume that the dealer inventories are all equal to  $Q$ . If inter-dealer trading is run as a limit-order book, the customer receives the following revenue in the first round of trading:

$$\begin{aligned} \tilde{R}'_1 = \tilde{V} & \\ = & \begin{cases} \frac{\rho_{\tau_v}^{-1}}{2}(\tilde{X} - \tilde{Y})^2 + \tilde{p}'_2 \tilde{z} + \frac{1}{2\gamma_2} \sum_{j=1}^N (\tilde{Y} - Q)^2 & \text{if } z \in [\underline{s}, 1], \\ \bar{v}\tilde{z} - \frac{\rho_{\tau_v}^{-1}}{2}(2Q\tilde{z} + \tilde{z}^2) & \text{if } z \in [0, \underline{s}]. \end{cases} \end{aligned} \quad (33)$$

where the ‘‘excess quantity’’ retained by the winning dealer,  $\tilde{X} - \tilde{Y}$ , is given by Eq. (30), and the equilibrium price during inter-dealer trading is  $\tilde{p}'_2$ , by Eq. (27).

*Proof.* It is straightforward to show:

$$\bar{v} - \tilde{p}'_2 - \frac{\rho_{\tau_v}^{-1}}{2}(\tilde{X} + \tilde{Y}) = \frac{\rho_{\tau_v}^{-1}}{2}(\tilde{X} - \tilde{Y}) \geq 0.$$

Eq. (33) follows from rewriting Eq. (31) using the last equation and Eq. (30).

Next we compare  $\tilde{R}'_1$  with the revenue that a customer receives from a one-shot benchmark model in Section V.A:

$$\tilde{R}_b = \tilde{p}_b \tilde{z} + \frac{1}{2\gamma_b} \sum_{j=1}^N \left( \frac{\tilde{z}}{N} \right)^2. \quad (34)$$

When  $\tilde{z} \in [0, \underline{s}]$ , we can show that  $\tilde{R}'_1 > \tilde{R}_b$ .<sup>28</sup> This can be viewed as an extreme case of quantity restriction by the winning dealer: the winner

28. To see this, we first show that  $\tilde{R}_1 - \tilde{R}_b = F(z)$  is a concave function of  $z$ . We then compute  $F(0) = 0$ ,  $F(\underline{s}) > 0$ , and  $F'(0) > 0$ ; thus  $F(z) > 0, \forall z \in [0, \underline{s}]$ .

gets the customer order and no inter-dealer trading takes place afterwards. In this situation, the customer is strictly better off selling to a single dealer who does not retrade rather than selling to  $N$  dealers simultaneously.

For customer orders that are not too small ( $\tilde{z} \in [\underline{s}, 1]$ ), there will be inter-dealer trading following the initial customer-dealer trade. Comparing the first line of Eq. (33) with Eq. (34), we note that  $\tilde{R}'_1$  has an extra term proportional to  $(\tilde{X} - \tilde{Y})^2$ . This is directly attributable to the winning dealer keeping a larger share of the customer order. A second reason why  $\tilde{R}'_1$  tends to be higher than  $\tilde{R}_b$  is that the equilibrium price during inter-dealer trading is higher than the corresponding price in one-shot limit-order book trading.<sup>29</sup> Finally, comparing the last terms in Eqs. (33) and (34), we find that there is a cost to two-stage trading related to the use of a flatter demand curve. The preceding discussion is summarized in the following result (see Figure 4 for an illustration).

**COROLLARY 7.** Assume that dealer inventories are all equal to  $Q$ . Comparing the customer's revenue from one-shot trading in a limit-order book,  $\tilde{R}_b$ , with the revenue from two-stage trading with a limit-order book,  $\tilde{R}'_1$ , we find that: (i) for small customer order sizes,  $\tilde{z} \in [0, \underline{s}]$ , it is always true that  $\tilde{R}'_1 > \tilde{R}_b$ . For  $\tilde{z} \in [\underline{s}, 1]$ , however, there are the following tradeoffs: (ii) the benefits to two-stage trading come from quantity restriction by the winning dealer and the size of the benefits is small for large outside orders; (iii) there is a cost to two-stage trading because a lesser amount of price discrimination surplus is extracted from the dealers.

As in Section III, the benefits of two-stage trading with a limit-order book are also directly related to the winning dealer's tendency to reduce trading quantity in order to raise price. There are two key differences, however. First, the cost of a two-stage trading with limit-order book trading manifests itself through a reduced amount of price discrimination surplus that the customer can extract from the competing dealers. Second, the benefits of two-stage trading with a book become less important for larger customer order sizes. This is quite different from the situation with a single-price inter-dealer trading, where the strategic effect is proportional to the customer order flow. This difference has its origin in the different ways that strategic bidding (i.e., departure from pricing according to marginal valuation) operates in a single-price auction versus a discriminatory auction: The extent of bid reduction decreases with quantity levels in a limit-order book, whereas it increases with quantity levels in a single-price clearing mechanism.

29. Comparison of Eq. (27) with Eq. (21) shows that:

$$\tilde{p}'_2 - \tilde{p}_b \geq 0.$$

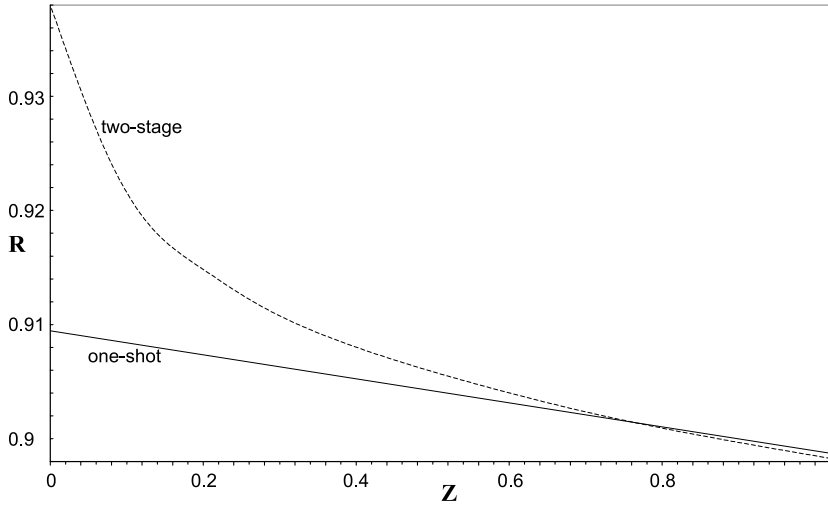


FIG. 4.—Average price received by the customer vs customer order size in a limit-order book market. The solid line is for one-shot trading. The dashed line is for the initial stage of a two-stage trading model. All dealer inventories are identical. The price for two-stage trading is generally higher than its one-shot counterpart at small customer order sizes, although at large customer order sizes the reverse is true.

**VI. Informative Customer Trades**

In this section, we study the impact of private customer information on inter-dealer trading. Conditional on the customer order size,  $\tilde{z}$ , the dealer is assumed to face a simple inference problem:

$$E[\tilde{v}|\tilde{z} = z] = \tilde{v} - \xi \rho \tau_v^{-1} z, \tag{35}$$

where  $\xi$  is a strictly positive parameter, i.e., a larger customer sell order implies a greater downward adjustment to the expected value of the asset.<sup>30</sup> Since our focus here is on asymmetric information, we set all dealer inventories to zero, i.e.,  $I_k = 0, \forall k = 1, 2, \dots, N$ . We assume no disclosure of information about previous trades, which is consistent with trading rules in foreign exchange and other institutional markets.

The above linear updating rule can be exploited to restate the two benchmark one-shot trading models in a convenient way. When the customer order is informative, the only change to the one-shot dealership model is to replace Eq. (1) with:

$$\gamma_d(\xi) = \frac{N - 2}{(N - 1)(N\xi + 1)\rho\tau_v^{-1}}. \tag{36}$$

30. For simplicity, we assume that other aspects of the asset value distribution, such as variance, do not change with  $\tilde{z}$ .

Similarly, in the one-shot limit-order book model (assuming the customer order is uniformly distributed, i.e.,  $\theta = 1$ ), we can use:

$$\gamma_b(\xi) = \frac{2N - 1}{(N - 1)(N\xi + 1)\rho\tau_v^{-1}}, \quad (37)$$

in place of Eq. (20). These results are quite intuitive because, with a worsening adverse selection problem (a larger  $\xi$  value), the dealers bid less aggressively by steepening their demand curves. From the preceding discussion, private customer information tends to make the one-shot trading less competitive.

#### A. Sequential Auctions

Next we explore the effect of private customer information on the sequential auction model. Since a linear updating rule is assumed for the customer-dealer trading stage, we conjecture that in subsequent inter-dealer trading the asset value has a similar correlation structure with the trading quantity. That is:

$$E[\tilde{v}|q_n] = \bar{v} - \rho\tau_v^{-1}\xi_n q_n, \quad (38)$$

in the  $(n, m)$  trading model. Note that, by definition,  $\xi_{N+1} \equiv \xi$  and  $q_{N+1} \equiv z$ .

Suppose inter-dealer trading prices take the form:

$$p_n(q_n) = \bar{v} - \rho\tau_v^{-1}\lambda_n q_n,$$

we have the following result.<sup>31</sup>

**PROPOSITION 8.** In the sequential trading model with private information, the liquidity parameter and information parameter are determined by the following iteration formulas:

$$\lambda_{n+1} = \frac{2\lambda_n(1 + 2\xi_{n+1}) - \xi_{n+1}^2}{2(1 + 2\lambda_n)} + \frac{\lambda_m - \xi_m - \frac{1}{2(m-1)}}{m-1} \left( \frac{\xi_n + 1}{\xi_m} \right)^2, \\ \forall N + 1 \geq n \geq m \geq 4, \\ \xi_{n+1} = \frac{\xi_n}{1 + 2\lambda_n - \xi_n},$$

with  $\lambda_m = (m - 2)[(m - 1)\xi_m + 1]/[(m - 1)(m - 3)]$  and  $\xi_{N+1} = \xi$ .

31. In contrast to Section IV, we omit the superscript  $m$  in this section to reduce the notational complexity. It is understood that the number of dealers in the last stage of the sequential game is  $m$ .

*Proof.* See Appendix VI.

COROLLARY 8. As inter-dealer trading progresses in a sequential trading game (i.e., as  $n$  becomes smaller), both the adverse selection problem and market liquidity worsen (i.e.,  $\xi_{n+1} < \xi_n$  and  $\lambda_{n+1} < \lambda_n$ ).

*Proof.* See Appendix VII.

In contrast to the one-shot models where linear strategy equilibria always exist, with sequential trading the existence of a linear strategy equilibrium is not assured and the presence of informed customer trades may lead to a market breakdown.<sup>32</sup>

PROPOSITION 9. When the informativeness of the order flow is sufficiently high, market breakdown will always occur in sequential auctions. Fixing the total number of dealers, the parameter region with market breakdown expands with the number of trading rounds.

*Proof.* See Appendix VIII.

The intuition for a market breakdown is as follows. Because only the winning dealer observes the customer order flow in the first round, he possesses information about the asset value that other dealers do not have. Recognizing the incentives of the informed dealer to sell a greater share when he perceives a lower asset value, the other dealers respond by steepening their demand curves. In a multi-auction trading environment, information asymmetry worsens along the auction path. For the same level of quantity traded, the dealers in later trading stages infer a lower asset value because this quantity must have resulted from a larger quantity sold by the customer (which is taken as a bad signal).

As trading progresses, market liquidity worsens and the dealers' demand curves become more inelastic. At high enough values of the initial adverse selection parameter  $\xi$ , the inference parameter in the last round ( $\xi_m$ ) becomes negative, indicating the nonexistence of a linear strategy equilibrium. Not surprisingly, the region of breakdown becomes larger when there are more trading rounds and is the smallest when there is one round of inter-dealer trading. The more rounds of trading, the worse is the market liquidity in the last round. With enough initial information asymmetry and enough rounds of trading, we find that the final round of trading collapses. If the initial information asymmetry is sufficiently high, more market breakdown occurs (i.e., market breakdown occurs not just in the last round, but in the final two rounds, final three rounds, etc.) This is discussed in Proposition 9 and illustrated in Figure 5.

Comparing the above result with Corollary 5, we see that tension exists between the strategic advantage implied by running more

32. This "no-trade" result is different from other examples of market breakdown in the literature (see, e.g., Glosten (1989), Bhattacharya and Spiegel (1991)) in that it occurs with two-stage trading but not with one-shot trading.

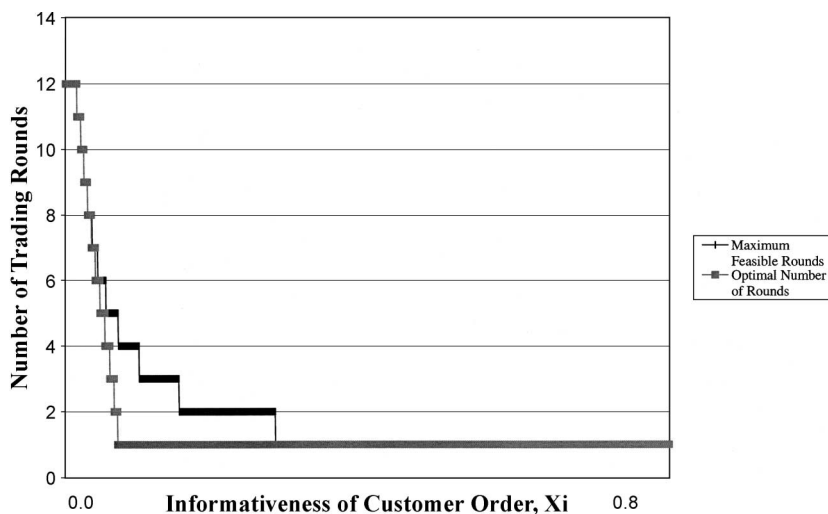


FIG. 5.—The maximum feasible and optimal numbers of trading rounds vs the information parameter,  $\xi$ , in a sequential auction inter-dealer market ( $N = 15$ ). In this figure, the higher “staircase” represents the maximum feasible rounds of trading; the lower “staircase” represents the optimal (from the customer’s perspective) rounds of trading. When there is little private customer information (when  $\xi$  is small), the number of rounds of inter-dealer trading is relatively high and the two staircases coincide. If the extent of private information is very large (e.g., when  $\xi$  is greater than 2), inter-dealer trading will be made impossible (i.e., the two staircases fall to zero level, which is not shown in the figure). At intermediate values of  $\xi$ , the optimal number of rounds is smaller than the maximum feasible rounds of trading.

auctions and the information disadvantage associated with more inter-dealer trading. In other words, without private information, more trading rounds benefit the customer because dealers compete for the opportunity to win and gain higher trading profits in subsequent trading. With private information, more trading rounds exacerbate the information asymmetry problem and have an offsetting effect on customer welfare. This implies that an interior number of rounds can be optimal (see Figure 5 for illustration).

### B. Two-Stage Limit-Order Book Trading

Given the increasing use of limit-order books in the context of inter-dealer trading, it is important to understand how limit-order book trading is affected by the presence of private information. For this purpose, we modify the model in Section V by adding a linear inference problem (Eq. (35)) to the customer-dealer trading stage.

PROPOSITION 10. Assume that customer orders are uniformly distributed over the interval  $[0,1]$ . If a limit-order book is used during

inter-dealer trading, there exists a linear strategy equilibrium in which the strategy for the winning dealer of the customer order is:

$$x^W(p, z) = \begin{cases} \frac{\bar{v} - \xi \rho \tau_v^{-1} z - p}{\rho \tau_v^{-1}} & \text{if } z \in [\underline{s}, 1], \\ z & \text{if } z \in [0, \underline{s}]. \end{cases} \quad (39)$$

The inter-dealer trading strategy for dealer  $L$  is  $x^L(p) = \mu - \gamma p$ , where

$$\begin{aligned} \gamma &= \frac{2(N-2) - N\xi + \sqrt{4N(N-2)(1+\xi) + N^2\xi^2}}{2(N-2)\rho\tau_v^{-1}(1+N\xi)}, \\ \mu &= \frac{\left(1 + \frac{\sigma}{\rho\tau_v^{-1}}\right)\bar{v} - \sigma(1+\xi)}{\rho\tau_v^{-1}(1+\xi) + (N-1)(\rho\tau_v^{-1}\xi - \sigma)}, \\ \underline{s} &= \frac{\gamma\bar{v} - \mu}{\rho\tau_v^{-1}(1+\xi)\gamma}, \\ \sigma &= \frac{1}{(N-2)\gamma + \frac{1}{\rho\tau_v^{-1}}}. \end{aligned} \quad (40)$$

*Proof.* See Appendix IX.

It is interesting to compare the way market makers in different inter-dealer trading systems respond to the problem of private information. In a dealership setting (e.g., a sequential auction), dealers use increasingly inelastic demand curves when adverse selection worsens. This phenomenon eventually leads to a market breakdown. With a limit-order book, however, the winning dealer in the customer-dealer round makes two adjustments in response to asymmetric information: he lowers the intercept of his demand curve, and he decreases the “no-trade” zone (i.e., increasing  $\underline{s}$ ). It turns out that, for customer orders above a certain threshold size, a linear strategy equilibrium always exists in limit-order book trading.

The above result demonstrates that, when inter-dealer trading takes place in a limit-order book, a linear strategy equilibrium exists even when there is a severe adverse selection problem. This contrasts with sequential auction’s susceptibility to private information. It suggests that, in environments where the concentration of informed traders is expected to be high, inter-dealer trading might well take the form of a limit-order book rather than a sequential auction.

### C. The Customer's Expected Revenue

To a large extent, the successes and failures of inter-dealer trading systems are measured by the customer welfare achievable under such systems. Thus, we are interested in comparing the expected customer revenue under both the sequential auction and the limit-order book structure when private information may be an issue. The revenue comparisons in this section are established through numerical computation based on the following relations (for simplification, we set all dealer inventories to be zero).

From Eq. (19), the expected customer revenue in a sequential auction is:

$$E[\tilde{R}_S] = \int_0^1 [\bar{v} - \rho\tau_v^{-1}\lambda_{N+1}z]zg(z)dz. \quad (41)$$

Using (33), the customer's expected revenue in limit-order book trading is:

$$E[\tilde{R}_B] = \int_{\underline{s}}^1 \left[ \frac{\rho\tau_v^{-1}}{2}(X - Y)^2 + pz + \frac{N}{2\gamma}Y^2 \right] g(z)dz + \int_0^{\underline{s}} \left( \bar{v} - \frac{\rho\tau_v^{-1}}{2}z \right) zg(z)dz, \quad (42)$$

where:

$$p = \frac{\bar{v} + (N - 1)\rho\tau_v^{-1}\mu - (1 + \xi)\rho\tau_v^{-1}z}{1 + (N - 1)\rho\tau_v^{-1}\gamma},$$

$$X = \frac{\bar{v} - \xi\rho\tau_v^{-1}z - p}{\rho\tau_v^{-1}},$$

$$Y = \mu - \gamma p.$$

The following results summarize the impact of inter-dealer trading system designs on customer welfare in the absence of asymmetric information. See Figures 6 and 7 for illustrations.

**PROPOSITION 11.** Suppose the customer trades do not carry private information and there is no public disclosure of trades. With two stages of trading, a risk-neutral customer prefers inter-dealer trading in a limit-order book to a sequential auction. With more than two stages of trading, a sequential auction is revenue preferred.

**PROPOSITION 12.** The customer's expected revenues in both the sequential auction and the limit-order book decrease when the customer trades contain more information about the asset's value. When the information asymmetry is significant (i.e., at larger values of  $\xi$ ), a risk-neutral customer favors the two-stage limit-order book over the sequential auction.

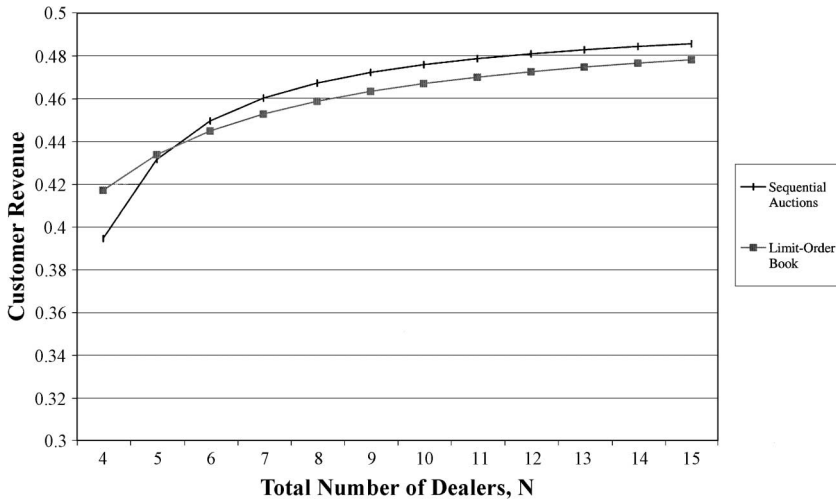


FIG. 6.—The customer’s expected revenue vs the total number of dealers in the absence of private customer information ( $\xi = 0$ ). Except at very small number of dealers, the customer revenue is higher under an inter-dealer trading system with a sequence of auctions than with a limit-order book. We assume that sequential trading takes the maximum rounds of auctions (i.e.,  $m = 4$ ).

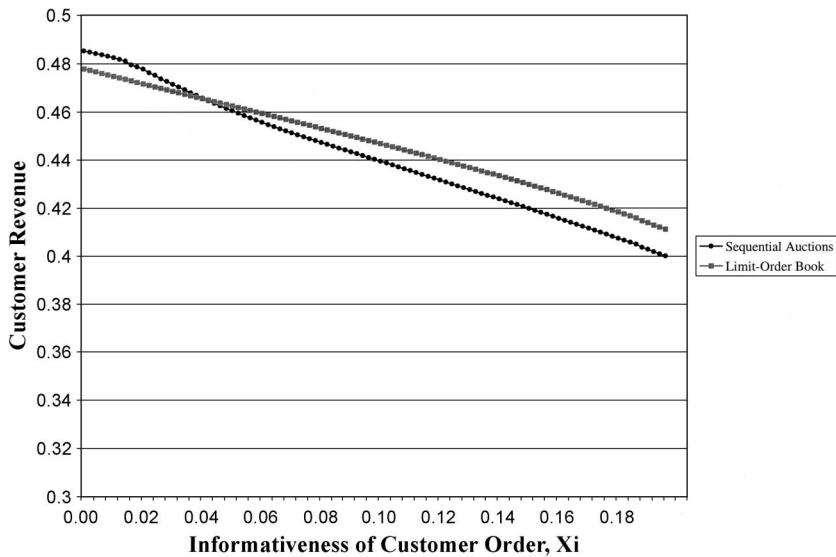


FIG. 7.—The customer’s expected revenue vs the information parameter,  $\xi$ . For the sequential inter-dealer market ( $N = 15$ ), we assume that sequential trading takes the optimal rounds of auctions. At small  $\xi$  values, the customer revenue in sequential trading is higher than the customer revenue from an inter-dealer trading system based on a limit-order book. At higher values of  $\xi$ , however, the customer prefers limit-order book trading.

From Figure 7, it is clear that both  $E[\tilde{R}_S]$  and  $E[\tilde{R}_B]$  are decreasing functions of  $\xi$ . That is, the presence of asymmetric information has a negative impact on customer welfare in the sequential auction and in the limit-order book. However, the extent to which private information adversely affects the customer's expected revenue differs depending on the structure of inter-dealer trading.

When private information is nonexistent or unimportant (zero or small  $\xi$  values), the auction market (two or more stages) tends to be favored by a risk neutral customer over the two-stage limit-order book. When private information is pervasive (large values of  $\xi$ ), the limit-order book is revenue superior to the sequential auction regardless of the number of auction rounds inter-dealer trading may take. Proposition 12 reinforces the notion that a limit-order book is a better venue for inter-dealer trading when there is a severe adverse selection problem.

## VII. Conclusion

In this paper, we study whether multi-stage trading mechanisms that involve inter-dealer trading provide welfare improvement for customers over the one-shot settings traditionally analyzed in the market microstructure literature. Important determinants of such a comparison include the pricing rules in the inter-dealer market, the size and distribution of the customer orders, the information content of the customer orders, and, in the case of sequential auctions, the number of rounds of trading.

We identified a key advantage of multi-stage trading that are absent in one-shot trading environments: the dealer who wins the customer order restricts the quantity he sells in subsequent rounds so as to raise the price. As a result, the winning dealer gets an above average fraction of the customer order, thus providing strong incentives for all dealers to compete more aggressively for the customer order. This intensified competition leads to improved customer welfare relative to a one-shot trading. Thus, multi-stage procedures are better able to deal with "collusive" bidding among market makers that occurs in a one-shot single price mechanism. The combination of an "all or nothing" first stage bidding and an active inter-dealer trading phase serves the dual purpose of facilitating competitive bidding and attaining risk-sharing among the dealers.

We analyzed a sequential auction model that approximates the traditional voice-brokering in foreign exchange trading. In the sequential auction, there is repeated bilateral trading between dealers. A back-of-the-envelope calculation shows that the model can generate inter-dealer volumes consistent with the trading volume reported in the literature. Absent private information in the order flow, sequential auctions yield higher revenue than limit-order books.

When informative order flows, in sequential auctions the winning dealer in any given trading round uses his private information in subsequent rounds, reducing the liquidity of the inter-dealer market. Although in equilibrium all information is revealed, the presence of private information distorts risk-sharing and works against the sequential auction. As a result, the customer's expected revenue in multi-stage markets decreases with increases in the informativeness of the customer order. Of the two kinds of inter-dealer trading, the sequential auction is more susceptible to market breakdowns than is the limit-order book. A market breakdown always occurs in sequential auctions when asymmetric information is high. This argues in favor of using the limit-order book in inter-dealer trading.

Our results provide strong support for inter-dealer mechanisms that use a sequential auction or a limit-order book. The first corresponds to the traditional voice-brokering services in the foreign exchange market (and its electronic successor, the Reuters 2000-1 System) while the second is closer to the electronic limit-order books like the Reuters Dealing 2002 System and the EBS Spot Dealing System.

## Appendix A

### Proof of Proposition 1

Given the information partition of the dealers, we conjecture the following linear equilibrium strategies:

$$x^W = \mu' - \gamma'p + \beta'(I^W + z),$$

$$x^L = \mu - \gamma p - \beta I^L, \forall L \neq W.$$

Note that dealer  $W$  can condition his strategy on  $z$ , but dealer  $L$  cannot because the actual customer order size is only known to dealer  $W$  in the first round of trading.

From the market clearing condition

$$I^W + z = x^W + \sum_{L \neq W}^N x^L,$$

we can back out dealer  $W$ 's residual supply curve:

$$p = \lambda \left[ x^W + (N-1)\mu - I^W - z - \beta \sum_{L \neq W}^N I^L \right],$$

where

$$\lambda = \frac{1}{(N-1)\gamma}. \quad (\text{A-1})$$

Since dealer  $W$  already possesses  $I^W + z$  units of the asset to begin with, he will receive a payment on the  $I^W + z - x^W$  units that he resells to the other dealers during inter-dealer trading. For any given set  $I^L, L \neq W$ , dealer  $W$ 's trading profit is therefore:

$$\hat{\pi}^W = \tilde{v}x^W + p(I^W + z - x^W). \quad (\text{A-2})$$

Expected utility maximization leads to the following first-order condition for dealer  $W$ :<sup>33</sup>

$$0 = \bar{v} - p + \lambda(I^W + z - x^W) - \rho\tau_v^{-1}x^W; \quad (\text{A-3})$$

thus,

$$x^W = \frac{\bar{v} - p + \lambda(I^W + z)}{\rho\tau_v^{-1} + \lambda}. \quad (\text{A-4})$$

Comparing Eq. (A-4) with the conjectured functional form, we have:

$$\mu' = \frac{\bar{v}}{\rho\tau_v^{-1} + \lambda}, \quad (\text{A-5})$$

$$\beta' = \frac{\lambda}{\rho\tau_v^{-1} + \lambda}, \quad (\text{A-6})$$

$$\gamma' = \frac{1}{\rho\tau_v^{-1} + \lambda}. \quad (\text{A-7})$$

Note that the solution obtained does not explicitly depend on  $I^L$ ; thus, it is optimal for all realizations of  $I^L, L \neq W$ .

The analysis of dealer  $L$ 's optimal strategy proceeds along similar lines; the only significant difference is that dealer  $L$  cannot condition his strategy on  $z$ . In the proof below we fix a specific realization of  $\tilde{z}$  and find the trading strategy for dealer  $L$ . Since this strategy is independent of  $z$ , it is an optimal strategy for all possible values of  $\tilde{z}$ .

From dealer  $L$ 's perspective, the market clearing condition

$$I^W + z = x^L + x^W + \sum_{k \neq L, W}^N x_k,$$

can be used to back out his residual supply curve:

$$p = \sigma \left[ x^L + (N - 2)\mu - I^W - z - \beta \sum_{k \neq L, W}^N I_k + \mu' + \beta'(I^W + z) \right],$$

33. Since the objective function is concave in  $x^W$ , the first-order condition is necessary and sufficient.

where

$$\sigma = \frac{1}{\gamma' + (N-2)\gamma}. \quad (\text{A-8})$$

If he acquires  $x^L$  units in inter-dealer trading to augment his initial inventory, dealer  $L$  will have the following trading profit:

$$\tilde{\pi}^L = \tilde{v}(I^L + x^L) - px^L. \quad (\text{A-9})$$

Expected utility maximization leads to the following first-order condition for  $L$ ;

$$0 = \bar{v} - p - \sigma x^L - \rho\tau_v^{-1}(I^L + x^L),$$

thus;

$$x^L = \frac{\bar{v} - p - \rho\tau_v^{-1}I^L}{\rho\tau_v^{-1} + \sigma}. \quad (\text{A-10})$$

Comparing Eq. (A-10) with the conjectured functional form, we have:

$$\mu = \frac{\bar{v}}{\rho\tau_v^{-1} + \sigma}, \quad (\text{A-11})$$

$$\beta = \frac{\rho\tau_v^{-1}}{\rho\tau_v^{-1} + \sigma}, \quad (\text{A-12})$$

$$\gamma = \frac{1}{\rho\tau_v^{-1} + \sigma}. \quad (\text{A-13})$$

The coefficients  $(\mu', \beta', \gamma')$  and  $(\mu, \beta, \gamma)$  can be solved from Eq.s (A-1), (A-5), (A-6), (A-7), (A-8), (A-11), (A-12), and (A-13). The solutions are summarized in the proposition.

## Appendix B

### Proof of Proposition 3

First we prove the following lemma which establishes that it is appropriate to use the difference in certainty equivalent utilities from the second stage (depending on whether the dealer wins or loses in the first round) as the private value for the unit-demand auction in the first round.

LEMMA 1. Optimal trading strategies in the first round can be determined by assuming that each market maker has the private value,  $\tilde{V}_k$ , which is equal to the difference in certainty equivalent utility levels when he wins the customer order and when he does not.

*Proof.* Because a dealer uses a demand curve as his trading strategy, the dealer can condition the total payment he makes to the customer (upon winning) on the size of the customer order. Consequently, in the following we will integrate over the true value of the asset,  $\tilde{v}$ , and other dealers' inventories,  $\tilde{I}_{-k}$ , but not over  $\tilde{z}$ .

Let  $\delta$  be an indicator function which takes on value 1 if dealer  $k$  wins the customer order and value 0 otherwise. Denoting the equilibrium bidding strategy in a second-price auction as  $B_k$  and the equilibrium price in the second-round as  $\tilde{p}_2$ , dealer  $k$ 's objective function in the first round is:

$$\begin{aligned}
& \mathbb{E}_{\tilde{v}, \tilde{I}_{-k}} \left[ -e^{-\rho \{ \delta [\tilde{v} \tilde{x}_k^W + \tilde{p}_2 (I_k + \tilde{z} - \tilde{x}_k^W) - B_k] + (1-\delta) [\tilde{v} (I_k + \tilde{x}_k^L) - \tilde{p}_2 \tilde{x}_k^L] \}} \right] \\
&= \mathbb{E}_{\tilde{v}, \tilde{I}_{-k}} \left[ -e^{-\rho \tilde{v} [\delta \tilde{x}_k^W + (1-\delta)(I_k + \tilde{x}_k^L)] - \rho \delta [\tilde{p}_2 (I_k + \tilde{z} - \tilde{x}_k^W) - B_k] - \rho (1-\delta) \tilde{p}_2 \tilde{x}_k^L} \right] \\
&= \mathbb{E}_{\tilde{I}_{-k}} \left[ -e^{-\rho \{ \tilde{v} [\delta \tilde{x}_k^W + (1-\delta)(I_k + \tilde{x}_k^L)] - \frac{\rho \tau_v^{-1}}{2} [\delta \tilde{x}_k^W + (1-\delta)(I_k + \tilde{x}_k^L)]^2 \}} \right. \\
&\quad \left. - \rho \delta [\tilde{p}_2 (I_k + \tilde{z} - \tilde{x}_k^W) - B_k] - \rho (1-\delta) \tilde{p}_2 \tilde{x}_k^L \right] \\
&= \mathbb{E}_{\tilde{I}_{-k}} \left[ -e^{-\rho \delta (\tilde{U}_k^W - B_k) - \rho (1-\delta) \tilde{U}_k^L} \right] \\
&= \mathbb{E}_{\tilde{I}_{-k}} \left[ -e^{-\rho \tilde{U}_k^L} \right] \mathbb{E}_{\tilde{I}_{-k}} \left[ e^{-\rho \delta (\tilde{U}_k^W - \tilde{U}_k^L - B_k)} \right].
\end{aligned}$$

Notice that  $\tilde{U}_k^L$  is independent of  $\tilde{I}_{-k}$  and thus we are left with an objective function which is proportional to the objective function of a bidder in a standard single-unit auction with private value of  $\tilde{U}_k^W - \tilde{U}_k^L$ .

The above proof assumes that the inter-dealer trading occurs at a single price. It can be easily adapted to show that the certainty equivalent approach is also valid when the customer order is informative about the value of the asset according to Eq. (35).

Suppose dealer  $l$  holds the smallest inventory among all dealers other than  $k$ . Let  $m$  be any dealer who is neither  $k$  nor  $l$ . A result from Milgrom and Weber (1982) (page 1114) states that the equilibrium bidding strategy in a second-price auction as a function of private values,  $B_k(x)$ , is the unique solution to:

$$\mathbb{E} \left[ -e^{-\rho [\tilde{V}_k - B_k(x)]} \mid I_k = x, I_l = x, I_m \leq x, \forall m \neq k, l \right] = -e^{-\rho x}, \quad (\text{A-14})$$

where  $\tilde{V}_k$  is bidder  $k$ 's private value of the asset, given by Eq. (11):

$$\tilde{V}_k = \tilde{z} \left\{ \tilde{v} - \frac{\rho \tau_v^{-1}}{(N-1)^2} \left[ I_k + N(N-2) \tilde{Q} + \left( N - \frac{3}{2} \right) \tilde{z} \right] \right\}.$$

For any given  $\tilde{z}$ , Eq. (A-14) can be inverted to give:

$$\begin{aligned}
 B_k(x) &= -\frac{1}{\rho} \ln E[e^{-\rho \tilde{v}_k} | I_k = x, I_l = x, I_m \geq x] \\
 &= -\frac{1}{\rho} \ln E \left[ e^{-\rho \tilde{z} \left\{ \tilde{v} - \frac{\rho \tau_v^{-1}}{(N-1)^2} \left[ \tilde{I}_k + (N-2) \left( \tilde{I}_k + \tilde{I}_l + \sum_{m \neq k, l} \tilde{I}_m \right) + (N - \frac{3}{2}) \tilde{z} \right] \right\}} \right] \\
 &\quad | I_k = x, I_l = x, I_m \geq x \Big] \\
 &= \tilde{z} \left\{ \tilde{v} - \frac{\rho \tau_v^{-1}}{(N-1)^2} \left[ x + 2(N-2)x + \left( N - \frac{3}{2} \right) \tilde{z} \right] \right\} \\
 &\quad - \frac{N-2}{\rho} \left[ \ln \int_x^{\tilde{I}} e^{-\frac{(N-2)\rho^2 \tau_v^{-1}}{(N-1)^2} \tilde{z} \psi} f(\psi) d\psi \right].
 \end{aligned}$$

Therefore, dealer  $k$ 's bidding strategy (i.e., total payment for a customer order of size  $\tilde{z}$ ) expressed as a function of his inventory is:

$$\begin{aligned}
 B_k(I_k) &= \tilde{z} \left[ \tilde{v} - \frac{\rho \tau_v^{-1}}{(N-1)^2} \left( N - \frac{3}{2} \right) (2I_k + \tilde{z}) \right] \\
 &\quad - \frac{N-2}{\rho} \left[ \ln \int_{I_k}^{\tilde{I}} e^{-\frac{(N-2)\rho^2 \tau_v^{-1}}{(N-1)^2} \tilde{z} \psi} f(\psi) d\psi \right].
 \end{aligned}$$

The seller's expected revenue takes the expectation of the above expression over  $\tilde{I}_k = \tilde{I}_2$  (the second-lowest inventory):

$$\begin{aligned}
 \tilde{R}_1 &= \tilde{z} \left[ \tilde{v} - \frac{\rho \tau_v^{-1}}{(N-1)^2} \left( N - \frac{3}{2} \right) (2\tilde{I}_2 + \tilde{z}) \right] - \frac{N(N-1)(N-2)}{\rho} \\
 &\quad \times \int_{\underline{I}}^{\tilde{I}} \left[ \ln \int_{\eta}^{\tilde{I}} e^{-\frac{(N-2)\rho^2 \tau_v^{-1}}{(N-1)^2} \tilde{z} \psi} f(\psi) d\psi \right] f(\eta) F(\eta) [1 - F(\eta)]^{N-2} d\eta.
 \end{aligned}$$

where  $\tilde{I}_2$  denotes the expected second-lowest inventory level  $N(N-1) \int_{\underline{I}}^{\tilde{I}} \eta f(\eta) F(\eta) [1 - F(\eta)]^{N-2} d\eta$ .

The above explicit solution involves the distribution characteristics of the dealer inventories. In general, the payment function  $B_k$  is not quadratic in the customer

order size  $\tilde{z}$ , i.e., the unit (bid) price is nonlinear in  $\tilde{z}$ .<sup>34</sup> Take the example of exponentially distributed dealer inventories with the parameter  $\mu > 0$ . That is,  $f(\eta) = \mu e^{-\mu\eta}$  and  $F(\eta) = 1 - e^{-\mu\eta}$ .

For a given  $\tilde{z}$ , the customer's expected revenue is:

$$\begin{aligned} \tilde{R}_1 = \tilde{z} & \left\{ \tilde{v} - \frac{\rho\tau_v^{-1}}{(N-1)^2} \left( N - \frac{3}{2} \right) \left[ \frac{2(2N-1)}{\mu N(N-1)} + \tilde{z} \right] \right\} \\ & + \frac{(N-2)}{\rho} \left[ \left( \frac{\mu - \tilde{\kappa}}{\mu} \right) \frac{2N-1}{N(N-1)} - \ln \left( \frac{\mu}{\mu - \tilde{\kappa}} \right) \right], \end{aligned}$$

where we require  $\tilde{\kappa} < \mu$ .

## Appendix C

### Proof of Proposition 4

Analyzing the selling dealer's maximization problem:

$$\max_{q_n} U_n = \tilde{v}(q_{n+1} - q_n) - \frac{\rho\tau_v^{-1}}{2}(q_{n+1} - q_n)^2 + q_n p_n(q_n),$$

provides the following optimal quantities.<sup>35</sup>

$$q_n = \frac{1}{1 + 2\lambda_n^m} q_{n+1}, \quad (\text{A-15})$$

$$U_n = q_{n+1} \left( \tilde{v} - \frac{\lambda_n^m}{1 + 2\lambda_n^m} \rho\tau_v^{-1} q_{n+1} \right). \quad (\text{A-16})$$

Notice that the above expected utility for the selling dealer is net of the price that he paid for the quantity  $q_{n+1}$  obtained in the previous round of trading. For each dealer to be just indifferent between receiving or not receiving  $q_{n+1}$ , it must be that:

$$U_n - q_{n+1} p_{n+1}(q_{n+1}) = V_n.$$

For dealers other than the seller, their expected utility is a constant at any stage of the game, i.e.,  $V_n = \text{Constant}, \forall n$ . To determine this constant, we examine the last inter-dealer trading stage (i.e., when  $n = m$ ). Using Eq. (2), the selling price is:

$$p_m(q_m) = \tilde{v} - \frac{(m-2)\rho\tau_v^{-1}}{(m-3)} \frac{q_m}{m-1} \equiv \tilde{v} - \rho\tau_v^{-1} \lambda_m^m q_m.$$

34. With known dealer inventories, the unit price is linear in  $\tilde{z}$ . See Eq. (12).

35. Since  $\lambda_n^m > 0$ , the second-order condition is always satisfied.

From the above formula,  $\lambda_m^m = \frac{m-2}{(m-1)(m-3)}$ . The expected utility for a dealer who buys in the last round is:

$$\begin{aligned} V_m &= \bar{v} \frac{qm}{m-1} - \frac{\rho\tau_v^{-1}}{2} \left( \frac{qm}{m-1} \right)^2 - p_m(q_m) \frac{qm}{m-1} \\ &= \frac{\lambda_m^m - \frac{1}{2(m-1)}}{m-1} \rho\tau_v^{-1} q_m^2. \end{aligned}$$

Thus, using Eq. (A-15) recursively, we have:

$$\begin{aligned} V_n &= V_m = \frac{\lambda_m^m - \frac{1}{2(m-1)}}{m-1} \rho\tau_v^{-1} q_m^2 = \frac{\lambda_m^m - \frac{1}{2(m-1)}}{m-1} \rho\tau_v^{-1} \left[ \frac{qm+1}{(1+2\lambda_m^m)} \right]^2 \\ &= \dots = \frac{\lambda_m^m - \frac{1}{2(m-1)}}{m-1} \rho\tau_v^{-1} \left[ \frac{qn+1}{(1+2\lambda_m^m)(1+2\lambda_{m+1}^m) \dots (1+2\lambda_n^m)} \right]^2. \end{aligned}$$

Finally, the price at the  $(n+1, m)$  trading game can be determined as follows:

$$\begin{aligned} p_{n+1}(q_{n+1}) &= \frac{1}{q_{n+1}} (U_n - V_n) \\ &= \bar{v} - \frac{\rho\tau_v^{-1} \lambda_n^m}{1+2\lambda_n^m} q_{n+1} - \frac{\lambda_m^m - \frac{1}{2(m+1)}}{m-1} \\ &\quad \times \frac{\rho\tau_v^{-1} q_{n+1}}{(1+2\lambda_m^m)^2 (1+2\lambda_{m+1}^m)^2 \dots (1+2\lambda_n^m)^2} \\ &\equiv \bar{v} - \rho\tau_v^{-1} \lambda_{n+1}^m q_{n+1}. \end{aligned}$$

Thus, we have the following iteration formula for the liquidity parameter in an  $(n+1, m)$  game:

$$\lambda_{n+1}^m = \frac{\lambda_n^m}{1+2\lambda_n^m} + \frac{\lambda_m^m - \frac{1}{2(m-1)}}{(m-1)(1+2\lambda_n^m)^2 (1+2\lambda_{m+1}^m)^2 \dots (1+2\lambda_n^m)^2}.$$

## Appendix D

### Proof of Proposition 5

Suppose the dealer  $L_s$  plays the strategy,  $x^L(p)$ , which does not depend on the realization of  $\tilde{z}$  (only dealer  $W$  observes the true customer order size going into the inter-dealer trading stage).

If dealer  $W$  intends to retain  $x^W < Q + z$  units after the second round of trading, then his inter-dealer trading profit is:

$$\tilde{\pi}^W = \tilde{v}x^W + p(Q + z - x^W) + \sum_{j \neq W}^N \int_p^{\bar{p}} x_j(\psi) d\psi, \quad (\text{A-17})$$

where  $\bar{p}$  is the intercept of dealer  $j$ 's demand schedule with the price axis.

Maximization of dealer  $W$ 's expected utility is equivalent to maximizing the following objective function:

$$\tilde{v}x^W + p(Q + z - x^W) - \frac{\rho\tau_v^{-1}}{2}(x^W)^2 + \sum_{j \neq W}^N \int_p^{\bar{p}} x_j(\psi) d\psi,$$

which leads to the first-order condition for dealer  $W$ :

$$\begin{aligned} 0 &= \tilde{v} - p + \frac{\partial p}{\partial x^W}(Q + z - x^W) - \rho\tau_v^{-1}x^W - \frac{\partial p}{\partial x^W} \sum_{j \neq W}^N x_j(p) \\ &= \tilde{v} - p - \rho\tau_v^{-1}x^W. \end{aligned}$$

The market clearing condition  $Q + z = x^W + \sum_{j \neq W}^N x_j(p)$  is used in the last step.

Thus far, we have proved that the trading strategy:

$$x^W(p) = \frac{\tilde{v} - p}{\rho\tau_v^{-1}},$$

is the equilibrium strategy for dealer  $W$ .

In the proof of Proposition 6, we will show that there exists a unique cutoff customer order size,  $\underline{z}$ , such that dealer  $W$  is a net seller during inter-dealer trading if and only if  $z \in [\underline{z}, 1]$ . Should dealer  $W$  decide to use the strategy  $x^W(p)$  when he observes a  $z$  value that is less than  $\underline{z}$  (based on the incorrect assumption that all other dealers get a positive amount), it can be shown that dealer  $L$ 's quantity allocation would be negative in that case. This cannot happen in equilibrium, therefore, dealer  $W$  retains all of the customer order he fills if it is sufficiently small (i.e., when  $z < \underline{z}$ ).

## Appendix E

### Proof of Proposition 6

This proof consists of three parts. By conjecturing that dealer  $W$  will trade in the inter-dealer market only if the customer order is greater than a threshold size  $0 < \underline{z} < 1$ , we first establish an ordinary differential equation (ODE) as the necessary condition for dealer  $L$ 's equilibrium trading strategies. Then we explicitly solve for a linear solution to the ODE. In the last step, we verify the existence of such a threshold customer order size,  $\underline{z}$ .

In the following we characterize dealer  $i$ 's optimal trading strategy,  $x_i(p)$  (we sometimes write dealer  $i$ 's upward-sloping residual supply curve as  $h(p)$ ) in response to the strategy used by the other dealers,  $x_j(p)$  ( $\forall j \neq i$ ). We use  $g(z)$  and  $G(z)$  to denote the pdf and cdf for the customer order size  $\tilde{z}$  when it falls within  $[\underline{z}, 1]$ .

We can write dealer  $i$ 's uncertain trading profit as follows:

$$\tilde{\pi}_i = \tilde{v}x_i(p) - TP_i, i \neq W,$$

where his total payment is:

$$\begin{aligned} TP_i &= (1 - \alpha)px_i(p) + \int_0^{x_i(p)} p(q)dq \\ &= (1 - \alpha)px_i(p) + \left[ \sum_{j=1}^N \int_0^{x_j(p)} p(q)dq - \sum_{j \neq i}^N \int_0^{x_j(p)} p(q)dq \right] \\ &\equiv (1 - \alpha)px_i(p) + [A - B]. \end{aligned}$$

We note the following relations for use in subsequent calculation:

$$\frac{\partial A}{\partial z} = \sum_{j=1}^N p \frac{\partial x_j}{\partial z} = p \frac{\partial(Q + z)}{\partial z} = p,$$

and

$$\frac{\partial B}{\partial p} = \sum_{j \neq i}^N p \frac{\partial x_j(p)}{\partial p} = -p \frac{\partial h(p)}{\partial p}, \quad (\text{A-18})$$

where we make use of the market clearing condition:

$$Q + z = x^w + \sum_{i \neq W}^N x_i.$$

We assume that dealer  $i$  chooses his optimal trading strategy by maximizing the following derived mean-variance utility function:

$$E_{\tilde{z}} \left[ \tilde{v}h(p) - \frac{\rho\tau_v^{-1}}{2}h^2(p) - TP_i \right].$$

Defining  $A(z)$  as the state variable and  $p(z)$  the control variable, we can analyze the problem using the following Lagrangian:

$$L = g(z) \left[ \tilde{v}h(p) - \frac{\rho\tau_v^{-1}}{2}h^2(p) - A(z) + B(p) \right] + \lambda p.$$

The optimality condition is:

$$0 = \frac{\partial L}{\partial p} = g(z)[\bar{v} - p - \rho\tau_v^{-1}h(p)]\frac{\partial h(p)}{\partial p} + \lambda. \quad (\text{A-19})$$

The adjoint equation is:

$$-\frac{\partial \lambda}{\partial z} = \frac{\partial L}{\partial A} = -g(z), \quad (\text{A-20})$$

with the transversality condition

$$\lambda(\underline{g}) = -1.$$

Using the transversality condition, the adjoint equation can be integrated to obtain

$$\lambda(z) = -[1 - G(z)]. \quad (\text{A-21})$$

Combining Eqs. (A-19) and (A-21), we have:

$$[\bar{v} - p - \rho\tau_v^{-1}h(p)]\frac{\partial h(p)}{\partial p} = \frac{1 - G(z)}{g(z)}, \quad (\text{A-22})$$

which is equivalent to:

$$\sum_{j \neq i}^N x'_j(p) = -\frac{[1 - G(z)]/g(z)}{\bar{v} - p - \rho\tau_v^{-1}x_i(p)}. \quad (\text{A-23})$$

Motivated by dealer  $W$ 's use of a linear bidding strategy, we now search for a linear strategy equilibrium of the form:

$$x_k = \mu - \gamma p, \quad (\text{A-24})$$

for dealers  $k$ . With this, the market clearing condition is:

$$Q + \tilde{z} = x^W + x_k + (N - 2)(\mu - \gamma p).$$

The hazard ratio for  $\tilde{z}$  over the interval  $[\underline{g}, 1]$  is:

$$\frac{1 - G(z)}{g(z)} = \theta(1 - z) = \theta[Q + 1 - x^W - x_k - (N - 2)(\mu - \gamma p)]. \quad (\text{A-25})$$

Plugging Eqs. (22), (A-24), and (A-25) into Eq. (A-23), we obtain:

$$\frac{1}{\rho\tau_v^{-1}} + (N-2)\gamma = \frac{\theta \left[ Q + 1 - \frac{\bar{v} - p}{\rho\tau_v^{-1}} - (N-1)(\mu - \gamma p) \right]}{\bar{v} - p - \rho\tau_v^{-1}(Q + x_k)},$$

which can also be written as:

$$x_k = \frac{1}{\rho\tau_v^{-1}} \left[ \left( 1 + \frac{\theta\sigma}{\rho\tau_v^{-1}} (\bar{v} - p) + \theta\sigma(N-1)(\mu - \gamma p) - \theta\sigma(Q + 1) - \rho\tau_v^{-1}Q \right) \right],$$

with

$$\sigma = \frac{1}{\frac{1}{\rho\tau_v^{-1}} + (N-2)\gamma}.$$

Because of the symmetry among the  $N - 1$  dealers other than dealer  $W$ , we must have:

$$\begin{aligned} \gamma &= \frac{1}{\rho\tau_v^{-1}} \left[ \left( 1 + \frac{\theta\sigma}{\rho\tau_v^{-1}} \right) + \theta\sigma(N-1)\gamma \right], \\ \mu &= \frac{1}{\rho\tau_v^{-1}} \left[ \left( 1 + \frac{\theta\sigma}{\rho\tau_v^{-1}} \right) \bar{v} + \theta\sigma(N-1)\mu - \theta\sigma(Q + 1) - \rho\tau_v^{-1}Q \right]. \end{aligned}$$

The solutions to the above equations are:

$$\begin{aligned} \gamma &= \frac{(N-1)(1+\theta) - 2 + \sqrt{(N-1)^2(1+\theta)^2 - 4\theta}}{2(N-2)\rho\tau_v^{-1}}, \\ \mu &= \gamma \left[ \bar{v} - \rho\tau_v^{-1}Q - \frac{2\rho\tau_v^{-1}\theta}{(N+1)(1+\theta) + 2\theta + \sqrt{(N-1)^2(1+\theta)^2 - 4\theta}} \right]. \end{aligned}$$

Given the above solutions, it can be checked that the following choice of threshold customer order size:

$$\underline{z} = \frac{2\theta}{(N-1)(1+\theta) + 2\theta + \sqrt{(N-1)^2(1+\theta)^2 - 4\theta}} \in (0, 1),$$

is a unique number that satisfies the requirements that  $Q + z \geq x^W$  and  $x_k \geq 0$ ,  $\forall k \neq W$  for  $\bar{z} \in [\underline{z}, 1]$ .

This last statement completes the proofs of Propositions 5 and 6.

## Appendix F

### Proof of Proposition 8

We can analyze the selling dealer's maximization problem:

$$\max_{q_n} U_n = (\bar{v} - \rho\tau_v^{-1}\xi_{n+1}q_{n+1})(q_{n+1} - q_n) - \frac{\rho\tau_v^{-1}}{2}(q_{n+1} - q_n)^2 + q_n p_n(q_n),$$

which provides the following optimal quantities:<sup>36</sup>

$$q_n = \frac{1 + \xi_{n+1}}{1 + 2\lambda_n} q_{n+1} \equiv \delta_n q_{n+1},$$

$$U_n = q_{n+1} \left[ \bar{v} - \frac{2\lambda_n(1 + 2\xi_{n+1}) - \xi_{n+1}^2}{2(1 + 2\lambda_n)} \rho\tau_v^{-1} q_{n+1} \right]. \quad (\text{A-26})$$

Using Eq. (36), the equilibrium strategy in the last stage of the game (where one dealer sells to  $m - 1$  other dealers) is:

$$x_k(\xi) = \gamma_d(\xi)(\bar{v} - p),$$

where

$$\gamma_d(\xi) = \frac{m - 3}{(m - 2)[(m - 1)\xi_m + 1]\rho\tau_v^{-1}}.$$

Thus, the equilibrium price there is:

$$p_m = \bar{v} - \rho\tau_v^{-1}\lambda_m q_m,$$

with

$$\lambda_m = \frac{(m - 2)[(m - 1)\xi_m + 1]}{(m - 1)(m - 3)} \quad (\text{A-27})$$

In addition, the expected utility for those bidding dealers is:

$$V_m = \frac{\lambda_m - \xi_m - \frac{1}{2(m-1)}}{m - 1} \rho\tau_v^{-1} q_m^2.$$

Using Eq. (A-26), the above can be rewritten as:

$$V_n = V_m = \frac{\lambda_m - \xi_m - \frac{1}{2(m-1)}}{(m - 1)} \rho\tau_v^{-1} (q_{m+1} \delta_m)^2$$

$$= \dots = \frac{\lambda_m - \xi_m - \frac{1}{2(m-1)}}{(m - 1)} \rho\tau_v^{-1} (q_{n+1} \delta_m \delta_{m+1} \dots \delta_n)^2.$$

36. The second-order condition,  $1 + 2\lambda_n > 0$ , is satisfied since  $\lambda_n > 0$  in equilibrium.

Note that the expected utility for the nonwinning dealers in each stage must be the same.

Thus, the price at the  $(n + 1)$ -th stage can be determined as follows:

$$\begin{aligned} p_{n+1}(q_{n+1}) &= \frac{1}{q_{n+1}}(U_n - V_n) \\ &= \bar{v} - \frac{2\lambda_n(1 + 2\xi_{n+1}) - \xi_{n+1}^2}{2(1 + 2\lambda_n)} \rho\tau_v^{-1} q_{n+1} \\ &\quad - \frac{\lambda_m - \xi_m - \frac{1}{2(m-1)}}{m-1} (\delta_m \delta_{m+1} \dots \delta_n)^2 \rho\tau_v^{-1} q_{n+1}. \end{aligned}$$

Thus, we derive the following iteration formula for the liquidity parameter:

$$\lambda_{n+1} = \frac{2\lambda_n(1 + 2\xi_{n+1}) - \xi_{n+1}^2}{2(1 + 2\lambda_n)} + \frac{\lambda_m - \xi_m - \frac{1}{2(m-1)}}{m-1} (\delta_m \delta_{m+1} \dots \delta_n)^2. \quad (\text{A-28})$$

Because the optimal quantity  $q_n$  is linear in  $q_{n+1}$  (the quantity traded in the previous round; see Eq. (A-26)),  $q_n$  must be linear in  $z$ , the original customer order. Hence, the information updating is linear:

$$\begin{aligned} \rho\tau_v^{-1} \xi_n &= -\frac{\text{Cov}(v, q_n)}{\text{Var}(q_n)} = -\frac{\text{Cov}(v, \delta_n \delta_{n+1} \dots \delta_N z)}{\text{Var}(\delta_n \delta_{n+1} \dots \delta_N z)} \\ &= -\frac{1}{\delta_n \delta_{n+1} \dots \delta_N} \frac{\text{Cov}(v, z)}{\text{Var}(z)} = \frac{\rho\tau_v^{-1} \xi_{N+1}}{\delta_n \delta_{n+1} \dots \delta_N}. \end{aligned}$$

Therefore, we have:

$$\xi_n = \frac{\xi_{N+1}}{\delta_n \delta_{n+1} \dots \delta_N}. \quad (\text{A-29})$$

Using the last relation, we have:

$$\delta_n = \frac{\xi_{n+1}}{\xi_n}. \quad (\text{A-30})$$

From Eq. (A-26),  $\delta_n$  is defined as:

$$\delta_n = \frac{1 + \xi_{n+1}}{1 + 2\lambda_n}. \quad (\text{A-31})$$

Thus,  $\xi_{n+1}$  can be solved from the last two relations as:

$$\xi_{n+1} = \frac{\xi_n}{1 + 2\lambda_n - \xi_n}, \quad (\text{A-32})$$

which provides the iteration formula for the information parameter of the model. Note that, using Eq. (A-30), we can also rewrite Eq. (A-28) as:

$$\lambda_{n+1} = \frac{2\lambda_n(1 + 2\xi_{n+1}) - \xi_{n+1}^2}{2(1 + 2\lambda_n)} + \frac{\lambda_m - \xi_m - \frac{1}{2(m-1)}}{m-1} \left( \frac{\xi_{n+1}}{\xi_m} \right)^2. \quad (\text{A-33})$$

## Appendix G

### Proof of Corollary 8

We assume that  $\xi_n > 0$ ,  $\forall n$ . Using Eq. (A-32), we note that the result  $\xi_{n+1} < \xi_n$  follows directly from  $\lambda_n > \xi_n$ . Thus, we first prove that  $\lambda_n > \xi_n$  by induction.

From Eq. (A-27), it is clear that  $\lambda_m > c_m$ . Now assume this is true for stage  $n$ , i.e.,  $\lambda_n > \xi_n$ . Then, we also have  $\lambda_n > \xi_n > \xi_{n+1}$ . Using this and Eq. (A-33) (the last term of which is easily shown to be positive), we obtain the following:

$$\begin{aligned} \lambda_{n+1} &> \frac{2\lambda_n(1 + 2\xi_{n+1}) - \xi_{n+1}^2}{2(1 + 2\lambda_n)} = \frac{2\lambda_n + 4\lambda_n\xi_{n+1} - \xi_{n+1}^2}{2(1 + 2\lambda_n)} \\ &> \frac{2\lambda_n + 4\lambda_n\xi_{n+1} - (2\lambda_n)\xi_{n+1}}{2(1 + 2\lambda_n)} = \lambda_n \frac{1 + \xi_{n+1}}{1 + 2\lambda_n} = \lambda_n \frac{\xi_{n+1}}{\xi_n}. \end{aligned}$$

From this, it follows that:

$$\frac{\lambda_{n+1}}{\xi_{n+1}} > \frac{\lambda_n}{\xi_n} > 1,$$

which proves that  $\lambda_{n+1} > \xi_{n+1}$ .

To prove  $\lambda_{n+1} < \lambda_n$ , we introduce the following variable:

$$\chi_n \equiv 2\lambda_n - \xi_n.$$

Using Eqs. (A-31) and (A-33), we have:

$$\begin{aligned} \lambda_{n+1} &= \frac{2\lambda_n(1 + 2\xi_{n+1}) - \xi_{n+1}^2}{2(1 + 2\lambda_n)} + B_m\delta_n \dots \delta_m \\ &= \frac{2\lambda_n(1 + \xi_{n+1}) + \xi_{n+1}[(1 + 2\lambda_n) - (1 + \xi_{n+1})]}{2(1 + 2\lambda_n)} + B_m\delta_n \dots \delta_m \\ &= \lambda_n\delta_n + \frac{\xi_{n+1}}{2}(1 - \delta_n) + B_m\delta_n \dots \delta_m \\ &= \frac{\delta_n}{2}(2\lambda_n - \xi_{n+1}) + \frac{\xi_{n+1}}{2} + B_m\delta_n \dots \delta_m, \end{aligned}$$

where  $B_m > 0$  is short for  $\lambda_m - \xi_m - 1/(2m - 2)]/(m - 1)$ .

Thus,

$$\begin{aligned} 2\lambda_{n+1} - \xi_{n+1} &= \delta_n(2\lambda_n - \xi_{n+1}) + 2B_m\delta_n \dots \delta_m, \\ &= \delta_n(2\lambda_n - \xi_n) + \delta_n(\xi_n - \xi_{n+1}) + 2B_m\delta_n \dots \delta_m, \end{aligned}$$

or:

$$\chi_{n+1} = \delta_n\chi_n + \delta_n(\xi_n - \xi_{n+1}) + 2B_m\delta_n \dots \delta_m,$$

From Eqs. (A-30) and (A-32),

$$\delta_n = \frac{\xi_{n+1}}{\xi_n} = \frac{1}{1 + \chi_n} < 1.$$

Thus,

$$\chi_{n+1} = \frac{\chi_n}{1 + \chi_n}(1 + \xi_{n+1}) + 2B_m\delta_n \dots \delta_m.$$

To prove by induction, we can explicitly verify that  $\chi_{m+1} < \chi_m$ . Furthermore, if we make the induction assumption  $\chi_n < \chi_{n-1}$ , the above equation can be used to show:

$$\chi_{n+1} < \frac{\chi_{n-1}}{1 + \chi_{n-1}}(1 + \xi_n) + 2B_m\delta_{n-1} \dots \delta_m = \chi_n.$$

Having proved  $\chi_{n+1} < \chi_n, \forall n$ , we can rewrite it as:

$$2\lambda_{n+1} - \xi_{n+1} < 2\lambda_n - \xi_n,$$

or

$$2(\lambda_{n+1} - \lambda_n) < \xi_{n+1} - \xi_n < 0.$$

This completes the proof that  $\lambda_{n+1} < \lambda_n$ .

## Appendix H

### Proof of Proposition 9

Substituting Eq. (A-27) into Eq. (A-32), we have:

$$\xi_{m+1} = \frac{1}{\frac{m-1}{m-3} + \frac{m^2-2m-1}{(m-1)(m-3)\xi_m}} < \frac{m-3}{m-1},$$

as long as  $\xi_m$  is positive and  $m \geq 4$ . Using Corollary 8, we then get  $\xi = \xi_{N+1} < \xi_N < \dots < \xi_{m+1} < (m-3)/(m-1)$ .

Thus, for any given sequential auction model, we can find an initial information parameter that violates the above relations (e.g., by choosing  $\xi \geq (m-3)/(m-1)$ ). This shows that a market breakdown will occur when there is a severe adverse selection problem at the customer-dealer trading stage.

As for the second statement, we observe that, with more trading rounds (smaller  $m$  values), there are more  $\xi$  values that satisfy the market breakdown relation  $\xi > (m-3)/(m-1)$ .

## Appendix I

### Proof of Proposition 10

The analysis of inter-dealer trading with a limit-order book under conditions of asymmetric information parallels the previous analysis without informed trades.

First, assume that dealer  $L$  gets a positive quantity allocation from inter-dealer trading; then dealer  $W$  uses the following equilibrium strategy (the derivation is very similar to that in Appendix IV):

$$x^W(p, z) = \begin{cases} \frac{\bar{v} - \xi \rho \tau_v^{-1} z - p}{\rho \tau_v^{-1}} & \text{if } z \in [\underline{z}, 1], \\ z & \text{if } z \in [0, \underline{z}], \end{cases} \quad (\text{A-34})$$

where  $\underline{z} \in [0, 1]$  will be determined later.

As in the analysis in Appendix V, dealer  $i$ 's ( $i \neq W$ ) strategies are described by the following first-order condition:

$$\begin{aligned} \sum_{j \neq i}^N x_j'(p) &= - \frac{[1 - G(z)]/g(z)}{\bar{v} - \xi \rho \tau_v^{-1} z - p - \rho \tau_v^{-1} x_i(p)} \\ &= - \frac{1 - z}{\bar{v} - \xi \rho \tau_v^{-1} z - p - \rho \tau_v^{-1} x_i(p)}. \end{aligned}$$

Conjecturing a linear solution  $x_i(p) = \mu - \gamma p$ , from the market clearing condition we can solve for:

$$z = \frac{\bar{v} - p + \rho \tau_v^{-1} (N-1)(\mu - \gamma p)}{\rho \tau_v^{-1} (1 + \xi)}.$$

Substituting the above into dealer  $i$ 's first-order condition:

$$\frac{1}{\sigma} = \frac{1 - \frac{\bar{v} - \xi \rho \tau_v^{-1} z - p}{\rho \tau_v^{-1}} - (N-1)(\mu - \gamma p)}{\bar{v} - \xi \rho \tau_v^{-1} z - p - \rho \tau_v^{-1} x_i},$$

where

$$\sigma = \frac{1}{(N-2)\gamma + \frac{1}{\rho \tau_v^{-1}}}.$$

Collecting terms to match the original conjecture, we have:

$$-\rho\tau_v^{-1}\gamma = \left(1 + \frac{\sigma}{\rho\tau_v^{-1}}\right) \frac{\xi}{\xi + 1} [1 + \rho\tau_v^{-1}(N-1)\gamma] - (N-1)\sigma\gamma - \left(1 + \frac{\sigma}{\rho\tau_v^{-1}}\right), \quad (\text{A-35})$$

$$\rho\tau_v^{-1}\mu = \left(1 + \frac{\sigma}{\rho\tau_v^{-1}}\right) \left\{ \bar{v} - \frac{\xi}{\xi + 1} [\bar{v} + \rho\tau_v^{-1}(N-1)\mu] \right\} - \sigma + \sigma(N-1)\mu.$$

Therefore, the solutions are:

$$\gamma = \frac{2(N-2) - N\xi + \sqrt{4N(N-2)(1+\xi) + N^2\xi^2}}{2(N-2)\rho\tau_v^{-1}(1+N\xi)},$$

$$\mu = \frac{\left(1 + \frac{\sigma}{\rho\tau_v^{-1}}\right) \bar{v} - \sigma(1+\xi)}{\rho\tau_v^{-1}(1+\xi) + (N-1)(\rho\tau_v^{-1}\xi - \sigma)}.$$

The requirements that  $z \geq x^W$  and  $x_i \geq 0, \forall i \neq W$  are satisfied with the following choice of threshold customer order size:

$$\underline{s} = \frac{\gamma\bar{v} - \mu}{\rho\tau_v^{-1}(1+\xi)\gamma}.$$

Finally, we can verify that  $\lambda > 0$ . Thus, the second-order condition is always satisfied and the solution constitutes an equilibrium strategy.

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