

# Collateral, Risk Management, and the Distribution of Debt Capacity\*

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## Abstract

When financing and risk management are subject to collateral constraints due to limited enforcement, more constrained firms engage in less risk management, and indeed may exhaust their debt capacity and abstain from risk management altogether, consistent with empirical evidence and in contrast to received theory. Conserving debt capacity to seize investment opportunities has an opportunity cost due to forgone investment, which is higher for more productive and less well capitalized firms. In downturns, when cash flows are low, such firms may be unable to seize investment opportunities and instead be forced to downsize, and capital may be less productively deployed as a result.

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We provide a dynamic model of collateralized firm financing in which firms have access to complete markets, subject to collateral constraints, and hence are able to engage in risk management. We study firms' decisions to conserve or exhaust debt capacity and the extent to which firms engage in risk management, and the implications of these decisions for the distribution of debt capacity in the cross section.

Firms may choose to conserve debt capacity to take advantage of future investment opportunities. Our model predicts that firms with less internal funds exhaust their debt capacity rather than conserving it, rendering them unable to seize investment opportunities, while firms with more internal funds conserve some of their debt capacity, allowing them to seize opportunities. Our model moreover implies that the more constrained firms hedge less and may not engage in risk management at all, because the financing needs for investment override hedging concerns. Thus, there is an important connection between firm financing and risk management, since both involve promises to pay by the firm, which are limited by collateral. The prediction of our model for corporate risk management is consistent with the evidence that smaller firms, which are likely more financially constrained, hedge less. This fact is considered a puzzle in the literature, since models, which take up front investment as given, predict that financially constrained firms are effectively risk averse and should hedge (see, for example, Froot, Scharfstein, and Stein (1993)). In contrast, our model suggests the absence of risk management by severely constrained firms should not be considered a puzzle.

The cost of conserving debt capacity is the opportunity cost of foregone investment. Conserving net worth in a state contingent way, by engaging in risk management or arranging for loan commitments, has a similar opportunity cost as it reduces the amount of net worth available for current investment. This opportunity cost is high for firms with few internal funds, because they operate at smaller scale and are hence productive at the margin. Relatedly, when firms differ in their productivity, the opportunity cost is higher for the more productive firms and such firms are hence more likely to exhaust their debt capacity and abstain from risk management.

This has important implications for the distribution of debt capacity which is endogenous in our model. Suppose that in downturns cash flows are low but investment opportunities arise since the price of capital is low at the same time. More productive firms, and firms which are less well capitalized, may be unable to take advantage of such investment opportunities and may indeed be forced to scale down investment during downturns, because their debt capacity is exhausted. Note that this is constrained efficient since firms optimally choose to exhaust their debt capacity and abstain from hedging in our model, and is not due to the fact that firms cannot hedge. In contrast,

less productive and more well capitalized firms are able to use their free debt capacity in such times to expand. The dynamics of the distribution of debt capacity may hence imply that capital is less productively deployed in downturns.

Our model allows us to consider the effect of an increase in firms' ability to collateralize claims. Such an increase raises firms' ability to borrow *ex ante* and hence allows higher leverage, but increased leverage reduces firms' net worth *ex post*. Firms which exhaust their debt capacity may then be forced to scale down investment even more due to their lower net worth *ex post*, that is, the contraction of such firms may be more severe. Thus, higher collateralizability may render the amount of capital deployed by productive and poorly capitalized firms more volatile. The effects highlighted in this paper may hence become more prominent as financial innovation increases the ability to collateralize claims, as it arguably has recently.

Finally, the minimum down payment requirements, or "lending standards," in our model vary endogenously with expected capital appreciation. When the price of capital is expected to rise, down payment requirements are low, because the higher expected collateral value allows greater borrowing. When the price of capital is expected to decline, firms are forced to deleverage. This prediction is empirically plausible and consistent with anecdotal evidence.

The collateral constraints in our model are derived from an explicit dynamic model with limited enforcement, which is the only friction in the model. Our assumptions on limited enforcement imply that firms have access to complete markets in one period ahead state contingent claims subject to state-by-state collateral constraints, that is, they can issue promises to pay against each state up to a fraction of the collateral value in that state. Importantly this allows firms in our model to engage in risk management. Thus, in contrast to most of the literature, we do not make an assumption that aggregate states are not contractible. Our collateral constraints are similar to the ones in Kiyotaki and Moore (1997), except that they are state contingent, and are derived in an economy with limited contract enforcement in the spirit of Kehoe and Levine (1993, 2001, 2008). We assume that firms have limited commitment and can default on their promises to pay and abscond with all cash flows and a fraction of capital. We assume that firms which default can be excluded from neither the market for capital nor from borrowing and lending. Kehoe and Levine and most of the subsequent literature assume instead that borrowers who default are excluded from intertemporal trade.<sup>1</sup> Deriving collateral constraints from a dynamic environment with limited commitment, as we do, allows the explicit analysis of

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<sup>1</sup>A notable exception is Lustig (2007) who considers limited enforcement similar to the one in our model in an endowment economy.

their dynamic effects without requiring “ad hoc” extensions of constraints motivated by a static contracting problem to a dynamic setting. Indeed, we think our model provides a useful framework to address many questions in dynamic corporate finance.

The paper proceeds as follows: Section 1 provides the model of collateral constraints due to limited enforcement, defines debt capacity and financial slack, and discusses the role of loan commitments. Section 2 studies how firms’ decisions to conserve or exhaust debt capacity vary with their productivity and considers the implications for the distribution of debt capacity in the cross section. Section 3 considers the effect of firm net worth on firms’ decisions to exhaust debt capacity and the implications for risk management and contrasts our conclusions to those of received theory. Section 4 discusses the related literature. Section 5 concludes. All proofs are in the Appendix.

## 1 Modeling Collateralized Borrowing

We propose a dynamic model of collateralized borrowing. We consider an economy with limited enforcement which constrains firms’ ability to make credible promises. We show that this economy is equivalent to an economy in which lending is subject to collateral constraints. We define *debt capacity* and *financial slack* explicitly and show how to interpret loan commitments in the context of the model. Finally, we study the dynamics of minimum down payment requirements.

### 1.1 Environment

There are 3 dates, 0, 1, and 2. There is a continuum of agents (of measure 1) which are risk neutral, subject to limited liability, and have preferences over (non-negative) dividends given by

$$E \left[ \sum_{t=0}^2 \beta^t d_t \right], \quad (1)$$

where  $\beta \leq 1$  is the rate of time preference. There are two goods in the economy, consumption goods and capital. Each agent is endowed with  $w_0$  units of the consumption good at time 0 and no capital. Agents also have access to a production technology described below. These agents can be interpreted as entrepreneurs or firms which have a financing need, and we refer to them throughout as *firms*.

The firms have access to a standard neoclassical production technology. An amount of capital  $k_0$  invested at time 0 returns a cash flow  $A_1(s)f_0(k_0)$  in consumption goods at time 1 in state  $s$ , where  $s \in \mathcal{S}$ , as well as the depreciated capital  $(1 - \delta)k_0$  where

$\delta \in (0, 1)$ . Entrepreneurs also have access to a production technology at time 1 which, for an investment of  $k_1(s)$ , returns a cash flow  $A_2(s)f_1(k_1(s))$  in consumption goods at time 2 as well as the depreciated capital  $(1 - \delta)k_1(s)$ . The production function  $f_t(k_t)$  is strictly increasing and (weakly) concave, and productivity is strictly positive in all states, that is,  $A_t(s) > 0, \forall s \in \mathcal{S}, t = 0, 1$ .

We assume that firms vary either in terms of their productivity or net worth. In Section 2 we assume that the production function has constant returns to scale and that firms differ in the productivity  $A_t(s)$  of their technology and analyze how the decision to conserve debt capacity depends on productivity. In Section 3 we assume decreasing returns instead and study the connection between net worth and risk management when firms differ in their time 0 net worth  $w_0$ . Thus, agents in our model are of different types, although the dependence (of productivity or net worth) on type is suppressed throughout.

In addition to the firms described above, there is also a continuum of lenders (of measure 1) in the economy which are unconstrained and risk neutral and discount the future at rate  $\beta \leq 1$  which equals the agents' rate of time preference. Lenders have a large endowment of funds in all dates and states. Lenders cannot run the production technology. Lenders have a large amount of collateral and hence are not subject to enforcement problems but rather are able to commit to deliver on their promises. Lenders are thus willing to provide any state-contingent loan at an expected rate of return  $R = 1/\beta$  subject to firms' enforcement constraints.

We assume that markets are complete but there is limited enforcement; firms can abscond with the cash flows from the production technology and with fraction  $1 - \theta$  of capital. Importantly we assume that entrepreneurs cannot be excluded from future borrowing or the market for capital. We show below that this is equivalent to assuming the following specification of financing constraints: firms can borrow in a state-contingent way, at time  $t$ , up to  $\theta \in (0, 1)$  times the resale value of capital against each state at time  $t + 1$ .<sup>2</sup>

Finally, we assume that consumption goods can be transformed into capital (and vice versa) at a cost  $q_0$  at time 0 per unit of capital and at a cost of  $q_t(s)$  at time  $t \in \{1, 2\}$  in state  $s \in \mathcal{S}$ , where state  $s$  has probability  $\pi(s)$ . We refer to  $q_t(s)$  as the *price* of capital. The assumption that the price of capital is exogenously determined by a technological rate of transformation allows us to focus on the corporate finance implications of our model, whereas much of the literature has focused on the endogenous determination of this price

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<sup>2</sup>Considering capital explicitly and separately from consumption goods is important for two reasons. First, this is the standard assumption and notation in the theory of investment and macroeconomics. Second, this makes the role of collateral in financing explicit which is central to our analysis and delivers a model which is empirically plausible in our view.

(see, most notably, Kiyotaki and Moore (1997)).<sup>3</sup> Moreover, our assumption effectively reduces our model to a one good economy and thus the allocation is constrained efficient.<sup>4</sup>

We do not need to put specific structure on the stochastic process for productivity or the price of capital. Nevertheless, we assume for simplicity that the state  $s$  is realized at time 1 and there is no further uncertainty.<sup>5</sup> One special case that is of particular interest is the case in which capital is relatively cheap when cash flows are low, that is, for all  $s, s^+ \in \mathcal{S}$ ,  $s^+ > s$ ,  $q_1(s^+) > q_1(s)$  but  $A_1(s^+) > A_1(s)$ . This is meant to capture the idea that states with lower  $s$  are states in which there is an economy wide downturn. The downturn implies low cash flows. But in a downturn capital is relatively cheap at the same time which implies that this may be a good time to invest. Thus, this is an important scenario in which cash flows and investment opportunities are negatively correlated. The model allows us to study which firms are likely to be able to take advantage of such opportunities and which firms are likely to be constrained in these times. That said, the model is interesting even in the simpler case in which the price of capital is constant.

## 1.2 Limited Enforcement

Suppose that enforcement of contracts is limited as follows: firms can default on their promises, that is walk away from their debt obligations and abscond with all cash flows and fraction  $1 - \theta$  of capital, and that lenders can seize only fraction  $\theta$  of the capital and do not have access to any other enforcement mechanism. In particular, firms cannot be excluded from further borrowing or from purchasing capital. Thus, enforcement is limited as in Kehoe and Levine (1993) but unlike in their model, firms cannot be excluded from intertemporal markets here.<sup>6,7</sup>

The firm chooses dividends  $\{d_0, d_t(s)\}$ , capital levels  $\{k_0, k_1(s)\}$ , loan amounts  $\{l_0, l_1(s)\}$  and state-contingent repayments  $\{b_t(s)\}$ ,  $\forall s \in \mathcal{S}, t \in \{1, 2\}$ , to maximize the expected

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<sup>3</sup>Endogenizing the price would not change our main conclusions, however.

<sup>4</sup>More specifically, we can show that any distribution of the initial net worth leads to a competitive equilibrium that is constrained efficient, that is, no other allocation can Pareto dominate the competitive allocation and satisfy the collateral constraints.

<sup>5</sup>Rampini and Viswanathan (2009) analyze a stationary version of this model with an infinite horizon and a constant price of capital.

<sup>6</sup>If  $\theta$  were equal to 0, that is, if the firm could abscond with all cash flows and all capital and would not be excluded from future lending, firms could not borrow at all (see Bulow and Rogoff (1989)).

<sup>7</sup>In practice, such exclusion is in fact rather limited. Moreover, this assumption implies a tractable dynamic model of collateral constraints with empirically plausible implications.

discounted value of dividends (1) subject to the budget constraints at time 0, 1, and 2,

$$w_0 + l_0 \geq d_0 + q_0 k_0 \quad (2)$$

$$A_1(s)f_0(k_0) + q_1(s)k_0(1 - \delta) + l_1(s) \geq d_1(s) + q_1(s)k_1(s) + Rb_1(s), \quad \forall s \in \mathcal{S}, \quad (3)$$

$$A_2(s)f_1(k_1(s)) + q_2(s)k_1(s)(1 - \delta) \geq d_2(s) + Rb_2(s), \quad \forall s \in \mathcal{S}, \quad (4)$$

the lender's participation constraint at time 0,

$$E \left[ \sum_{t \in \{1,2\}} R^{-(t-1)} b_t \right] \geq l_0 + E [R^{-1} l_1], \quad (5)$$

the enforcement constraints at time 1 and 2,

$$d_1(s) + \beta d_2(s) \geq \hat{d}_1(s) + \beta \hat{d}_2(s), \quad \forall s \in \mathcal{S}, \quad (6)$$

$$d_2(s) \geq A_2(s)f_1(k_1(s)) + q_2(s)k_1(s)(1 - \theta)(1 - \delta), \quad \forall s \in \mathcal{S}, \quad (7)$$

limited liability constraints, and non-negativity constraints on capital,

$$d_0 \geq 0, \quad d_t(s) \geq 0, \quad k_0 \geq 0, \quad k_1(s) \geq 0, \quad \forall s \in \mathcal{S} \text{ and } t \in \{1, 2\}, \quad (8)$$

where  $\{\hat{d}_t(s)\}_{t \in \{1,2\}}$  are the dividends that the firm could achieve after absconding, that is,  $\{\hat{d}_t(s), \hat{k}_1(s), \hat{b}_2(s)\}_{t \in \{1,2\}}$  maximize

$$\sum_{t \in \{1,2\}} \beta^{t-1} d_t(s) \quad (9)$$

subject to

$$A_1(s)f_0(k_0) + q_1(s)k_0(1 - \theta)(1 - \delta) + b_2(s) \geq d_1(s) + q_1(s)k_1(s), \quad (10)$$

the time 2 budget constraint (4), the time 2 enforcement constraint (7), and the non-negativity constraints (8). The enforcement constraints (6) and (7) require that the firm attains a value, when it keeps its promises, that is at least as high as the value attained by absconding. The firm's problem after absconding at time 1 in state  $s$  is identical to the continuation problem at time 1 in state  $s$ , when it does not default, except that the firm has net worth  $A_1(s)f_0(k_0) + q_1(s)k_0(1 - \theta)(1 - \delta)$  after default, as opposed to net worth  $A_1(s)f_0(k_0) + q_1(s)k_0(1 - \delta) - Rb_1(s)$ , when it does not default.

### 1.3 Collateral Constraints due to Limited Enforcement

We show that the model with limited enforcement is equivalent to a model with state-contingent one period debt subject to state-contingent collateral constraints. Thus, our

model implies intuitive collateral constraints which facilitate the analysis of optimal dynamic collateralized financing.

The equivalence obtains in two steps. First, we show that state-contingent one period debt is sufficient. Intuitively, the enforcement constraints imply that the firm can only credibly promise payment streams with present value less than or equal to the value of capital the firm cannot abscond with. Any long term debt contract which satisfies this restriction can be implemented with a sequence of one period debt contracts. Hence, long term debt is irrelevant.<sup>8</sup> Second, state-contingent one period promises are only credible if they are less than or equal to the value of capital the firm cannot abscond with.<sup>9</sup>

**Proposition 1** *Enforcement constraints (6) and (7) are equivalent to collateral constraints*

$$q_1(s)\theta k_0(1 - \delta) \geq Rb_1(s), \quad \forall s \in \mathcal{S}, \quad (11)$$

$$q_2(s)\theta k_1(s)(1 - \delta) \geq Rb_2(s), \quad \forall s \in \mathcal{S}, \quad (12)$$

where  $b_1(s)$  and  $b_2(s)$  are state-contingent one period debt with  $l_0 = E[b_1]$  and  $l_1(s) = b_2(s)$ ,  $\forall s \in \mathcal{S}$ .

Lustig (2007) considers a similar outside option in an endowment economy and Lorenzoni and Walentin (2007) consider collateral constraints with a similar motivation in an economy with constant returns to scale. The original formulation of the enforcement constraints is in the same spirit as the one used to endogenize debt constraints in Kehoe and Levine (1993), although the limits on enforcement are different here. Kehoe and Levine assume that borrowers who default are excluded from intertemporal markets whereas we assume that firms cannot be excluded. Proposition 1 shows that, given our assumptions about the limits on enforcement, the constraints can equivalently be formulated as collateral constraints in the spirit of Kiyotaki and Moore (1997), but, importantly, are state contingent.

The equivalent formulation has the important advantage that the implementation of the optimal dynamic lending contract is rather simple: firms have access to state-contingent secured loans only.<sup>10</sup> Such lending arrangements are hence decentralized rel-

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<sup>8</sup>In contrast, when firms can be excluded from intertemporal trade, long term contracts are not irrelevant in general.

<sup>9</sup>Note that the productivity at which a firm is able to deploy capital does not affect the collateral constraints. This is a natural consequence of the enforcement problem in our model. A firm's ability to deploy capital productively results in higher cash flows, but the firm is unable to pledge these since it can abscond with them.

<sup>10</sup>Another advantage of this equivalent formulation is that the constraint set (2)-(4), (8), and (11)-(12) is convex. We study this problem henceforth.

atively easily by defining an equilibrium with collateral constraints with trade in state-contingent one period loans which are subject to a state-contingent collateral constraint equal to fraction  $\theta$  times the resale value of capital.<sup>11</sup>

## 1.4 Collateral Constraints

To summarize, we now restate the firm's problem restricting attention to state-contingent one period debt and replacing the enforcement constraints (6) and (7) with the collateral constraints (11) and (12). The firm chooses  $\{d_0, d_t(s)\}$ , capital levels  $\{k_0, k_1(s)\}$ , and state-contingent one period borrowing  $\{b_t(s)\}$  for all  $(s, t) \in \mathcal{S} \times \{1, 2\}$  to maximize (1) subject to the budget constraints,

$$w_0 + E[b_1] \geq d_0 + q_0 k_0 \quad (13)$$

$$A_1(s)f_0(k_0) + q_1(s)k_0(1 - \delta) + b_2(s) \geq d_1(s) + q_1(s)k_1(s) + Rb_1(s), \quad \forall s \in \mathcal{S}, \quad (14)$$

$$A_2(s)f_1(k_1(s)) + q_2(s)k_1(s)(1 - \delta) \geq d_2(s) + Rb_2(s), \quad \forall s \in \mathcal{S}, \quad (15)$$

the collateral constraints (11) and (12), and the limited liability and non-negativity constraints (8). Note that if the firm promises to pay  $Rb_1(s)$  in state  $s$  at time 1, it receives an amount of funds  $\pi(s)b_1(s)$  at time 0. This guarantees the lender an expected return of  $R$  on the loan. Moreover, note that the amount that the firm can credibly promise to repay at time  $t$  in state  $s$  is limited to a fraction  $\theta$  of the resale value of capital in that state.

## 1.5 Definition of Debt Capacity and Financial Slack

Our model allows us to provide a precise definition of debt capacity and financial slack. Given an amount of capital  $k_{t-1}$ , the firm has *debt capacity*  $q_t(s)\theta k_{t-1}(1 - \delta)$  for state  $s$  at time  $t$  and can issue promises  $Rb_t(s)$  up to that amount. By promising less than its debt capacity, the firm can keep *financial slack*  $h_t(s)$  where

$$h_t(s) \equiv q_t(s)\theta k_{t-1}(1 - \delta) - Rb_t(s)$$

for state  $s$  at time  $t$ . Note that using this definition we can write the collateral constraints (11) and (12) simply as non-negative financial slack constraints, that is,

$$h_t(s) \geq 0, \quad \forall s \in \mathcal{S}, t \in \{1, 2\}.$$

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<sup>11</sup>Similarly, Alvarez and Jermann (2000) define an equilibrium with solvency constraints to decentralize optimal allocations in an environment with limited commitment as in Kehoe and Levine (1993). The solvency constraints in their model are agent and state specific in contrast to the simple collateral constraints here.

The firm's net worth in state  $s$  at time  $t$  can now be written in two equivalent ways as

$$\begin{aligned} w_t(s) &= A_t(s)f_{t-1}(k_{t-1}) + q_t(s)k_{t-1}(1 - \delta) - Rb_t(s) \\ &= A_t(s)f_{t-1}(k_{t-1}) + q_t(s)k_{t-1}(1 - \theta)(1 - \delta) + h_t(s). \end{aligned}$$

The firm can conserve net worth in a state-contingent way by not exhausting debt capacity (as the former expression above suggests) or, in other words, by keeping financial slack (as the latter expression suggests). Similarly, not keeping any financial slack is equivalent to exhausting the debt capacity.

Since borrowing against a particular state reduces the firm's net worth in that state, which in turn constrains investment going forward, the firm has a debt overhang problem in the spirit of Myers (1977). However, in our model the firm can optimally choose its debt overhang for each state and we analyze how the firm's choice of its optimal debt overhang for each state depends on the firm's productivity and net worth at time 0.

Finally, a firm's capital  $k_{t-1}$  and hence its debt capacity are of course not exogenous. Rather, the firm chooses its capital  $k_{t-1}$  endogenously, and thus investment, financing, and risk management are all jointly endogenously determined.

## 1.6 The Role of Loan Commitments

In our model, firms have access to one period state-contingent loans or, equivalently, complete markets in one period Arrow claims, subject to collateral constraints. We now show how to interpret loan commitments in the context of our model. In particular, we show that taking out loan commitments is one way firms can conserve debt capacity in a state-contingent way, which is important in practice.

Define a *loan commitment* as a binding agreement to provide a loan of a particular size in state  $s$  at time 1 for a fee paid up front. Clearly, a loan which has zero net present value to the lender when extended at time 1 requires neither ex ante commitment by the lender nor up front fees. Indeed, so far we assume that all loans are of this type, which is without loss of generality given Proposition 1.

Now consider a loan commitment  $\{c_0(s), l_1(s), b_2(s)\}$  in which for an up front fee  $c_0(s)$  to be paid at time 0, the lender agrees to provide a loan  $l_1(s) > b_2(s)$  in state  $s$  at time 1 such that

$$c_0(s) + \pi(s)R^{-1}\{-l_1(s) + R^{-1}Rb_2(s)\} = 0,$$

which means that the loan commitment has zero net present value at time 0 due to competition in the market for loan commitments. In contrast, the net present value to the lender of a loan commitment in state  $s$  at time 1 is  $NPV_1(s) = -l_1(s) + R^{-1}Rb_2(s) < 0$ ,

that is, negative, which is why it does in fact require a commitment. By taking out such a loan commitment, the firm effectively increases its net worth in state  $s$  at time 1 by  $NPV_1(s)$ . However, this comes at an up front cost of  $c_0(s) = -\pi(s)R^{-1}NPV_1(s)$  in terms of fees. But then the firm could equivalently buy state  $s$  Arrow claims in the amount of  $c_0(s)$  which would increase its net worth in state  $s$  at time 1 by the same amount.

The key insight is that lining up loan commitments requires internal funds up front and thus has a cost in terms of reduced investment up front. Arranging for loan commitments or contingent financing is akin to conserving contingent net worth. Firms which choose to exhaust their debt capacity thus do not arrange for loan commitments either.

## 1.7 Dynamics of Minimum Down Payments

This model of collateralized borrowing has the property that the minimum down payment is lower when the price of capital is expected to rise. This property seems empirically plausible and is consistent with anecdotal evidence that down payment requirements (or “lending standards”) vary inversely with expected capital appreciation. To see this, define the minimum down payment  $\varphi_0$  as the minimum amount that a firm needs to pay down per unit of the asset which is the price of the asset minus the collateralizable fraction of the discounted expected resale value, that is, minus the maximum amount that the firm can borrow against the asset,

$$\varphi_0 \equiv q_0 - R^{-1}E[q_1]\theta(1 - \delta),$$

and analogously for  $\varphi_1(s)$ . The minimum down payment as a fraction of the price of capital at time 0, for example, is  $\varphi_0/q_0 \equiv 1 - R^{-1}E[q_1]/q_0\theta(1 - \delta)$  and thus is decreasing in the expected capital appreciation  $E[q_1]/q_0$ . Thus, expectations about future asset prices have an important effect on current down payment requirements. We are not aware of other models that predict such variation in down payment requirements.

## 2 The Distribution of Debt Capacity

In this section we study the distribution of debt capacity and the dynamics of investment by different firms. We also analyze the effect of collateralizability and asset prices on the extent to which constrained firms might downsize, that is, scale down their investment. We show that more productive firms may exhaust their debt capacity since the opportunity cost of conserving debt capacity, which is foregone investment earlier on, is higher for them. This implies that in states where asset prices and cash flows are low, capital

may be less productively deployed on average, since more productive firms, which have exhausted their debt capacity, downsize relative to less productive firms.

## 2.1 Conserve or Exhaust Debt Capacity?

Consider how a firm's decision to conserve or exhaust its debt capacity depends on the firm's productivity. In order to abstract from net worth effects for now, we assume that investment exhibits constant returns to scale, that is,  $f_t(k_t) = k_t$  and hence  $f'_t(k_t) = 1$ . Define the return on the firm's internal funds when it invests by making the minimum down payment (that is, by choosing maximal leverage)  $R_1(k_0, s)$  as

$$R_1(k_0, s) \equiv \frac{A_1(s)f'_0(k_0) + q_1(s)(1 - \theta)(1 - \delta)}{\varphi_0} \quad (16)$$

and define  $R_2(k_1(s), s)$  analogously. With constant returns to scale,  $R_1(k_0, s)$  does not depend on  $k_0$  and we hence simplify the notation to  $R_1(s)$  (and similarly we write  $R_2(s)$  instead of  $R_2(k_1(s), s)$ ). Moreover, we assume that investment at time 1 is sufficiently productive, namely that

**Assumption 1**  $R_2(s) > R, \forall s \in \mathcal{S}$ .

This simplifies the analysis by implying that firms are constrained at time 1 and do not pay dividends before time 2.

Our first main result is that, depending on how productive investment is at time 0, firms either invest as much as they can and exhaust their debt capacity against all states at time 1 or stay on the sidelines at time 0 and conserve all their net worth and debt capacity for state  $s'$  at time 1, at which point they invest the maximal amount. The state  $s'$  is the state where the return is the highest, that is,  $s' \in \arg \max_{s \in \mathcal{S}} R_2(s)$ .<sup>12</sup>

**Proposition 2** *Under Assumption 1 and with constant returns to scale, productive firms invest at time 0 and exhaust their debt capacity, that is, if*

$$E[R_1 R_2] > \max_s \{R R_2(s)\},$$

*then  $k_0 = w_0/\varphi_0$ . Less productive firms stay on the sidelines and conserve their net worth to invest in the most productive state  $s'$  at time 1, that is, if the condition is not met,  $k_0 = 0$ ,  $w_1(s') = R/\pi(s')w_0$ , where  $s' \in \arg \max_s \{R_2(s)\}$ .*

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<sup>12</sup>Given the assumption of constant returns to scale, the bang-bang nature of the solution is expected.

The condition for investment is  $E[R_1 R_2] > \max_s \{RR_2(s)\}$  and thus firms with higher productivity in the first period, say higher  $E[R_1]$ , are more likely to invest and exhaust their debt capacity, all else equal. Moreover, the correlation between returns in the first period and returns in the second period, that is, investment opportunities, of course also matters. Higher autocorrelation of returns makes investment more likely. Hence, firms are more likely to exhaust their debt capacity when returns are more persistent.

## 2.2 Downsizing of Productive Firms

Now consider a firm which invests at time 0 and exhausts its debt capacity. Such a firm may not be able to deploy as much capital at time 1 as it deploys at time 0, thus, it may be “forced to” downsize. This occurs in a state  $s$  in which cash flows  $A_1(s)f_0(k_0)$  are sufficiently low and, importantly, occurs despite the fact that the firm could arrange for contingent financing. The firm chooses not to do so because the opportunity cost is too high.

**Proposition 3** *Firms are “forced to” downsize, that is,  $k_1(s) < k_0$ , if  $A_1(s)$  sufficiently low.*

Proposition 3 implies that productive firms may downsize when less productive firms, which did not previously invest, expand. If firms’ productivity is persistent, average productivity may hence decline in such states.

## 2.3 Effect of Collateralizability on Contraction

When the collateralizability  $\theta$  increases, firms which invest at time 0 may downsize by more. Thus, financial innovation, which increases the collateralizability, may result in more severe contractions of firms which exhaust their debt capacity. This means that the effects we stress in this paper may become even more important over time as the ability to collateralize increases, consistent with recent events in financial markets.

**Proposition 4** *With higher collateralizability, firms which exhaust their debt capacity may be forced to downsize by more. Suppose the parameters are as in Proposition 3 such that  $k_1(s)/k_0 < 1$ . Then  $\frac{\partial}{\partial \theta} (k_1(s)/k_0) < 0$  as long as  $q_1(s)/q_2(s) > k_1(s)/(Rk_0)$ .*

This condition is satisfied for example when  $q_1(s) = q_2(s)$ . A higher  $\theta$  has two effects. First, the firm is able to pledge more funds at time 0 and hence has less net worth left at time 1. Second, the firm has a greater ability to borrow at time 1 going forward and hence requires a smaller down payment requirement in terms of net worth then. The two

effects go in opposite directions, but as long as the price of capital is not too much higher at time 2, the first effect dominates: higher leverage due to higher pledgeability leads to a more severe contraction in capital.

## 2.4 Effect of Asset Prices on Contraction

The price of capital  $q_1(s)$  at time 1 affects the extent of the contraction since it affects both firms' net worth as well as the down payment required to purchase capital:

**Proposition 5** *Firms, which exhaust their debt capacity, downsize by more when asset prices fall by less, that is,  $\frac{\partial}{\partial q_1(s)}(k_1(s)/k_0) < 0$ .*

A higher price of capital at time 1 in state  $s$  has two effects, raising the net worth, since the firm retains fraction  $1 - \theta$  of the resale value of capital, while at the same time raising the down payment requirement  $\varphi_1(s)$ . The second effect dominates the first. The higher the price of capital, the more capital needs to be reduced as more net worth is required to purchase capital.

## 3 Net Worth and Risk Management

This section considers the effect of firm net worth and the implications of our model for risk management. The model shows that there is an important connection between firm financing and risk management. In particular, we show that more financially constrained firms optimally do less risk management, consistent with the data and in contrast to the extant results in the literature. The most constrained firms in our model choose not to hedge, as the financing needs for investment override the hedging concerns.

### 3.1 Role of Firm Net Worth

Firms' net worth plays no role in the analysis of Section 2 due to the assumption of constant returns to scale. To study the effect of firm net worth, we drop this assumption and instead assume decreasing returns to scale.

**Assumption 2**  *$f_t(k_t)$  is strictly increasing, strictly concave, and satisfies  $\lim_{k_t \rightarrow 0} f'_t(k_t) = \infty$  and  $\lim_{k_t \rightarrow \infty} f'_t(k_t) = 0$ .*

To make the point as simply as possible, suppose there are strictly decreasing returns at time 0 only, and that the technology has constant returns to scale at time 1 as before. The firm chooses  $k_0 > 0$  at time 0 and hence has positive net worth in all states at

time 1. Moreover, under Assumption 1, the firm invests all its net worth at time 1 in all states which implies  $k_1(s) > 0$ . If the firm hedges, it conserves net worth for the most productive state at time 1 only. However, the extent to which firms engage in risk management depends on their net worth. Indeed, the firms do not hedge at all if their net worth is below some threshold:

**Proposition 6** *Under Assumption 1 and Assumption 2 at time 0 only, firms with net worth below  $\underline{w}_0$  exhaust their debt capacity and do not engage in risk management, while firms with net worth exceeding the threshold conserve some net worth for the state with the highest productivity at time 1, that is, for state  $s' \in \arg \max_s \{R_2(s)\}$ .*

This result is closely related to the result in Proposition 2 that productive firms exhaust their debt capacity. With decreasing returns, firms with low net worth operate at lower scale and hence are more productive at the margin, leading them to exhaust their debt capacity. Firms which are sufficiently well capitalized, conserve additional net worth for the most productive state at time 1 and keep their time 0 investment unchanged.

We now show that this result obtains generally with decreasing returns to scale in both periods.<sup>13</sup> Firms optimally exhaust their debt capacity against all states and hence abstain from risk management if their net worth is sufficiently low.

**Proposition 7 (Optimal Absence of Risk Management)** *Under Assumption 2 at time 0 and 1, firms with sufficiently low net worth do not engage in risk management.*

Intuitively, for very low net worth, the return on investing the firm's net worth  $R_1(k_0, s)$  becomes so high, that it must eventually exceed the return on conserving debt capacity  $R/\pi(s)$  for all states. For very low net worth the primary concern must be financing investment, not risk management.

Firms which are sufficiently well capitalized, on the other hand, engage in complete risk management at time 0, that is, hedge to the point where the marginal value of net worth is equalized across all states at time 1.

**Proposition 8** *Suppose  $A_2(s)$  is sufficiently high,  $\forall s \in \mathcal{S}$ , such that the firm is constrained in all states at time 1. (i) Capital levels  $k_0$  and  $k_1(s)$ ,  $\forall s \in \mathcal{S}$ , are increasing in net worth  $w_0$  and strictly increasing as long as at least one collateral constraint binds at time 0, that is,  $\lambda_1(s) > 0$  some  $s \in \mathcal{S}$ , and  $k_0$  is constant otherwise. (ii) There exists a threshold level of net worth  $\bar{w}_0$  such that all firms with net worth exceeding the threshold engage in complete risk management at time 0, that is,  $\mu_1(s) = \mu_1(\hat{s})$ ,  $\forall s, \hat{s} \in \mathcal{S}$ .*

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<sup>13</sup>Note that Proposition 7 does not require that the firm is constrained in the second period.

To illustrate Proposition 7 and 8, we compute a numerical example. For simplicity, we assume that there are 10 equally probable states at time 1, that the price of capital is constant in all dates and states, and that the productivity at time 2 is constant across states. Thus, the only reason that investment at time 1 varies across states in this example is because firms may optimally not engage in complete risk management. The parameters of the example and results are reported in Figures 1 and 2. Panel A of Figure 1 shows investment  $k_0$ , state-contingent borrowing  $b_1(s)$ , and financial slack  $h_1(s)$  as a function of the firm’s net worth  $w_0$  at time 0. Investment is strictly increasing in net worth below a threshold and constant above the threshold. The firm exhausts its debt capacity against all states for very low net worth. As the firm’s net worth increases, the firm no longer exhausts its debt capacity against the state with the lowest cash flow at time 1 and instead keeps some financial slack for that state. As the firm’s net worth increases further, the firm starts to conserve debt capacity for progressively more states, until it keeps some slack for all states. If, at a given level of wealth, the firm conserves debt capacity for a particular state, then it conserves debt capacity for all states with lower cash flow than that. Moreover, the firm has the same net worth at time 1 for all states for which it keeps financial slack.

Panel B of Figure 1 shows investment  $k_1(s)$  and state-contingent borrowing  $b_2(s)$  at time 1. Investment is again increasing in time 0 net worth. Borrowing is now strictly increasing in net worth as, given the parameters, the firm is constrained in all states at time 1. The firm has the same net worth in all states for which it keeps financial slack at a given wealth level and hence investment and borrowing are the same in all these states. Figure 2 shows the multipliers on the collateral constraints at time 0 and 1. The time 0 multipliers  $\lambda_1(s)$  are decreasing in time 0 net worth  $w_0$  and drop to 0 one by one as the firm starts to conserve debt capacity for progressively more states at time 1. The time 1 multipliers  $\lambda_2(s)$  are decreasing and strictly positive in our example and, for a given net worth, coincide for all states for which the collateral constraint is slack. The example thus illustrates our main conclusion that more constrained firms do less risk management.

### 3.2 Reconsidering Risk Management

The state-contingent loans in our model allow firms to engage in “corporate risk management.” Conserving state  $s$  contingent debt capacity amounts to buying state  $s$  Arrow claims, that is, partially hedging the amount of net worth in that state. The main theory of risk management, formalized by Froot, Scharfstein, and Stein (1993), is based on the effective risk aversion of firms subject to financial constraints. The rationale for hedging in this theory is that when firms are subject to financial constraints, hedging ensures that

firms have sufficient internal funds to take advantage of investment opportunities. Importantly, this intuition suggests that financially constrained firms should hedge as they are effectively risk averse. In practice, however, large firms, which are arguably less financially constrained, hedge while small firms, which are likely more financially constrained, often do not engage in risk management. Thus, this fact presents an important puzzle from the vantage point of received theory. Our theory resolves this “risk management puzzle,” since it predicts that the more constrained firms, that is, the more productive or less well capitalized firms, exhaust their debt capacity and hence do not hedge. In our model, firms’ ability to credibly promise to pay is limited, and firms have an incentive to hedge net worth in the low state for the usual reasons. However, investment up front is endogenous in our model and the overriding concern may be to finance up front investment. Indeed, the more constrained the firm, the more likely it is that investment financing needs override the hedging concerns. This is the main implication of our model for risk management. Thus, we expect that smaller firms, which are likely more financially constrained, hedge less and, as a result, are more sensitive to aggregate fluctuations than larger firms, consistent with empirical evidence. In contrast, Froot, Scharfstein, and Stein (1993) take up front investment as exogenously given in their model, in effect making risk management the only concern, and thus reach the opposite conclusion.

Indeed, the results in Froot, Scharfstein, and Stein (1993) can be interpreted as a special case of our environment. From time 1 onward, the problem is identical to the problem with decreasing returns described above. In particular, firms are subject to financing frictions, which we model with collateral constraints and Froot, Scharfstein, and Stein (1993) model as convex financing costs. At time 0 however, there is no investment and instead the firm simply has an exogenous stochastic net worth  $w_1(s)$  in state  $s$  at time 1. Moreover, the firm has access to complete frictionless markets at time 0 which allow hedging of stochastic net worth at time 1. There are two critical differences between our model and Froot, Scharfstein, and Stein (1993). First, in our model, hedging is subject to the same collateral constraints as financing itself. Second, we allow for investment in the first period and hence the collateral requirements for financing investment compete with the collateral requirements for hedging. Below we solve the problem in Froot, Scharfstein, and Stein (1993) using our notation and then amend it by imposing collateral constraints on both hedging and financing. Amending the problem further by allowing for investment in the first period brings us back to the general problem with decreasing returns analyzed in Propositions 7 and 8.

The problem in Froot, Scharfstein, and Stein (1993) can be written as follows:

$$\max_{\{k_1(s), b_1(s)\}} \beta^2 E [A_2 f_1(k_1) + k_1(1 - \theta)(1 - \delta)] \quad (17)$$

subject to

$$w_1(s) \geq (1 - R^{-1}\theta(1 - \delta))k_1(s) - Rb_1(s), \quad \forall s \in \mathcal{S}, \quad (18)$$

$$E[b_1] \geq 0, \quad (19)$$

assuming for simplicity that  $A_2(s)$  is sufficiently high in all states such that the collateral constraints bind at time 1 in all states and that  $q_t(s) = 1, \forall s \in \mathcal{S}, t \in \{1, 2\}$ . Note that there is only investment at time 1 ( $k_1(s), s \in \mathcal{S}$ ), that net worth at time 1  $w_1(s)$  is exogenous, and that there are frictionless markets allowing transfers across states ( $b_1(s), s \in \mathcal{S}$ ). The first order conditions with respect to  $b_1(s), s \in \mathcal{S}$ , imply

$$\mu_1(s) = \mu_1(\hat{s}), \quad \forall s, \hat{s} \in \mathcal{S}, \quad (20)$$

that is, the marginal value of net worth is equalized across states. This equation is equivalent to equation (28) in Froot, Scharfstein, and Stein (1993). If furthermore investment opportunities are non-stochastic, that is,  $A_2(s) = A_2, \forall s \in \mathcal{S}$ , then investment is independent of net worth  $k_2(s) = E[w_1]/(1 - R^{-1}\theta(1 - \delta)), \forall s \in \mathcal{S}$ , again paralleling their results. Figure 3 illustrates the prediction of this model in the two state case.

In our environment enforcement is limited in all periods, not just in the second period. With limited enforcement of payments at time 1, the problem in equations (17)-(19) needs to be solved subject to additional collateral constraints on time 1 payments, which in the absence of investment in capital at time 0 simply require that promises are non-positive, that is,

$$0 \geq Rb_1(s), \quad \forall s \in \mathcal{S}. \quad (21)$$

Equations (19) and (21) imply that  $b_1(s) = 0, \forall s \in \mathcal{S}$ , that is, limited enforcement implies no hedging at all.<sup>14</sup> Investment is  $k_1(s) = w_1(s)/(1 - R^{-1}\theta(1 - \delta))$ , and thus higher net worth in state  $s^+$  ( $w_1(s^+) > w_1(s)$ ) implies higher investment  $k_1(s^+) > k_1(s)$ . Moreover, generically the marginal value of net worth is not equalized across states, unlike in Froot, Scharfstein, and Stein (1993). Indeed, if the marginal revenue product given the net worth is lower in higher states, that is,  $A_2(s^+)f'(w_1(s^+)/(1 - R^{-1}\theta(1 - \delta))) < A_2(s)f'(w_1(s)/(1 - R^{-1}\theta(1 - \delta)))$ , then the marginal value of net worth is lower in higher states  $\mu_1(s^+) < \mu_1(s)$  and the collateral constraint will bind for the highest state (at least) but not the lowest state,  $\lambda_1(s_{max}) > 0 = \lambda_1(s_{min})$ . The collateral constraint on the lowest state is slack because the firm is trying to hedge by shifting additional funds into the lowest state. Nevertheless the model predicts what one might call maximal hedging,

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<sup>14</sup>The assumption here that the firm has no capital at time 0 and hence no collateral is for simplicity only. The results can easily be extended to a positive (and exogenous) amount of capital (and hence collateral) at time 0.

in the sense that the firm transfers net worth across states to the maximal extent possible, which is similar to the prediction in Froot, Scharfstein, and Stein (1993).

The prediction is however overturned when the problem is further amended to allow for investment at time 0 as we do above. In this dynamic environment, there are competing demands on the firm's ability to promise. The financing needs for investment may override the hedging concerns. As illustrated in Figure 4 for the case with two states, if the need to shift funds to time 0 is sufficiently strong, then no funds are shifted across states at time 1, so there is no risk management. Indeed, by Proposition 7, financing needs do necessarily override the hedging concerns if a firm's net worth is sufficiently low. Thus, the absence of risk management by severely constrained firms should not be considered a puzzle.

### **3.3 Hedging with Options, Forwards, and Futures**

Our model entails state-contingent borrowing and thus has complete markets for Arrow claims, subject to collateral constraints. These markets for Arrow claims can be implemented with complete options markets.

At first blush, forwards and futures seem to get around the financing constraint as they require no payment at time 0. However, this intuition is misleading, because forwards and futures involve promises to pay at time 1 and such promises are limited by collateral constraints and hence have a shadow cost which in turn is determined by financing needs for investment at time 0. In fact, because ours is a model with complete markets subject to collateral constraints, these different contracts are just different implementations of the unique optimal allocation.

Froot, Scharfstein, and Stein (1993) recognize the importance of these intertemporal aspects of risk management without formally addressing them. They point out that futures contracts may result in substantial margin fluctuations and hence substantial variation in cash available for investment, while forwards do not entail such margin fluctuations but may involve substantial credit risk. Our model provides an explicit analysis of dynamic enforcement constraints and highlights their importance for the understanding of risk management.

### **3.4 Risk Management of Households**

Reinterpreting our model in terms of household finance, the prediction is that less well-off, and hence likely more constrained, households insure less and are more vulnerable to economic downturns. This prediction seems consistent with the evidence in the insurance

literature and certainly with anecdotal evidence. Received theory, by contrast, would again have the prediction that less well-off households insure more, which we think is counterfactual.

## 4 Related Literature

We provide a dynamic model in which both financing and risk management are limited by collateral constraints which are derived explicitly from limits on enforcement. Dynamic models with limited commitment are used extensively in the literature to study optimal risk sharing<sup>15</sup> and asset pricing with heterogeneity,<sup>16</sup> for example. Albuquerque and Hopenhayn (2004) and Hopenhayn and Werning (2007) analyze the implications for dynamic firm financing and Cooley, Marimon, and Quadrini (2004) and Jermann and Quadrini (2007) consider the aggregate implications of firm financing with limited commitment.

The collateral constraints we derive are similar to the ones in Kiyotaki and Moore (1997), albeit in our model they are state contingent. This is important because in our model firms can arrange additional financing contingent on states in which they require funding and would otherwise be constrained, which is the case in practice but is typically ruled out in theoretical models. That is, in our model firms are able to engage in risk management by accessing complete markets, subject to collateral constraints. Kiyotaki and Moore motivate their collateral constraints with an incomplete contracting model based on Hart and Moore (1994) and do not consider state-contingent borrowing. Several authors study models with collateral constraints with a similar motivation as in Kiyotaki and Moore. For example, Krishnamurthy (2003) studies a model in which both borrowers and lenders have to collateralize their promises and considers situations where lenders' collateral is scarce.<sup>17</sup> In contrast, we focus on firms incentives to arrange contingent financing when lenders have abundant funds and collateral. Most closely related to our model are Lorenzoni and Walentin (2007) who study a model with similar collateral constraints. Their focus is on the relation between investment, Tobin's  $q$ , and cash flow, and they do not consider aggregate shocks. Moreover, they restrict attention to the case in which firms always exhaust their debt capacity, whereas we analyze the incentives to conserve debt capacity and the implications for the cross-sectional distribution of debt

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<sup>15</sup>See, e.g., Kocherlakota (1996), Ligon, Thomas, and Worrall (1997), Kehoe and Perri (2002, 2004), and Krueger and Uhlig (2006).

<sup>16</sup>See, e.g., Alvarez and Jermann (2000, 2001), Lustig (2007), and Lustig and van Nieuwerburgh (2007).

<sup>17</sup>See also, Iacoviello (2005) who studies a business cycle model with collateral constraints; and Eisfeldt and Rampini (2007, 2009) who study firm financing subject to collateral constraints.

capacity.

Shleifer and Vishny (1992) study debt capacity and the choice of optimal leverage in a model with aggregate states. They argue that debt may result in forced liquidations in bad times which in turn may limit the leverage that firms choose. They do not consider contingent financing, which is the focus here.

This paper is also related to the emerging literature on contracting models of dynamic firm financing, see Bolton and Scharfstein (1990), Gromb (1994), and, more recently, Clementi and Hopenhayn (2006), DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007a, 2007b), Biais, Mariotti, Plantin, and Rochet (2007), DeMarzo, Fishman, He, and Wang (2007), and Atkeson and Cole (2008) in addition to the papers mentioned above. These papers consider dynamic financing in the presence of private information or moral hazard, whereas we, and the literature discussed above, consider dynamic financing with limited commitment.

Finally, several other roles of collateral have been considered in the literature. When cash flows are private information, collateral may be used to induce borrowers to repay loans (see Diamond (1984), Lacker (2001), and Rampini (2005)). It has also been argued that collateral affects the interest rate that borrowers pay (see Barro (1976)), alleviates credit rationing due to adverse selection (see Bester (1985))<sup>18</sup>, reduces underinvestment problems (see Stulz and Johnson (1992)), provides lenders with an incentive to monitor (see Rajan and Winton (1995)), and renders markets more complete (see Dubey, Geanakoplos, and Shubik (2005) and Geanakoplos (1997)).

## 5 Conclusion

We provide a dynamic model of collateralized lending in which collateral constraints are endogenously derived based on limited enforcement. In the model, firms have access to complete markets, subject to collateral constraints, and thus are able to engage in risk management. We show that there is an important connection between firm financing and risk management since both involve promises to pay by the firm, which are limited by collateral. Our model predicts that firms with low net worth exhaust their debt capacity and hedge less, since financing needs override hedging concerns, consistent with the empirical evidence. In contrast, this evidence is considered a puzzle from the vantage point of the standard theory of risk management, which takes investment as given.

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<sup>18</sup>See also Chan and Kanatas (1985), Besanko and Thakor (1987a, b), and Chan and Thakor (1987), who study the role of collateral in models with adverse selection, and Berger and Udell (1995) and Boot, Thakor, and Udell (1991), who study the role of collateral in models with moral hazard.

The cost of conserving debt capacity is the opportunity cost of foregone investment. This cost is higher for firms with low net worth since they operate at smaller scale and are hence more productive at the margin. When firms differ in their productivity, more productive firms are more constrained and hence exhaust their debt capacity rather than keeping financial slack to take advantage of future investment opportunities. This has important implications for the cross-sectional distribution of debt capacity, which is endogenous in our model. In downturns, when cash flows are low but investment opportunities arise because the price of capital is low, more productive and less well capitalized firms may hence not be able to seize these opportunities because their debt capacity is exhausted. Indeed, due to their lack of financial slack, they may be forced to scale down investment in such times. More productive and less well capitalized firms are hence likely more vulnerable to economic downturns since they optimally keep less financial slack. As a result, capital may be less productively deployed in such times.

Higher collateralizability allows firms to borrow more ex ante and thus increases leverage, but leaves them with less net worth ex post. When capital is more collateralizable, firms which exhaust their debt capacity may hence be forced to scale down investment by more. Thus, the amount of capital deployed by more productive and low net worth firms may be more volatile in that case. If collateralizability increases over time, as arguably it has recently, the effects stressed in this paper become even more important.

# Appendix

**Proof of Proposition 1.** To show that considering state-contingent one period debt is sufficient, note that  $Rb_2(s)$  is the total payment from the firm to the lender at time 2, and there is no need to distinguish payments due to funds lent at time 0 ( $l_0$ ) and at time 1 in state  $s$  ( $l_1(s)$ ). Moreover, the program only determines the net payment  $Rb_1(s) - l_1(s)$ ,  $\forall s \in \mathcal{S}$ , see equations (3) and (5), and thus we are free to set  $l_1(s) = b_2(s)$ ,  $\forall s \in \mathcal{S}$ . Equation (5) then simplifies to  $E[b_1] \geq l_0$  and using the fact that this equation holds with equality we can substitute for  $l_0$ .

Next we show that equations (6) and (7) are equivalent to (11) and (12). Notice that (4) holds with equality due to non-satiation. Substituting for  $d_2(s)$  in (7) using (4) and canceling terms implies (12). Conversely, (12) together with (4) at equality implies (7).

To obtain (11), assume that  $Rb_1(s) > q_1(s)\theta k_0(1-\delta)$ . Let  $X(s) \equiv \{d_t(s), k_1(s), b_2(s)\}_{t \in \{1,2\}}$  be the allocation from time 1 onward in state  $s$ . Consider default at time 1 to an allocation  $X'(s) = X(s)$ . Note that (4) implies

$$\begin{aligned} A_1(s)f_0(k_0) &+ q_1(s)k_0(1-\theta)(1-\delta) + b_2(s) \\ &> A_1(s)f_0(k_0) + q_1(s)k_0(1-\delta) - Rb_1(s) + b_2(s) \geq d_1(s) + q_1(s)k_1(s), \end{aligned}$$

and hence  $X'(s)$  is feasible. Moreover  $d'_1(s)$  can be increased which violates (6), a contradiction. Conversely, (11) implies that the optimal allocation after default,  $\hat{X}(s)$  say, is a feasible allocation and hence the contractual allocation  $X(s)$  must attain at least that value, implying that (6) is satisfied.  $\square$

**Proof of Proposition 2.** The first order conditions of the problem of maximizing (1) subject to (8) and (11)-(15), which are necessary and sufficient, are

$$\mu_0 = 1 + \nu_0^d, \quad (22)$$

$$\mu_t(s) = \beta^t + \nu_t^d(s), \quad \forall t \in \{1, 2\}, \forall s \in \mathcal{S}, \quad (23)$$

$$\mu_0 = R\mu_1(s) + R\lambda_1(s), \quad \forall s \in \mathcal{S}, \quad (24)$$

$$\mu_1(s) = R\mu_2(s) + R\lambda_2(s), \quad \forall s \in \mathcal{S}, \quad (25)$$

$$q_0\mu_0 = E[(A_1f'_0(k_0) + q_1(1-\delta))\mu_1 + q_1\theta(1-\delta)\lambda_1] + \nu_0^k \quad (26)$$

$$q_1(s)\mu_1(s) = (A_2(s)f'_1(k_1(s)) + q_2(s)(1-\delta))\mu_2(s) + q_2(s)\theta(1-\delta)\lambda_2(s) + \nu_1^k(s), \quad \forall s, \quad (27)$$

where  $\pi(s)\lambda_1(s)$ ,  $\pi(s)\lambda_2(s)$ ,  $\mu_0$ ,  $\pi(s)\mu_1(s)$ , and  $\pi(s)\mu_2(s)$  are the multipliers on constraints (11)-(15), and  $\nu_0^d$ ,  $\nu_t^d(s)$ ,  $\nu_0^k$ , and  $\nu_1^k(s)$  are the multipliers on the constraints in (8).

Using the return definitions (16) and equations (24) and (25), (26) and (27) can be written as

$$\mu_0 = E[R_1(k_0)\mu_1] + \frac{1}{\wp_0}\nu_0^k \quad (28)$$

$$\mu_1(s) = R_2(k_1(s), s)\mu_2(s) + \frac{1}{\wp_1(s)}\nu_1^k(s). \quad (29)$$

Using (23), (25), (29), and Assumption 1,  $R\mu_2(s) + R\lambda_2(s) = \mu_1(s) \geq R_2(s)\mu_2(s) > R\mu_2(s)$  and thus  $\lambda_2(s) > 0$ ,  $\forall s \in \mathcal{S}$ . Moreover,  $\mu_0 \geq R\mu_1(s) \geq R^2(\mu_2(s) + \lambda_2(s)) >$

$R^2\mu_2(s) \geq R^2\beta^2 = 1$ . Then (22) and (23) imply  $\nu_0^d > 0$  and  $\nu_1^d(s) > 0, \forall s \in \mathcal{S}$ , that is, dividends at time 0 and time 1 are zero.

Since  $d_1(s) = 0$  and using (3) and (12) at equality we have  $k_1(s) = w_1(s)/\wp_1(s)$ , where  $w_1(s) \equiv A_s(s)k_0 + q_1(s)k_0(1 - \delta) - Rb_1(s)$  is the net worth at time 1 in state  $s$ . Moreover, (4) and (12) at equality imply that  $d_2(s) = (A_2(s) + q_2(s)(1 - \theta)(1 - \delta))k_1(s)$  and hence the value attained by a firm at time 1 in state  $s$  with a net worth  $w_1(s)$  is  $V_1(w_1(s), s) = d_1(s) + \beta d_2(s) = \beta R_2(s)w_1(s), \forall s \in \mathcal{S}$ .

Suppose  $k_0 = 0$ . Then  $w_1(s) = -Rb_1(s)$  and using above characterization we have

$$V_0(w_0) \equiv \max_{\{b_1(s)\}_{s \in \mathcal{S}}} \beta^2 E[-RR_2b_1]$$

subject to  $w_0 \geq -E[b_1]$  and  $-Rb_1(s) \geq 0, \forall s \in \mathcal{S}$ . If  $s' \in \arg \max_s \{R_2(s)\}$ , then  $b_1(s') = -w_0/\pi(s')$ ,  $w(s') = R/\pi(s')w_0$ , and  $V_0(w_0) = \beta^2 RR_2(s')w_0$ .

Suppose  $k_0 > 0$ . Then  $w_1(s) \geq (A_1(s) + q_1(s)(1 - \theta)(1 - \delta))k_0 > 0$ , which implies that  $k_1(s) > 0$  (and  $\nu_1^k(s) = 0$ ) and  $d_2(s) > 0$  (and  $\mu_2(s) = \beta^2$ ). From (29),  $\mu_1(s) = \beta^2 R_2(s)$ , and (24) and (28) can be written as

$$\mu_0 = \beta^2 RR_2(s) + R\lambda_1(s), \quad \forall s \in \mathcal{S}, \quad (30)$$

$$\mu_0 = \beta^2 E[R_1R_2]. \quad (31)$$

Since  $\lambda_1(s) \geq 0$ , equations (30) and (31) imply  $E[R_1R_2] \geq RR_2(s), \forall s \in \mathcal{S}$ , and hence  $E[R_1R_2] \geq \max_s \{RR_2(s)\}$ . Moreover, the case where the inequality is an equality is not generic and hence generically  $\lambda_1(s) > 0, \forall s \in \mathcal{S}$ . But then (11) implies  $b_1(s) = R^{-1}q_1(s)\theta k_0(1 - \delta)$  and (2) implies  $k_0 = w_0/\wp_0$ . Using the characterization of the second period problem above we get  $V_0(w_0) = \beta^2 E[R_1R_2]w_0$ .

Thus, if  $E[R_1R_2] > \max_s \{RR_2(s)\}$ ,  $k_0 > 0$  attains a higher value and the optimal  $k_0$  and value attained are as stated in the proposition. Otherwise,  $k_0 = 0$  attains a higher value and is hence optimal.  $\square$

**Proof of Proposition 3.** Suppose  $E[R_1R_2] > \max_s \{RR_2(s)\}$ . Then, by Proposition 2,  $k_0 = w_0/\wp_0 > 0$ ,  $w_1(s) = (A_1(s) + q_1(s)(1 - \theta)(1 - \delta))k_0$ , and  $k_1(s) = w_1(s)/\wp_1(s)$ . Thus,  $k_1(s)/k_0 = (A_1(s) + q_1(s)(1 - \theta)(1 - \delta))/\wp_1(s)$ , which is less than 1 as long as  $A_1(s) < q_1(s)\delta + (q_1(s) - R^{-1}q_2(s))\theta(1 - \delta)$ . Any  $A_1(s) < \min\{q_1(s)\delta, q_1(s) - R^{-1}q_2(s)(1 - \delta)\}$  satisfies this condition. Moreover, the condition is satisfied for some  $A_1(s) \geq 0$  as long as  $q_1(s) - R^{-1}q_2(s)(1 - \delta) > 0$ .  $\square$

**Proof of Proposition 4.** Note that  $\frac{\partial}{\partial \theta} (k_1(s)/k_0) \propto ((q_2(s)/q_1(s))k_1(s)/(Rk_0) - 1) < 0$  as long as the condition in the statement of the proposition is satisfied.  $\square$

**Proof of Proposition 5.** Differentiating  $k_1(s)/k_0$  with respect to  $q_1(s)$  gives

$$\frac{\partial}{\partial q_1(s)} \left( \frac{k_1(s)}{k_0} \right) = \frac{(1 - \theta)(1 - \delta)}{\wp_1(s)} \left( 1 - \frac{\frac{A_1(s)}{(1 - \theta)(1 - \delta)} + q_1(s)}{q_1(s) - R^{-1}q_2(s)\theta(1 - \delta)} \right) < 0 \quad \square$$

**Proof of Proposition 6.** Under Assumption 1 and with decreasing returns to scale at time 0,  $\nu_0^k = \nu_1^k(s) = 0$  and (27) reduces to  $\mu_1(s) = \mu_2(s)R_2(s)$ . Hence,  $\lambda_2(s) > 0$ ,  $\forall s \in \mathcal{S}$ , which together with (25) and (24) implies that  $\nu_1^d(s) > 0$ ,  $\forall s \in \mathcal{S}$ , and  $\nu_0^d > 0$ . Since  $k_1(s) > 0$ ,  $\forall s \in \mathcal{S}$ ,  $\nu_2^d(s) = 0$  and  $\mu_2(s) = \beta^2$ , implying that  $\mu_1(s) = \beta^2 R_2(s)$ . From (25),  $\mu_0 = \beta^2 R R_2(s) + R \lambda_1(s)$  and thus  $\lambda_1(s') = 0$  at most for state  $s'$  such that  $s' \in \arg \max_s R_2(s)$ . Thus, the firm hedges at most the state with the highest productivity.

Suppose now that  $\lambda_1(s) = 0$ ,  $\forall s \in \mathcal{S}$ , and thus  $k_0 = w_0/\varphi_0$  and (28) reduces to

$$\mu_0 = \beta^2 E [R_1(w_0/\varphi_0)R_2] \quad (32)$$

and for  $s'$  (24) reduces to

$$\mu_0 = \beta^2 R R_2(s'). \quad (33)$$

By strict concavity, (32) is strictly decreasing in  $w_0$ , and, given the assumptions on the production function, goes to  $+\infty$  as  $w_0$  goes to 0, and goes to 0 as  $w_0$  goes to  $+\infty$  while (33) is constant. Thus, there is a  $\underline{w}_0$  such that (32) and (33) coincide and  $\lambda_1(s') > (=) 0$  for  $w_0 < (\geq) \underline{w}_0$ .  $\square$

**Proof of Proposition 7.** Using (11) and (13), capital  $k_0$  can be bounded above as  $k_0 \leq w_0/\varphi_0$ , and hence as  $w_0 \rightarrow 0$ ,  $k_0 \rightarrow 0$ . Since  $\lim_{k_0 \rightarrow 0} f'_0(k_0) \rightarrow \infty$ ,  $R_1(k_0, s) \rightarrow \infty$  as  $w_0 \rightarrow 0$ ,  $\forall s \in \mathcal{S}$ . From (28),

$$1 = E \left[ R_1(k_0) \frac{\mu_1}{\mu_0} \right] = \sum_{s \in \mathcal{S}} \pi(s) R_1(k_0, s) \frac{\mu_1(s)}{\mu_0} \geq \pi(s) R_1(k_0, s) \frac{\mu_1(s)}{\mu_0}$$

and thus as  $w_0 \rightarrow 0$ ,  $\mu_1(s)/\mu_0 \rightarrow 0$ , and, from (24),  $\lambda_1(s)/\mu_0 = R^{-1} - \mu_1(s)/\mu_0 \rightarrow R^{-1}$  implying that  $\lambda_1(s) > 0$ ,  $\forall s \in \mathcal{S}$ .  $\square$

**Proof of Proposition 8.** Part (i): If  $\lambda_1(s) > 0$ ,  $\forall s \in \mathcal{S}$ , then  $k_0 = w_0/\varphi_0$  and  $k_1(s) = (A_1(s)f(k_0) + q_1(s)k_0(1 - \theta)(1 - \delta))/\varphi_1(s)$  and the result is trivial. Suppose  $\exists \hat{s} \in \mathcal{S}$  such that  $\lambda(\hat{s}) = 0$ . Then dividing  $\mu_0 = \beta^2 E [R_1(k_0)R_2(k_1)]$ , from (28), by  $\mu_0 = \beta^2 R R(k_1(\hat{s}), \hat{s})$ , from (24), we obtain

$$R = \sum_{\{s|\lambda_1(s)>0\}} \pi(s) R_1(k_0, s) \frac{R_2(k_1(s), s)}{R_2(k_1(\hat{s}), \hat{s})} + \sum_{\{s|\lambda_1(s)=0\}} \pi(s) R_1(k_0, s) \frac{R_2(k_1(s), s)}{R_2(k_1(\hat{s}), \hat{s})}. \quad (34)$$

Suppose that  $w_0^+ > w_0$  and that the optimal capital levels  $k_0^+ \leq k_0$ . Then for  $s$  such that  $\lambda_1(s) > 0$ ,  $k_1^+(s) \leq k_1(s)$  while for  $s$  such that  $\lambda_1(s) = 0$   $k_1^+(s) > k_1(s)$  since more net worth must have been conserved for these states. Observe that  $R_1(k_0^+, s) \geq R_1(k_0, s)$ , that for  $s$  such that  $\lambda_1(s) > 0$ ,  $R_2(k_1^+(s), s) \geq R_2(k_1(s), s)$ , while for  $s$  such that  $\lambda_1(s) = 0$ ,  $R_2(k_1^+(\hat{s}), \hat{s}) < R_2(k_1(\hat{s}), \hat{s})$ . Noting that for  $s$  such that  $\lambda_1(s) = 0$  the ratio  $R_2(k_1(\hat{s}), \hat{s})/R_2(k_1(\hat{s}), \hat{s}) = 1$  at both  $w_0$  and  $w_0^+$ , the right hand side of (34) strictly increases as long as  $\{s|\lambda_1(s) > 0\}$  is non-empty, a contradiction. If  $\lambda_1(s) = 0$ ,  $\forall s \in \mathcal{S}$ , then  $R = E[R_1(k_0)]$  and is hence constant.

Part **(ii)**: Suppose that, at  $w_0$ ,  $\lambda_1(s) = 0, \forall s \in \mathcal{S}$ , but that at  $w_0^+ > w_0$ ,  $\exists \hat{s} \in \mathcal{S}$ , such that  $\lambda_1^+(\hat{s}) > 0$ . Then  $R\mu_1^+(s)/\mu_1^+ \leq 1$  with strict inequality at  $\hat{s}$ , which together with (28) implies that

$$R = E \left[ R_1(k_0^+) \frac{R\mu_1^+}{\mu_0^+} \right] = \sum_{s \in \mathcal{S}} \pi(s) R_1(k_0^+, s) \frac{R\mu_1^+(s)}{\mu_0^+} < \sum_{s \in \mathcal{S}} \pi(s) R_1(k_0^+, s)$$

and thus  $k_0^+ < k_0$ . This implies that

$$k_1^+(\hat{s}) = \frac{A_1(\hat{s})f(k_0^+) + q_1(\hat{s})k_0^+(1 - \theta)(1 - \delta)}{\wp_1(\hat{s})} < \frac{A_1(\hat{s})f(k_0) + q_1(\hat{s})k_0(1 - \theta)(1 - \delta)}{\wp_1(\hat{s})} \leq k_1(\hat{s}),$$

and hence

$$\mu_0^+ = \beta^2 R R_2(k_1^+(\hat{s}), \hat{s}) + R \lambda_1^+(\hat{s}) > \beta^2 R R_2(k_1^+(\hat{s}), \hat{s}) > \beta^2 R R_2(k_1(\hat{s}), \hat{s}) = \mu_0,$$

which contradicts concavity of the value function.  $\square$

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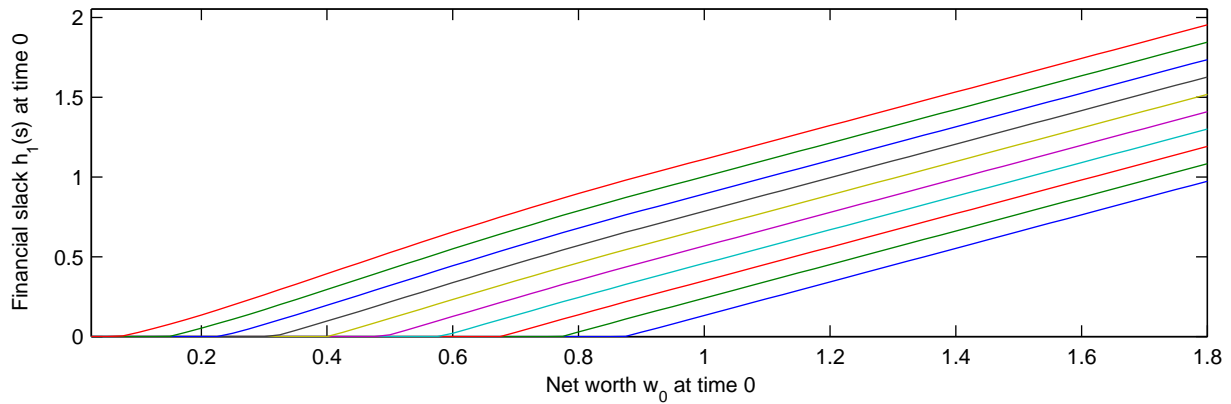
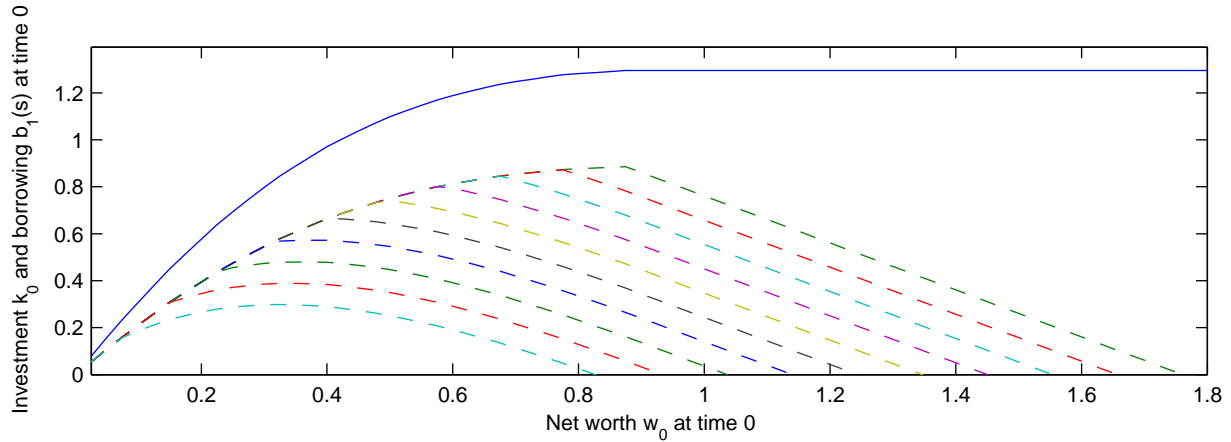
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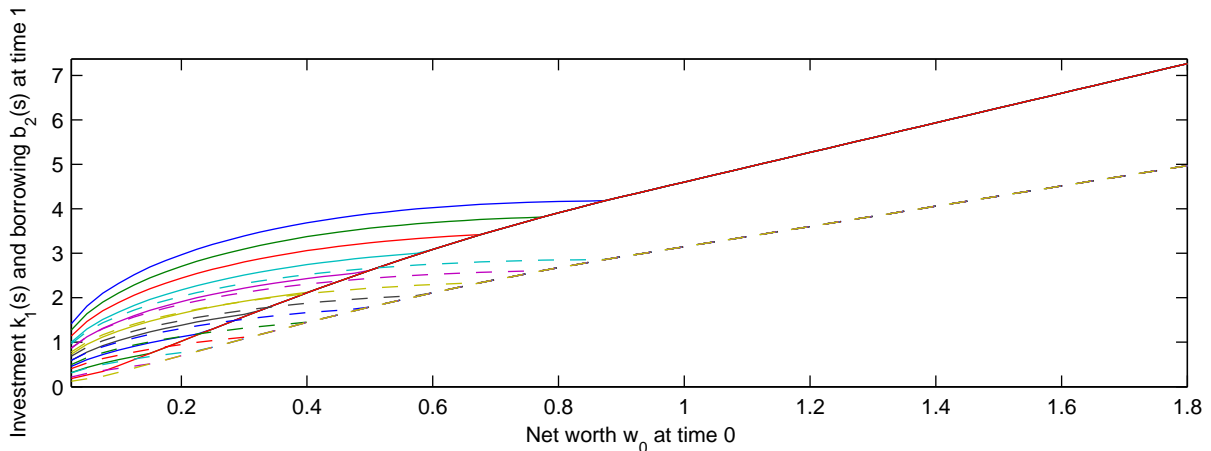
## Figure 1: Investment, State-contingent Borrowing, and Risk Management

Panel A: Time 0 investment in capital (solid, top panel), state-contingent borrowing (dashed) and financial slack (solid, bottom panel) as a function of time 0 net worth. Panel B: Time 1 investment in capital (solid) and state-contingent borrowing (dashed) as a function of time 0 net worth for all states  $s$  at time 1. Parameter Values: Preferences:  $\beta = 0.95$ ; Technology:  $f_t(k) = k^\alpha$  with  $\alpha = 0.33$ ,  $\delta = 0.1$ ,  $\mathcal{S} = \{1, \dots, 10\}$ ,  $\pi(s) = 1/10$ ,  $A_1(s) = s/10$ ,  $A_2(s) = 1.75$ ,  $q_0 = q_1(s) = q_2(s) = 1$ ,  $\forall s \in \mathcal{S}$ ; Collateralization Rate:  $\theta = 0.80$ .

### Panel A: Investment $k_0$ , Borrowing $b_1(s)$ , and Financial Slack $h_1(s)$



### Panel B: Investment $k_1(s)$ and Borrowing $b_2(s)$



**Figure 2: Multipliers on Collateral Constraints**

Top Panel: Time 0 multipliers on collateral constraints for each state at time 1 (solid) as a function of net worth at time 0. Bottom Panel: Time 1 multipliers on collateral constraints for each state at time 2 (solid) as a function of net worth at time 0 for all states  $s$  at time 1. Parameter Values: Preferences:  $\beta = 0.95$ ; Technology:  $f_t(k) = k^\alpha$  with  $\alpha = 0.33$ ,  $\delta = 0.1$ ,  $\mathcal{S} = \{1, \dots, 10\}$ ,  $\pi(s) = 1/10$ ,  $A_1(s) = s/10$ ,  $A_2(s) = 1.75$ ,  $q_0 = q_1(s) = q_2(s) = 1$ ,  $\forall s \in \mathcal{S}$ ; Collateralization Rate:  $\theta = 0.80$ .

