

# Financing auction bids

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In many auctions, bidders do not have enough cash to pay their bid. If bidders have asymmetric cash positions and independent private values then auctions will be inefficient. However, what happens if bidders have access to financial markets? We characterize efficient auctions and show that in an efficient auction the information rent that a bidder earns depends generally on both his valuation and his cash position. In contrast a competitive capital market that is efficient must have information rents that only depend on valuation. This tension between information rents in an efficient auction and zero profits in a competitive equilibrium implies that most often, competitive financing is not efficient.

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# 1 Introduction

■ The majority of auctions worldwide require cash bids. Yet in many auctions, bidders do not have cash equal to the sum they wish to bid. As a result, bidders finance part of their bids. This financing may come from the financial markets or from the seller. For example, financing was the norm in the Federal Communications Commission (FCC) bandwidth auctions, in which the government sold sections of the radio spectrum. In the FCC Class C auctions the FCC itself financed the winning bidders through an installment payment agreement. Ex-post, many of the winning bidders could not make the payments, leading to considerable litigation and finally re-auctioning of licences. In the third generation (3G) European wireless spectrum auctions the winning bidders have borrowed billions of dollars. In most commercial real-estate sales bidders borrow from banks to pay their bid. Bankrupt firms are often sold through cash auctions with financed bids.<sup>1</sup> Many mergers and acquisitions, especially leveraged buy-outs are auctions that involve financed bids. Privatizations also involve financed bids.<sup>2</sup> In almost any auction that sells an object of substantial value, bidders finance a portion of their bid. This paper explores the interaction between the auction and the financial markets.

Should financing have any effect on the auction? Consider an auction in which bidders have different private values and different amounts of available cash. Maskin and Riley (2000) and Maskin (2000) show that auctions with asymmetric bidders and private values will be inefficient, but they do not consider the role of financial markets. We show that access to capital *can* restore efficiency (thus the asymmetry (budget constraint) itself does not cause inefficiency). However, we show that *competitive* financial markets cannot restore efficiency.

If bidders all have access to competitive financial markets, then it would seem that budget constraints would not matter and thus that the auction would behave as predicted by the seminal work in auction theory.<sup>3</sup> As Aghion, Hart, and Moore (1992) state: “*Auctions work well if raising cash for bids is easy...*” (p527). Baird (1986) and Jensen (1991) imply that with competitive capital markets auctions will yield allocative efficiency. We show that when access to capital markets is easy, i.e., capital markets are competitive, efficiency fails. Maskin (2000) also suggests that the ability to “*pay for the asset out of future earnings...*” (p672)<sup>4</sup> might get around the inefficiency associated with capital constraints. Hart (1995) says that, “*In a world of perfect capital markets, a cash auction would (presumably) be the ideal bankruptcy procedure.*” Our work suggests that competitive capital markets will not lead to efficient auctions.

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<sup>1</sup>See Strömberg (2000) and Eckbo and Thorburn (2000) for empirical work on this issue. A theoretical evaluation of such bankruptcy auctions is Rhodes-Kropf and Viswanathan (2000).

<sup>2</sup>See Bolton and Roland (1992).

<sup>3</sup>Vickrey (1961), Harris and Raviv (1981), Myerson (1981), Riley and Samuelson (1981), Milgrom and Weber (1982), and all of the work that follows from these.

<sup>4</sup>This is the function of competitive capital markets.

Our work suggests the following tension between efficiency and competitive equilibrium. In an efficient auction, the probability of winning depends only on the valuation of the object and not the bidder's cash position. In such an efficient auction, each bidder receives an information rent (as defined by Baron and Myerson (1982)) and in general this rent will depend both on the bidder's valuation and cash position. This is because the marginal increase in the bidder's payoff for a small change in valuation depends both on the bidder's valuation and cash position. For example, with debt securities, a bidder with more cash will receive a higher portion of the ex-post payoff and thus his increase in payoff for a small change in valuation will be higher. Hence, in an efficient auction, bidders with more cash (keeping the valuation fixed) will have higher information rents. In contrast, the zero-profit requirement in a competitive equilibrium implies that in an efficient auction bidders with the same valuation must have the same bidder payoff and hence the same information rent. It is this tension between efficiency and competitive equilibrium that drives our results.

We consider the following situation: each bidder has a different independent private expected value for the object for sale, and each bidder also has a different amount of cash. Thus, bidders differ in two dimensions (although they are ex-ante symmetric).<sup>5</sup> Bidders finance the amount needed to cover their bid. Bidder financing is either debt or equity or state contingent financing, so the payment made to the financial market depends on the resulting value of the object. Thus, we explicitly model the ex post uncertainty of the valuation of the object for sale. This innovation is critical in our analysis of the financing of auction bids. In a standard auction with risk neutral bidders and no financial constraints, what is important in determining a bidder's bid is his/her expected value of the object; hence the ex post uncertainty of the valuation can be abstracted away. Ex post uncertainty is important and realistic since the payoffs of financial claims depend differently on the final realizations of payoffs.

Furthermore, we also consider various criteria for the financing rate. First we consider financing where the rate is set independent of the auction. This occurs when the rate depends on the assets being purchased (e.g. all bidders for a house will get the same mortgage rate). Next we consider pre-auction financing that uses information about the bidders but does not condition on the bid itself. This would occur if the financier completed due diligence before the auction and could determine the type of the bidder (although, one wonders why a seller cannot do this). Finally, we consider financing that conditions the rate on the bids when financiers do not know bidder valuations. This occurs when bidders attract financing after (or conditional on) winning the auction.

Our results are driven by the tension between efficiency and competitive equilibrium that we discussed above. In many of the cases we consider, this tension cannot be resolved. This includes a single interest rate (pooling equilibrium, no efficiency) and debt securities (whether the interest rate is set pre-auction or conditional on the auction). With equity, there is no efficiency with contracting conditional on the auction.

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<sup>5</sup>Solving optimal auctions with two-dimensional information is an important goal in auction theory. This paper is a step forward in that results are found with bidders who have different continuous values and different continuous amounts of cash.

We do obtain efficiency when we have equity financing if the rate is set pre-auction and when we are allowed to use contingent securities of a particular kind. However the contingent securities that yield efficiency often require payments by the lender to the borrower in some states of the world and are not robust to the presence of bidders who do not have to borrow to make their bids.

In an important paper, Che and Gale (1998) explore an auction in which bidders face budget constraints. They show that certain auction rules are revenue superior when bidders are cash constrained. While the authors allow for credit, they do not allow for ex-post uncertainty in payoffs, nor formally model financial markets. Instead they use a reduced form approach in which the marginal cost of financing is increasing in the capital borrowed. Auctions are always inefficient in their set up. The question remains whether it is the lack of internal capital or the poor functioning of financial markets that yields inefficiency in the auction. We address this question by examining multiple forms of financing and different levels of financial market competition. Our work suggests that there is a tension between the information rents in an efficient auction and the zero profit requirement in competitive markets and hence competitive financial markets cannot yield efficient auctions.

Our work is also related to two other papers. Rhodes-Kropf and Viswanathan (2000) consider an auction model with debt where different bidders have different valuations but the same amount of an asset to pledge in the event of bankruptcy.<sup>6</sup> Zheng (2001) studies a first price auction with bidders who have identical valuations for the object but differ in their cash positions and must finance their bids using debt at a fixed interest rate. Zheng (2001) shows that when the interest rate is low (high) bids decrease (increase) with cash positions.<sup>7</sup> The assumption of identical valuations removes the possibility that the auction is inefficient.

In contrast to the above papers, we allow bidders to differ both in their valuations and in the amount of cash. Further we consider more general securities (debt, equity, contingent claims) and more general contracting by financial markets. Our work on fixed interest rates generalizes the insight of Zheng (2001) and shows that this will occur with any fixed interest and with equity, i.e. it is a feature of the cross-subsidization implicit in any pooling equilibrium in financial markets. More importantly, we consider situations where financial contracts can be written on observables like the bid or cash and ask whether efficiency can be attained.

Our paper is organized as follows. Section 2 lays out the basic model while Section 3 provides an overview of our intuition and the foundation for the results in subsequent sections. Section 4 considers bids with a single interest rate, Section 5 considers rates that condition on the valuation while Section 6 considers rates conditioned on the bid. With each type of rate we examine first debt and then equity financing. The end of Section 6 considers complete state contingent claims. Section 7 consider some variations to our model while Section 8 concludes.

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<sup>6</sup>In Rhodes-Kropf and Viswanathan (2000), the asset is in the form of a non-pecuniary penalty that is paid if bankruptcy occurs. It can equally be interpreted as cash held by bidders.

<sup>7</sup>Zheng's model has a different timing convention, the winning bidder pledges his cash as collateral, finds the state of the world and then decides whether or not to default on his payment.

## 2 The Model

■ The model is a two-stage game with private information. In the first stage,  $n$  potential risk-neutral bidders bid with cash for a firm whose value is not known with certainty. The seller uses a first price cash auction, and the firm is awarded to the high bidder. In order to pay the bid, the bidder may access the securities market and raise money. The financing can be thought of as occurring before the auction (if the financing rate does not depend on the bids) or after the auction (if the financing rate depends on the bids), but must occur before the final stage in which the value of the firm is revealed, and all claims are settled. However, the terms of financing are known to all bidders before they submit their bids.

The final value of the firm for sale will be either a high value  $H$  or a low value  $L$ . In the first stage, each bidder has private information about  $\lambda_i$ , their ability to manage the firm. If bidder  $i$  runs the firm then with probability  $\lambda_i$  the firm will be worth  $H$  and with probability  $(1 - \lambda_i)$  the firm will be worth  $L$ . The  $\lambda$ s of each bidder are independently and identically distributed and drawn from the distribution  $F(\lambda)$  with  $F(\underline{\lambda}) = 0$ ,  $F(\bar{\lambda}) = 1$ .  $F(\lambda)$  is strictly increasing and differentiable over the interval  $[\underline{\lambda}, \bar{\lambda}]$ .  $\underline{\lambda}$  must be greater than some lower bound, which will be defined separately for each financial market.<sup>8</sup> Each bidder also has private information about the amount of cash  $c_i$  that he has. In general, we will assume that the amounts of cash held by the bidders are independently and identically distributed and drawn from the distribution  $\Psi(c)$  with  $\Psi(\underline{c}) = 0$ ,  $\Psi(\bar{c}) = 1$ .  $\Psi(c)$  is strictly increasing and differentiable over the interval  $[\underline{c}, \bar{c}]$ , and  $\underline{c} \geq 0$  (restrictions on  $\bar{c}$  will be defined for each financial market).

To establish a benchmark we will first examine the case when each bidder has enough cash to cover their entire bid ( $\underline{c} >$  highest bid). We will then consider three different models of the financial market. With each model we analyze bidding behavior and efficiency when bidders use their cash and access debt markets, then their cash and access equity markets. Towards the end of the paper, we allow bidders to finance bids using state contingent securities.

When every bidder has enough cash to cover their entire bid, the bidder's problem is

$$\max_{b_i} [\{H\lambda_i + L(1 - \lambda_i) - b_i\} Prob[b_i > \max_{j \neq i} b_j]]. \quad (1)$$

Standard techniques (see Krishna (2002)) reveal that the unique symmetric equilibrium has bidders bid according to

$$b_c(\lambda_i) = H\lambda_i + L(1 - \lambda_i) - (H - L) \int_{\underline{\lambda}}^{\lambda_i} \frac{F^{n-1}(s) ds}{F^{n-1}(\lambda_i)}, \quad (2)$$

where the subscript  $c$  signifies the standard cash auction. We will use this benchmark to understand when financed bids can implement the cash auction.

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<sup>8</sup>We need to impose some restrictions on the parameters to ensure that every bidder borrows and in the case of debt, that there is insolvency in the low state.

### 3 A General Model and Our Intuition

■ In what follows, we consider a variety of different sub-cases involving different securities (debt, equity, contingent auctions) and a variety of different financing rules (fixed interest rate, rate contingent on true value and cash but not bid, rate contingent on bid). In all these cases, the bidding function will (in general) depend on both the valuation ( $\lambda$ ) and the cash position ( $c$ ). While multi-dimensional auctions are generally intractable, our approach is to exploit the special structure of our problem to reduce our optimization to a single-index auction or to exploit the restrictions implied by efficiency to obtain results (see Goeree and Offerman (2000) for an example of a single-index auction when there are private and common components in valuation).

Importantly, we wish to understand the requirements for efficiency in an auction with a given security and whether the competitive outcome can support an efficient outcome. Towards this end, we prove the following three lemmas that we use repeatedly in our subsequent results.

The general bidding problem that we consider across all securities is

$$\max_b [\lambda_i H + (1 - \lambda_i)L - c_i - q_i(b, \lambda_i, c_i)] Prob[b > \max_{\forall j \neq i} b_j] \quad (3)$$

where  $c_i$  is the cash payment and  $q_i(b, \lambda_i, c_i)$  is the expected payment to the lender of the security for the amount  $b - c_i$  that is financed. We are assuming at this stage that it is optimal for the bidder not to finance more than his bid and that all bidders have to finance their bids. These details are provided in later sections of the paper.

If we multiply the objective of the optimization problem in Equation (3) by a positive constant  $a(\lambda_i, c_i)$  (since  $\lambda_i$  and  $c_i$  are in the information set of bidder  $i$ , we can do this) we do not change the optimal bid. After rearranging terms (the details of which vary for each security), the new optimization that we consider is

$$\max_b [\nu(\lambda_i, c_i) - p(b, \lambda_i, c_i)] Prob[b > \max_{\forall j \neq i} b_j], \quad (4)$$

here,

$$\nu(\lambda_i, c_i) - p(b, \lambda_i, c_i) = a(\lambda_i, c_i)[\lambda_i H + (1 - \lambda_i)L - c_i - q_i(b, \lambda_i, c_i)] \quad (5)$$

Now bidder  $i$ 's bid  $b$  wins when it exceeds the bids of all other agents, bidder receives a “value”  $\nu(\lambda_i, c_i)$  and makes a “payment”  $p(b, \lambda_i, c_i)$ . We use quotation marks because the optimization problem in Equation (4) is a rescaling of the original problem in Equation (3) that leaves the optimal bids unchanged. Because of financing, both the “value” and the “payment” depend on the valuation ( $\lambda_i$ ) and the cash position ( $c_i$ ). We next prove the following three lemmas based on standard arguments in auction theory (see Krishna (2002) for example).

First we ask when we can reduce the optimization problem to a standard single index first price auction. In such a case efficiency reduces to checking whether the index does not (or does) depend on the cash position.

*Lemma 1.* If  $p(b, \lambda_i, c_i) = b$ , let  $\nu_i = \nu(\lambda_i, c_i)$  and we have a standard first price auction in  $\nu_i$ . Let  $G(\cdot)$  be the cumulative distribution function of  $\nu_i$ . Define

$$f(x|\nu) = [\nu - b(x)] \frac{dG^{n-1}(x)}{dx} - b'(x)G^{n-1}(x) \quad (6)$$

and suppose that  $f(x|\nu)$  is strictly increasing in  $\nu$ .<sup>9</sup> Then the unique symmetric equilibrium of the first price auction is:

$$b(\nu_i) = \nu_i - \int_{\underline{\nu}}^{\nu_i} \frac{G^{n-1}(s)}{G_{n-1}(\nu_i)} ds \quad (7)$$

and efficiency requires that  $\frac{d\nu_i}{dc_i} = 0$ , i.e., the optimal bids do not depend on cash.

*Proof.* Most of the proof is standard and is omitted (see Krishna (2002) Section 2.3). The statement on efficiency is obvious. Q.E.D.

Lemma 1 applies only to cases where we can rescale the problem so as to make it isomorphic to a first price auction in  $\nu_i$ . In a number of cases we study, we are able to recast the model as a single index model and use standard results. In general, this cannot be done, i.e.,  $p(b_i, \lambda_i, c_i) \neq b_i$ .

Even when we cannot recast the problem as a single index auction, we can learn about the nature of the efficient auction by imposing efficiency and using a direct revelation approach.

In an efficient auction,  $b(\lambda_i, c_i) = b(\lambda_i)$ , i.e., bids do not depend on cash. Using a direct revelation approach, the optimization in Equation (4) reduces to

$$\max_x [\nu(\lambda_i, c_i) - p(x, \lambda_i, c_i)] F^{n-1}(x) \quad (8)$$

and standard methods yield the ‘‘payment’’ function

$$p(\lambda_i, \lambda_i, c_i) = \nu(\lambda_i, c_i) - \int_{\underline{\lambda}}^{\lambda_i} [\nu_1(x, c_i) - p_2(x, x, c_i)] dx \quad (9)$$

where the subscript  $k$  represents the partial derivative with respect to the  $k^{th}$  argument.<sup>10</sup>

Substituting the payment function in Equation (9), we obtain the information rent that accrues to an bidder with type  $(\lambda_i, c_i)$  as (see Baron and Myerson (1982), Krishna pages 73, 146)

$$\frac{1}{a(\lambda_i, c_i)} \left[ \int_{\underline{\lambda}}^{\lambda_i} \nu_1(x, c_i) - p_2(x, x, c_i) dx \right] F^{n-1}(\lambda_i) \quad (10)$$

where  $a(\lambda_i, c_i)$  is the constant which we used to rescale the objective of the optimization in Equation (3). This immediately yields Lemma 2.

<sup>9</sup>If  $f(x|\nu)$  is strictly increasing in  $\nu$ , then it has the following strict quasi-monotone property: if  $f(x|\nu) \geq 0$ , then  $f(x|\nu') > 0$  for all  $\nu' > \nu$ . Using this property, we know that for  $x < \nu$ ,  $f(x|\nu) > 0$  since  $f(x|x) = 0$  and for  $x > \nu$ ,  $f(x|\nu) < 0$  since otherwise  $f(x|x) > 0$ . Hence  $f(x|\nu)$  is a single-peaked function with the peak obtained at  $x = \nu$ . This is the standard argument that ensures sufficiency of the first order condition.

<sup>10</sup>In Section 6, we show how to use Equation (9) to find the optimal payment function and bid function in an efficient auction.

*Lemma 2.* The information rent is independent of the cash position  $c_i$  if and only if  $\frac{1}{a(\lambda_i, c_i)}[\nu_1(x, c_i) - p_2(x, x, c_i)]$  is independent of  $c_i$  for almost all  $(x, \lambda_i)$ .

*Proof.* This follows directly from Equation (10). Q.E.D.

Lemma 2 characterizes the conditions for the information rent to be independent of the cash position. Even though the bids do not depend on the cash position in an efficient auction, the payment and the information rent will depend on the cash position unless there is a special structure to the problem. For many standard securities, the information rent will depend on the cash position because the ex-post contingent payment depends on the cash position. For example, with debt, a bidder with more cash will make a lower ex-post contingent payment. Hence the marginal change in the bidder's payoff to a change in valuation is higher for such a bidder. This would make the information rent higher for bidders with higher cash (keeping the valuation constant).

A key objective of the paper is to understand when efficient auctions can be supported by competitive equilibria. Towards this end, we must understand the implications of having efficiency and perfect competition.

*Lemma 3.* In an efficient auction that is consistent with a competitive equilibrium, the objective value (and hence information rent) does not depend on the cash position  $c_i$ .

*Proof.* The objective for bidder  $i$  is

$$\max_b [\lambda_i H + (1 - \lambda_i)L - c_i - q_i(b, \lambda_i, c_i)] \text{Prob}[b > \max_{\forall j \neq i} b_j] \quad (11)$$

where  $c_i$  is the cash with the bidder that is paid and  $q_i(b, \lambda_i, c_i)$  is the expected payment to the lender for the amount financed,  $b - c_i$ . By zero profit condition of perfect competition we have that

$$q_i(b, \lambda_i, c_i) = b(\lambda_i, c_i) - c_i, \quad (12)$$

the expected payment is equal to the amount borrowed. Using Equation (12) and the fact that in an efficient auction  $b(\lambda_i, c_i) = b(\lambda_i)$ , the equilibrium bidder payoff simplifies to

$$[\lambda_i H + (1 - \lambda_i)L - b(\lambda_i)] F^{n-1}(\lambda_i) \quad (13)$$

which is independent of  $c_i$ . Hence information rents cannot depend on  $c_i$  in an efficient auction that can be supported by a competitive financial market. Q.E.D.

Lemmas 2 and 3 present the key argument that is used repeatedly in this paper. Using this argument, we check whether a given financial instrument and a given contracting approach can support a competitive financial market that is efficient. Further, Lemmas 2 and 3 suggest a tension between efficiency and competitive equilibrium. Efficiency requires higher information rents for the bidder whose payoff is more sensitive to a change in valuation. In general, this marginal effect is not independent of the cash position. This is because the amount of cash affects the ex-post contingent payoff that the bidder receives. In contrast, the zero profit

requirement in perfect competition implies that information rents do not depend on cash positions. This requirement is hard to meet and hence we obtain that (except in certain special cases) competitive financial markets cannot yield efficient outcomes.

In what follows we consider three kinds of contracting arrangements. In the first, the lender charges a fixed rate for all borrowers. Since this is a pooling arrangement, it is unlikely to be efficient. However, this mimics the arrangement in the FCC Class C auction where the FCC allowed the winning bidders to make installment payments. In the second, the lender charges a rate based on the valuation  $\lambda_i$  and the cash position  $c_i$  but not the bid. This contracting arrangement, which we refer to as pre-auction financing, requires the securities market to determine the type of the bidder independent of the auction, and to commit not to use the information in the bids. Much of the small business lending in the United States that is based on scoring rules fits this setup. Note that the seller does not have the information available to the financial market (otherwise an auction would not be required). In the third, we allow the lender to charge a rate based on the cash position  $c_i$  and the bid  $b_i$ . We refer to this contractual arrangement as post-auction financing. After the auction the securities market learns the bid of the winning bidder and may be able to determine the bidder's cash if it is incentive compatible for the bidder to reveal their cash.

A key difference between pre-auction and post-auction contracting is that the interest rate is affected in post-auction contracting by the bid and this creates additional incentives for bidders to distort their bidding. In pre-auction contracting, the financier knows the valuation of the bidder and commits not to use the bid, so this incentive is not present. As we show below, the results for the two contracting arrangement differ considerably.

Since we analyze a variety of different cases, we summarize the main results of the paper in Tables 1, 2 and 3. The interested reader may refer to these tables for an overview of our results.

## 4 Cash Bids With Fixed Rate Financing

### □ Cash Bids with Debt Financing

If the seller requires cash bids paid in full, then bidders who do not have enough cash to pay for the object for sale require financing. In this section, the winning bidder uses debt to finance the portion of his bid that he does not currently hold in cash at a fixed interest rate (the interest rate is the same for each borrower no matter what amount is borrowed). Every bidder is given equal access to debt financing and no limits on the amount he can borrow. While efficiency is not the main issue in this sub-section, this simpler model allow us to provide an explanation for what happened in the FCC C Class auctions where most licence holders defaulted on their payments.

Every bidder can borrow or lend money in a competitive debt market at an interest rate  $r$ . The possibility of default ensures that although  $r > 0$ , the lender's expected return is zero (our results hold for arbitrary

$r > 0$ ).<sup>11</sup> Thus, bidders borrow an amount  $b_i - c_i$  to make up the difference between their cash,  $c_i$ , and their bid,  $b_i$ , and owe the lender  $(1 + r)(b_i - c_i)$ . It may seem that bidders would like to borrow their entire bid rather than just  $b_i - c_i$ . In a moment we will prove that simply assuming the bidders cannot steal is enough to ensure that bidders do not benefit from borrowing more than  $b_i - c_i$ .

The bidder's problem now becomes

$$\begin{aligned} & \max_{b_i} [\{H\lambda_i + (1 - \lambda_i)L - \min[b_i, c_i] - \lambda_i(1 + r) \max[b_i - c_i, 0] \\ & - (1 - \lambda_i) \min[L, \max[(1 + r)(b_i - c_i), 0]]\} \text{Prob}[b_i > \max_{\forall j \neq i} b_j]]. \end{aligned} \quad (14)$$

The complication from the multiple *min* and *max* functions stems from the different possible amounts of cash. The first *min* and the second and third *max* are necessary if it is possible that the bidder did not borrow. Since the interesting case is when bidders do borrow, we will assume that bidders bid more than their current cash. The second *min* checks whether the bidder earns enough in the low state to fully repay the lender, i.e. no default. Again, the interesting case is with default since debt without default is like cash.<sup>12</sup> Furthermore, without default the interest rate from a competitive market must be zero.<sup>13</sup> Therefore, we assume that bidders have cash, but must borrow enough that they cannot meet their obligations in the low state. In this case  $L$  can be thought of as the secured part of the loan. Default in the low state requires that  $(b_i - c_i)(1 + r) > L$ , for all  $i$ , later we show the restrictions on the parameters  $(H, L, \underline{\lambda}, \bar{c})$  that ensure this.

The final assumption is that bidders cannot steal. That is, if bidders have cash at the end of the auction then that cash is still in the firm when the outcome of  $H$  or  $L$  is realized.<sup>14</sup> Thus, the cash is available to help repay the loan during bankruptcy. The following lemma proves that the inability to steal ensures that bidders only borrow  $b_i - c_i$ .

*Lemma 4. If bidders cannot steal then they do not benefit from borrowing more than  $b_i - c_i$ .*

*Proof.* See the Appendix.

Therefore, the bidder's problem simplifies to

$$\max_{b_i} \left[ \{H\lambda_i - c_i - \lambda_i(1 + r)(b_i - c_i)\} \text{Prob}[b_i > \max_{\forall j \neq i} b_j] \right]. \quad (15)$$

In general the probability of winning depends on two variables, the bidder's cash position and the bidder's valuation of the object. Multiplying the maximization by  $a(\lambda_i, c_i) = \frac{1}{\lambda_i(1+r)}$  does not change the optimal bid and allows us to reduce the problem to a single-index auction.

The bidder's problem then becomes

<sup>11</sup>The assumption of zero systematic risk is without loss of generality.

<sup>12</sup>Che and Gale (1998) have no default. Thus, their debt is just two certain cash payments, one now and one next period. However cash payments in the future are costly.

<sup>13</sup>In debt markets the premium over the risk free rate is called the default spread. In the real world all borrowers have a probability of default and must pay a default spread.

<sup>14</sup>Even if the bidder invests the cash, the investment's present value is the value of the cash.

$$\max_{b_i} \left[ \left\{ \frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i - b_i \right\} Prob[b_i > \max_{\forall j \neq i} b_j] \right]. \quad (16)$$

Define

$$\nu_i = \nu(\lambda_i, c_i) = \frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i, \quad (17)$$

and the bidder's problem reduces to

$$\max_{b_i} \left[ \{ \nu_i - b_i \} Prob[b_i > \max_{\forall j \neq i} b_j] \right]. \quad (18)$$

From Lemma 1, we know the answer to this optimization problem. However, we do need to identify whether  $\nu(\underline{\lambda}, c_i)$  is increasing or decreasing in  $c_i$  (for an interest  $r$  that is set competitively, we will show that  $\nu(\underline{\lambda}, c_i)$  is decreasing in  $c_i$ ) and to find the conditions on the parameters that ensure default. We provide these results in Proposition 1 (see Table 1 for a summary of the results).

*Proposition 1.* If the interest rate,  $r$ , is set such that either (A)  $1 < 1+r < \frac{1}{\underline{\lambda}} < \frac{(H-L)}{\bar{c}}$  or (B)  $\frac{1}{\underline{\lambda}} < 1+r < \frac{H-L}{\bar{c}-\underline{c}} - \frac{\underline{c}}{(\bar{c}-\underline{c})\underline{\lambda}}$  then there exists a unique symmetric equilibrium s.t.

$$b(\lambda_i, c_i) = b(\nu_i) = \nu_i - \int_{\underline{\nu}}^{\nu_i} \frac{G^{n-1}(s) ds}{G^{n-1}(\nu_i)}, \quad (19)$$

with

$$b(\underline{\nu}) = \underline{\nu} = \frac{H}{1+r} - \frac{\bar{c}}{\underline{\lambda}(1+r)} + \bar{c}, \quad (20)$$

if condition (A) holds, or with

$$b(\underline{\nu}) = \underline{\nu} = \frac{H}{1+r} - \frac{\underline{c}}{\underline{\lambda}(1+r)} + \underline{c}, \quad (21)$$

if condition (B) holds.

If the lending market is perfectly competitive, then there exists an equilibrium competitive rate  $r$  such that  $1 < 1+r < \frac{1}{\underline{\lambda}}$  (Case A). Finally,  $\frac{\partial \nu_i}{\partial \lambda_i} = \frac{c}{\lambda_i^2(1+r)} > 0$  and  $\frac{\partial \nu_i}{\partial c_i} = 1 - \frac{1}{\lambda_i(1+r)}$ . Hence  $\nu(\lambda_i, c_i)$  is not independent of  $c_i$  except for  $\lambda_i = \frac{1}{(1+r)}$ . Thus the auction is not efficient.

*Proof.* See the Appendix.

Much of Proposition 1 follows from Lemma 1. Two cases arise. In Case (A),  $1 < 1+r < \frac{1}{\underline{\lambda}}$  and there exist bidders with low valuations relative to the market interest rate. In this case bidders with high valuations ( $\lambda_i(1+r) > 1$ ) face too high a cost of borrowing (the market over-assesses their default risk). Thus these bidders find external capital to be expensive and have bids that increase in valuations and cash. In contrast, bidders with low valuations ( $\lambda_i(1+r) < 1$ ) face too low a cost of borrowing (the market under-assesses their default risk). Such bidders find external capital to be cheap and have bids that increase in valuations and decrease in cash. Note that the necessary and sufficient condition for all bidders to borrow is  $(b(\underline{\nu}) - \bar{c})(1+r) > L$ . Since the lowest type ( $\underline{\nu}$ ) is associated with the lowest valuation ( $\underline{\lambda}$ ) and the highest cash position ( $\bar{c}$ ), we can simplify the condition for default to  $(H-L)\underline{\lambda} > \bar{c}$ . This condition requires that the

difference in payoffs between the high and low state must be relatively large compared to the most cash the worst manager could have, and the probability of the high state for the lowest type should be high enough.

It is interesting to note that bidders with lower valuations ( $\lambda_i(1+r) < 1$ ) may have  $\nu_i$  greater than  $\lambda_i H + (1 - \lambda_i)L$  which is their “true” valuation of the object without financing distortions.<sup>15</sup> That is, these bidders may be willing to pay more than their “true” valuation without financing distortions. These bidders are willing to pay ‘too much’ because the lender is subsidizing them with a low interest rate. A bidder may or may not actually bid higher than his “true” valuation depending on the distribution of types and the level of competition. Equation (19) shows that bid shading is determined by the term  $\int_{\underline{\nu}}^{\nu_i} \frac{G^{n-1}(s)ds}{G^{n-1}(\nu_i)}$  that goes to zero as  $n \rightarrow \infty$ . Thus, in environments with a large number of bidders, low valuation bidders will bid more than their expected value because the financial market under-assesses their default risk.

In Case (B),  $1+r > \frac{1}{\lambda}$ , and all bidders face an interest rate that is too high, their true default risk is lower than that implied in the market interest rate. Consequently, they find external capital to be expensive and their bids are increasing in valuation and in cash positions. The lowest type ( $\underline{\nu}$ ) is associated with the lowest valuation ( $\underline{\lambda}$ ) and the lowest cash position ( $\underline{c}$ ), we can simplify the condition for default to  $1+r < \frac{H-L}{\underline{c}-\underline{c}} - \frac{\underline{c}}{(\underline{c}-\underline{c})\underline{\lambda}}$ .

The intuition in Case (A) is close to that in the classic paper of Akerlof (1970) and its application in finance by Myers and Majluf (1984). Undervalued firms view internal capital (cash position) as a resource while overvalued firms view internal capital as a cost; this occurs because the financial market provides capital at a single rate. Thus, a single financing rate results in significant inefficiencies. The ability of low value bidders who have less cash to outbid higher value bidders who have more cash may explain the ex-post bankruptcy of winners in the FCC C Class auction.<sup>16</sup> These winning bidders may have been low value low cash bidders since the FCC essentially offered the same financing terms to all winning bidders.<sup>17</sup>

Why is a competitive fixed interest rate inefficient? The inefficiency of the competitive fixed interest rate occurs because all bidders are charged the same interest rate and thus adverse selection occurs. Since this

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<sup>15</sup>The exact condition is an interest rate such that

$$1+r < \frac{1}{\lambda_i} \frac{\lambda_i H - c_i}{\lambda_i H + (1 - \lambda_i)L - c_i}.$$

This condition cannot hold if  $1+r \geq 1/\lambda_i$ , but if  $1+r < 1/\lambda_i$  then a small enough  $L$  will make the condition true.

<sup>16</sup>Zheng (2001) suggests a similar possibility in his model with one dimensional information (the cash position) but a different timing convention. In his model, a correctly set interest rate yields efficiency while our model (with two dimensional uncertainty) yields inefficiency with a single interest rate.

<sup>17</sup>If bidders are allowed to pay dividends before the auction, in Case (A) low types ( $\nu_i < \nu^* = H/(1+r)$ ) benefit from having low cash, these bidders dump their cash until their  $\nu_i = \nu^*$ , and they pool at  $b(\nu^*) = \frac{H}{1+r}$ . The remaining bidders now have the same chance of winning the auction as they did before, but the bidders who pool at  $\nu^*$  function as a kind of reserve price. So, the high  $\nu_i$  bidders now increase their bid to

$$b(\nu_i) = \nu_i - \int_{\nu^*}^{\nu_i} \frac{G^{n-1}(s)ds}{G^{n-1}(\nu_i)}. \quad (22)$$

pooling occurs with all securities, a similar inefficiency arises with equity financing, we show this next.

□ **Cash Bids with Equity Financing**

In this section the winning bidder sells equity to finance the portion of his bid that he does not currently hold in cash. Thus, the winning bidder will obtain some of the income in both the high and the low states. Our key point is that the source of the adverse selection is the pooling in the financial market and not the particular security that is considered.

Every bidder now has equal access to unlimited equity capital from a perfectly competitive securities market. For every dollar they need to finance they sell a fraction  $\phi$  of their firm. Thus,  $\phi$  is the reciprocal of the market capitalization. Assuming that the bidder needs to finance a portion of his bid  $b_i > \bar{c} \forall i$  (which must be checked in equilibrium), the bidder's problem becomes

$$\max_{b_i} \left[ \{H\lambda_i + (1 - \lambda_i)L - c_i - \phi(b_i - c_i)(H\lambda_i + (1 - \lambda_i)L)\} Prob[b_i > \max_{\forall j \neq i} b_j] \right]. \quad (23)$$

We multiply the maximization by  $a(\lambda_i, c_i) = \frac{1}{\phi(H\lambda_i + (1 - \lambda_i)L)}$  and define

$$\nu_i = \nu(\lambda_i, c_i) = \frac{1}{\phi} - \frac{c_i}{\phi(H\lambda_i + (1 - \lambda_i)L)} + c_i, \quad (24)$$

The bidder's problem reduces to

$$\max_{b_i} \left[ \{\nu_i - b_i\} Prob[b_i > \max_{\forall j \neq i} b_j] \right]. \quad (25)$$

and we can use Lemma 1 to obtain the following theorem (see Table 2 for a summary):

*Proposition 2.* If the price of equity,  $\phi$ , is set such that either (A)  $1/\phi > H\underline{\lambda} + (1 - \underline{\lambda})L > \bar{c}$  or (B)  $H\underline{\lambda} + (1 - \underline{\lambda})L > 1/\phi > \frac{(\bar{c} - \underline{c})(H\underline{\lambda} + (1 - \underline{\lambda})L)}{H\underline{\lambda} + (1 - \underline{\lambda})L - \underline{c}}$  then  $\exists$  a unique symmetric equilibrium s.t.

$$b(\nu_i) = \nu_i - \int_{\underline{\nu}}^{\nu_i} \frac{G^{n-1}(s)ds}{G^{n-1}(\nu_i)}. \quad (26)$$

with

$$b(\underline{\nu}) = \underline{\nu} = \frac{1}{\phi} - \frac{\bar{c}}{\phi(H\underline{\lambda} + (1 - \underline{\lambda})L)} + \bar{c}, \quad (27)$$

if condition (A) holds, or with

$$b(\underline{\nu}) = \underline{\nu} = \frac{1}{\phi} - \frac{\underline{c}}{\phi(H\underline{\lambda} + (1 - \underline{\lambda})L)} + \underline{c}, \quad (28)$$

if condition (B) holds. Further the auction is inefficient.

*Proof.* See the Appendix.

Once again, a bidder bids above his expected value if the price obtained for outside equity is high enough (the bidder is overvalued) and enough other bidders compete in the auction. Further Lemma 1 implies that equity also yields an inefficient auction. Hence pooling in the financial market and the consequent adverse selection creates inefficiencies that result in overbidding by low value bidders.

## 5 Pre Auction Financing

■ In this section each bidder's rate depends on their type and/or cash, but not the bid in the auction. This requires the securities market to determine the type of the bidder independent of the auction and to commit not to use the information contained in the bids. Thus there is complete information but incompleteness in contracting. Much of the small business and mortgage lending in the United States that is based on proprietary credit scoring rules has this flavor. While the financial market knows the valuation of the bidder, the seller does not (otherwise an auction would not be necessary). It could be argued that banks specialize in determining the type of the borrower, and equity markets aggregate information well. This seems to be the view expressed in Jensen (1991) and others who argue in favor of auctions for firms in bankruptcy. The assumption of full information in the securities market is somewhat extreme since the market's information is probably not complete. Later, we consider the alternative view that the valuation is the bidder's private information and the interest rate depends on the bid.

### □ Pre Auction Debt Financing

Lenders now have the skill to determine the type  $\lambda_i$  and the cash  $c_i$  of the bidder before the auction. The lenders commit to use only  $\lambda_i$  and  $c_i$  to determine an interest rate for each bidder,  $r(\lambda_i, c_i)$ , hence contracting on the bid is not possible. Assuming the bidder borrows and is insolvent in the bad state,  $(b_i - c_i)(1 + r(\lambda_i, c_i)) > L$ ,  $\forall i$ , (which requires assumptions on  $c$  and  $\lambda$  that must be checked in equilibrium) the bidder's problem is

$$\max_b \left[ \{H\lambda_i - c_i - \lambda_i(1 + r(\lambda_i, c_i))(b - c_i)\} Prob[b > \max_{\forall j \neq i} b_j] \right]. \quad (29)$$

Since the interest rate depends only on the private information of the borrower and not the bid, we multiply by  $a(\lambda_i, c_i) = \frac{1}{\lambda_i(1+r(\lambda_i, c_i))}$  and define  $\nu_i$  as

$$\nu_i = \nu(\lambda_i, c_i) = \frac{H}{1 + r(\lambda_i, c_i)} - \frac{c_i}{\lambda_i(1 + r(\lambda_i, c_i))} + c_i, \quad (30)$$

and the bidder's problem reduces to

$$\max_b \left[ \{\nu_i - b_i\} Prob[b > \max_{\forall j \neq i} b_j] \right]. \quad (31)$$

Given some restrictions that ensure that all bidders borrow and that default occurs in the low state, the solution follows from Lemma 1. From Lemma 1, an efficient auction exists if  $\frac{d\nu_i}{dc_i} = 0$ . The restrictions on the interest rate that ensure efficiency plus the conditions that ensure a positive interest rate and default in the low state are stated in Proposition 3 (see Table 1 for a summary).

*Proposition 3.* If the lender knows  $\lambda_i$  and  $c_i$  and must set an interest rate for each bidder without conditioning on the bid, then the only interest rate function which results in an efficient auction is

$$1 + r(\lambda_i, c_i) = \frac{1}{\lambda_i} \frac{\lambda_i H - c_i}{\nu_r(\lambda_i) - c_i} \quad (32)$$

where  $\nu_r(\cdot)$  is a monotone increasing function subject to  $H - \frac{\bar{c}}{\lambda} + \bar{c} \geq \nu_r(\bar{\lambda})$ ,  $\nu_r(\underline{\lambda}) > L + \bar{c}$ , where the subscript  $r$  denotes debt.

*Proof.* Since the bid is increasing in  $\nu_i$ , Lemma 1 shows that for the auction to be efficient  $\nu_i$  must be increasing in  $\lambda_i$  and not change with  $c_i$ ,  $\frac{d\nu_i}{dc_i} = 0$ . Therefore,  $\nu_i$  must equal a function that depends only on  $\lambda_i$ . Setting  $\nu_i = \nu_r(\lambda_i)$  and solving we obtain that

$$1 + r(\lambda_i, c_i) = \frac{1}{\lambda_i} \frac{\lambda_i H - c_i}{\nu_r(\lambda_i) - c_i}. \quad (33)$$

Since the interest rate,  $r(\lambda_i, c_i)$ , must be positive,  $H - \frac{c_i}{\lambda_i} + c_i \geq \nu_r(\lambda_i) \Rightarrow H - \frac{\bar{c}}{\lambda} + \bar{c} \geq \nu_r(\bar{\lambda})$ . Furthermore, since  $\nu_i = \nu_r(\lambda_i)$ , efficiency requires that  $\nu_r'(\cdot) > 0$ . Finally, we need to ensure that every bidder borrows and is possibly insolvent,  $(b(\lambda_i) - c_i)(1 + r(\lambda_i, c_i)) > L$ . It is easy to show that the bid function is

$$b(\lambda_i) = \nu_r(\lambda_i) - \int_{\underline{\lambda}}^{\lambda_i} \frac{\nu_r'(x) F^{n-1}(x) dx}{F^{n-1}(\lambda_i)}. \quad (34)$$

Thus, if  $\nu_r(\lambda)$  is too small, then the interest rate will be too large and some bidders will not borrow enough to be insolvent in the low state. The condition,  $\nu_r(\underline{\lambda}) > L + \bar{c}$  is sufficient to ensure that bidders are insolvent in the low state as long as the interest rate is positive. Q.E.D.

If we set  $\nu_r(\lambda_i) = \lambda_i H$ , then we obtain  $1 + r(\lambda_i, c_i) = 1/\lambda_i$  as a special case where the interest rate does not depend on  $c_i$ . Furthermore, if we set  $\nu_r(\lambda_i) = \lambda_i H + (1 - \lambda_i)L$  then the auction bids will be same as the standard cash auction (however the payment function is not identical to that in a cash auction). Finally, if  $\nu_r(\lambda_i)$  is set high enough and the number of bidders,  $n$ , is large enough then bidders bid above their expected value. This is because a high  $\nu_r(\lambda_i)$  lowers the interest rate and thus implicitly overvalues the debt.

In our discussion of Lemmas 2 and 3, we have emphasized that there is a tension between the information rents that are required in an efficient auction and the zero profit condition for a competitive equilibrium. We use this intuition again to show that a competitive equilibrium cannot be consistent with an efficient debt auction with pre-auction financing.

*Corollary to Proposition 3* If the debt market sets rates without conditioning on the bid and is perfectly competitive then the auction is inefficient.

*Proof.* If the debt market is perfectly competitive and the lender knows both  $\lambda_i$  and  $c_i$ , then each bidder must pay an interest rate that depends on their type and their cash, while the lender earns nothing. In order for the lender to break even the interest rate,  $r(\lambda_i, c_i)$ , must be set such that

$$\lambda_i(b(\lambda_i) - c_i)(1 + r(\lambda_i, c_i)) + (1 - \lambda_i)L = b(\lambda_i) - c_i, \quad (35)$$

where  $b(\cdot)$  is the equilibrium bid function (which the rational lender can ascertain). Therefore,  $r(\lambda_i, c_i)$  does not change if the bidder chooses to bid out of equilibrium. This can be rewritten as

$$1 + r(\lambda_i, c_i) = \left(1 - \frac{(1 - \lambda_i)L}{b(\lambda_i) - c_i}\right) \frac{1}{\lambda_i}. \quad (36)$$

For the competitive auction to be efficient, this interest rate must equal Equation (33), or

$$\frac{\lambda_i H - c_i}{\nu_r(\lambda_i) - c_i} = \frac{b(\lambda_i) - c_i - (1 - \lambda_i)L}{b(\lambda_i) - c_i}. \quad (37)$$

which rearranges to:

$$\frac{(1 - \lambda_i)L}{\nu_r(\lambda_i) - \lambda_i H} = \frac{b(\lambda_i) - c_i}{\nu_r(\lambda_i) - c_i} \quad (38)$$

The right hand side is independent of  $c_i$  only if  $b(\lambda_i) = \nu_r(\lambda_i)$  which is impossible as there are information rents in the auction (the seller does not know  $\lambda_i$ ), i.e.,  $b(\lambda_i) < \nu_r(\lambda_i)$  a contradiction. Q.E.D.

The intuition behind this corollary is similar to that emphasized in our discussions of Lemmas 2 and 3. A competitive financing market requires that information rents are independent of the cash position. However, the restrictions implied by efficiency lead to the result that a competitive financing market that is efficient must have no bid-shading. But that is impossible in an efficient auction as higher valuations must earn information rents and thus must shade their bid relative to their valuation.

#### □ **Pre Auction Equity Financing**

The equity market determines the  $\lambda_i$  and  $c_i$  of the bidder before the auction and commits to use only this information in determining the market capitalization,  $1/\phi(\lambda_i, c_i)$ . Assuming that the bidder needs to finance a portion of their bid,  $b_i > \bar{c} \forall i$  (which must be checked in equilibrium), the bidder's problem becomes

$$\max_{b_i} \left[ \{H\lambda_i + (1 - \lambda_i)L - c_i - \phi(\lambda_i, c_i)(b_i - c_i)(H\lambda_i + (1 - \lambda_i)L)\} Prob[b_i > \max_{\forall j \neq i} b_j] \right]. \quad (39)$$

Define  $\nu_i$

$$\nu_i = \frac{1}{\phi(\lambda_i, c_i)} - \frac{c_i}{\phi(\lambda_i, c_i)(H\lambda_i + (1 - \lambda_i)L)} + c_i, \quad (40)$$

and we can use Lemma 1 to solve for the optimal bid provided the restriction,  $b(\underline{\nu}) - \bar{c} > 0$  that all bidders raise capital in the equity markets is satisfied. From Lemma 1, an efficient auction exists if  $\frac{d\nu_i}{dc_i} = 0$ . The restrictions on the valuation function  $\phi(\lambda_i, c_i)$  that ensure this plus the conditions that result in default in the low state are stated in Proposition 4 (see Table 2 for a summary of Proposition 4).

*Proposition 4.* If the lender knows  $\lambda_i$  and  $c_i$  and must set a market capitalization for each bidder without conditioning on the bid, then the only market capitalization which results in an efficient auction is

$$1/\phi(\lambda_i, c_i) = [H\lambda_i + (1 - \lambda_i)L] \frac{\nu_\phi(\lambda_i) - c_i}{H\lambda_i + (1 - \lambda_i)L - c_i}, \quad (41)$$

with  $\nu_\phi(\underline{\lambda}) > \bar{c}$  and  $\nu_\phi(\lambda_i) > 0$ , where the subscript  $\phi$  denotes Equity.

*Proof.* Lemma 1 shows that for the auction to be efficient  $\nu_i$  must be increasing in  $\lambda_i$  and not change with  $c_i$ .

Therefore,  $\phi(\lambda_i, c_i)$  can be set such that

$$\frac{1}{\phi(\lambda_i, c_i)} - \frac{c_i}{\phi(\lambda_i, c_i)(H\lambda_i + (1 - \lambda_i)L)} + c_i = \nu_\phi(\lambda_i), \quad (42)$$

where  $\nu'_\phi(\lambda_i) > 0$ . Rearranging yields Equation (41). Since  $\phi(\lambda_i, c_i)$  must be positive,  $\nu_\phi(\underline{\lambda}) > \bar{c}$ . Q.E.D.

We note that setting  $1/\phi(\lambda_i, c_i) = H\lambda_i + (1 - \lambda_i)L$  results in  $\nu_\phi(\lambda_i) = H\lambda_i + (1 - \lambda_i)L$  and the same bid as the cash bid in a standard cash auction. We show next that this is consistent with perfectly competitive financial markets.

*Corollary to Proposition 4* If the equity market sets the rate without conditioning on the bid and is perfectly competitive then the auction is efficient.

*Proof.* If the equity market is perfectly competitive and the equity market knows both  $\lambda_i$  and  $c_i$ , then each bidder must sell equity at a rate that depends on their type, while the equity market earns nothing. In order for the equity market to break even the rate,  $\phi(\lambda_i, c_i)$ , must be set such that

$$\phi(\lambda_i, c_i)(H\lambda_i + (1 - \lambda_i)L) = 1, \quad (43)$$

which is identical to one of the rates that results in an efficient auction. Q.E.D.

The key difference between equity and debt is that the break even condition for competitive equilibrium for equity does not involve the bid, for debt it does (compare Equations (35) and (43)). In a competitive debt market if the rate is set before the bids then bidders wish to over bid since increasing their bid does not increase their rate and if the rate was competitive at the old bid it is now too low for the increased bid (this is true unless they have already bid their value). If the market is not competitive the lender can set a high rate that counters this incentive. In equity markets the price of stock is set correctly for any amount of shares sold so there is no overbidding incentive.

## 6 Post Auction Financing

■ We now consider what happens when the securities market learns the bid of the winning bidder after the auction and may be able to determine the bidder's cash if it is incentive compatible for the bidder to reveal their cash (or by auditing if it is not). Thus, the securities market could use the information from the auction to determine the rate. The rate could be a function of  $b_i$  and/or  $c_i$ ; with debt  $r(b_i, c_i)$  or with equity  $\phi(b_i, c_i)$ .<sup>18</sup> With this general functional form for the rate, we cannot find a closed form solution for the bid function (Lemma 1 does not apply). However, Lemma 2 will allow us to derive some general conclusions.

In the above section we allowed the rate to depend on the valuation but not the actual bid. One could argue that the bidder's type is unverifiable and thus, banks are unlikely to be able to do this. Furthermore, if the type is easily determined then the seller should not need to use an auction. In this section, the lender can use the bidder's actions in the auction to help determine the bidder's type. Again, the important questions are whether the lender is able to use this information to set an rate function that results in an efficient auction, and whether a competitive equilibrium can yield an efficient rate.

<sup>18</sup>Engelbrecht-Wiggans and Kahn (1991) and Rothkopf et al (1990) show that auction outcomes can have effects outside the auction. Katzman and Rhodes-Kropf (2000) generalize how bid information can affect secondary games such as obtaining financing.

A key difference between the results on pre-auction financing and this section on post-auction financing is that contracting on the bid and cash is different from contracting on the valuation and cash. The bid  $b_i$  now plays two roles: that of winning the auction and that of obtaining a better interest rate. The second role did not exist in the section on pre-auction financing where the financial market could observe the valuation ( $\lambda_i$ ).

□ **Post Auction Debt Financing**

Assuming the bidder still borrows and is still insolvent in the bad state (Proposition 5 shows that the restriction  $(H - L)\underline{\lambda} > \bar{c}$  ensures the bidder borrows and is insolvent in the bad state), the bidder's problem is

$$\max_{b_i} \left[ \{H\lambda_i - c_i - \lambda_i(1 + r(b_i, c_i))(b_i - c_i)\} \text{Prob}[b_i > \max_{\forall j \neq i} b_j] \right]. \quad (44)$$

Because the interest rate is a function of the bid, we cannot rearrange this maximization and use Lemma 1. Now bidders use the bid both to win the auction and to improve the interest rate obtained from the lender. However, we can use Lemma 2 to understand the restrictions required for an efficient auction (see Table 1 for a summary).

*Proposition 5.* If the lender does not know  $\lambda_i$  or  $c_i$  but can commit to an interest rate that depends on the bid and the amount of cash the bidder chooses to reveal<sup>19</sup>, then the only interest rate which results in an efficient auction is

$$1 + r(b_i, c_i) = [H - \frac{c_i}{b_r^{-1}(b_i)} - \int_{\underline{\lambda}}^{b_r^{-1}(b_i)} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(b_r^{-1}(b_i))} dx] \frac{1}{b_i - c_i}, \quad (45)$$

where  $b_i$  is the bid of bidder  $i$ ,  $c_i$  is the amount of cash the bidder has (we will show the bidder reveals this truthfully) and where  $b_r(\cdot)$  is any function chosen by the lender which satisfies,  $b_r'(\cdot) > 0$ ; here  $b_r(\cdot)$  is the resulting equilibrium bid function. To ensure all bidders borrow we require  $(H - L)\underline{\lambda} > \bar{c}$ .

*Proof.* To prove this proposition we will assume that there is an interest rate that depends on the bid and the revealed cash and results in an efficient auction. Further, we will assume that because bidders cannot steal they truthfully reveal their cash. Then we will determine what interest rate function is required, and show that with that interest rate the auction is efficient and that bidders do indeed truthfully reveal their cash. To begin we will use the direct revelation mechanism and write  $b_i$  as  $b(x)$  where  $x$  is a selection from  $[\underline{\lambda}, \bar{\lambda}]$ . Since we are searching for an efficient mechanism, the bid  $b(x)$  and the probability of winning the auction  $F^{n-1}(x)$  do not depend on  $c$ . Furthermore,  $1 + r(b_i, c_i)$  can be written as  $1 + r(b(x), c_i)$ , where  $c_i$  is the assumed truthfully revealed cash. Therefore, the bidder's problem is

$$\max_x \left[ \{H\lambda_i - c_i - \lambda_i(1 + r(b(x), c_i))(b(x) - c_i)\} F^{n-1}(x) \right]. \quad (46)$$

We multiply the maximization by  $a(\lambda_i, c_i) = \frac{1}{\lambda_i}$  and set bid. Set

$$p(x, c_i) = (1 + r(b(x), c_i))(b(x) - c_i), \quad (47)$$

$$\nu(\lambda_i, c_i) = H - \frac{c_i}{\lambda_i}. \quad (48)$$

<sup>19</sup>The bidder's revelation must be  $c \in [\underline{c}, \bar{c}]$ .

Hence we can use Equation (9) to obtain the payment function<sup>20</sup>

$$p(\lambda_i, c_i) = H - \frac{c_i}{\lambda_i} - \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx. \quad (50)$$

Individual rationality follows since  $0 \leq H\lambda_i + (1 - \lambda_i)L - c_i - \lambda_i(1 + r(b_i, c_i))(b_i - c_i) - (1 - \lambda_i)L$  as long as  $p(\lambda_i, c_i) \leq H - \frac{c_i}{\lambda_i}$ . It must also be the case that the borrowers default in the bad state,  $p(\lambda_i, c_i) > L$ . Since  $p_1(\lambda_i, c_i) > 0$  and  $p_2(\lambda_i, c_i) < 0$  the default condition can be reduced to the sufficient and necessary condition  $(H - L)\underline{\lambda} > \bar{c}$ .

Using the definition of  $p(\lambda_i, c_i)$  we find that in equilibrium, the efficient interest rate is

$$1 + r(b(\lambda_i), c_i) = [H - \frac{c_i}{\lambda_i} - \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx] \frac{1}{b(\lambda_i) - c_i}. \quad (51)$$

Since the lender sets the interest rate before the auction and without knowledge of  $\lambda_i$ , we conjecture that that

$$1 + r(b_i, c_i) = [H - \frac{c_i}{b_r^{-1}(b_i)} - \int_{\underline{\lambda}}^{b_r^{-1}(b_i)} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(b_r^{-1}(b_i))} dx] \frac{1}{b_i - c_i}. \quad (52)$$

where  $b_r(\cdot)$  is the function used by the lender to invert the chosen bid  $b_i$ . Reconsidering the bidder's optimization problem again, we substitute the interest rate function (52) into the bidder's objective function, Equation (46) and get

$$\max_x \left[ \left\{ \frac{-c_i}{\lambda_i} + \frac{c_i}{b_r^{-1}(b(x))} \right\} F^{n-1}(b_r^{-1}(b(x))) + \int_{\underline{\lambda}}^{b_r^{-1}(b(x))} \frac{c_i F^{n-1}(w)}{w^2} dw \right]. \quad (53)$$

Quasi-concavity is easily shown and the resulting FOC is  $\frac{c_i}{\lambda_i} = \frac{c_i}{b_r^{-1}(b(x))}$ . Therefore, bidders will choose  $x$  such that  $b(x) = b_r(\lambda_i)$ . For the auction to be efficient the function  $b_r(\cdot)$  must be chosen such that  $\frac{\partial b_r(\lambda_i, c_i)}{\partial \lambda_i} > 0$ . Finally we need to show that bidders truthfully report their cash positions,  $c_i$ , to the lender, this is shown in the Appendix. Q.E.D.

Notice that in the efficient auction, the lender receives

$$\lambda_i(b(\lambda_i) - c_i)(1 + r(b(\lambda_i), c_i)) + L(1 - \lambda_i), \quad (54)$$

and the bidder pays this plus  $c_i$ . Substituting in for  $p(\lambda_i, c_i)$  from Equation (50) yields

$$H\lambda_i + L(1 - \lambda_i) - c_i - \lambda_i \int_{\underline{\lambda}}^{\lambda_i} \frac{c_i F^{n-1}(x)}{x^2 F^{n-1}(\lambda_i)} dx. \quad (55)$$

Note that the bidder's payment is the amount in Equation (55) plus  $c_i$ . Therefore, the bidder's payment and the lender's receipts are unaffected by the interest rate. However, the total payment made by the bidder is decreasing in  $c_i$  and thus, having higher cash yields higher utility to the bidders.

<sup>20</sup>It is easy to show that

$$f(x|\lambda) = [\nu(\lambda, c) - p(x, \lambda, c)] \frac{dF^{n-1}(x)}{dx} - p_1(x, \lambda, c) F^{n-1}(x) \quad (49)$$

is strictly increasing in  $\lambda$  for all  $c$ , so the first order condition is sufficient.

The seller receives the bid,  $b_r(\cdot)$ , that is set by the lender. Thus, if  $b_r(\cdot)$  is set lower, then the interest rate is higher, the bidder bids less and the seller does worse. If the bidder bids less, then the lender must lend less. Since Equation (55) shows that the lender's total receipts are unaffected, his rate of return must increase.

The intuition for why bidders with higher cash positions earn higher information rents follows from Lemma 2 and Proposition 5. Bidders with more cash borrow less and hence keep a larger portion of the ex-post payoff. Thus a small change in valuation has a higher effect on bidder payoffs. Consequently, their total payment must be lower and their information rent higher.

Given the characterization in Proposition 5, we use Lemmas 2 and 3 to ask whether a competitive equilibrium can be efficient.

*Corollary to Proposition 5* The interest rate that is a function of the bid and results in an efficient auction is not the result of perfect competition in the lending market.

*Proof.* We calculate

$$\frac{1}{a(\lambda_i, c_i)}[\nu_1(x, c_i) - p_2(x, x, c_i)] = \frac{\lambda_i c_i}{x^2} \quad (56)$$

which is not independent of the cash position  $c_i$ . By Lemmas 2 and 3, the competitive equilibrium is not efficient. Q.E.D.

The intuition for the result is the tension between efficiency and competitive equilibrium that we have been emphasizing throughout the paper. The efficient auction requires bidders with the same valuation but different cash position to pay the same bid but actually make different expected payments. Thus a bidder with a larger cash position will receive an bigger contingent payoff. Thus a marginal change in valuation is worth more for such a bidder, i.e., the information rent has to be higher for a bidder with a larger cash position (keeping the valuation fixed). In contrast, the zero profit condition of a competitive equilibrium requires that the information rent not depend on the cash position in an efficient auction. Therefore, debt financing from a competitive lending market will not yield efficient auctions.

#### □ **Post Auction Equity Financing**

Although competitive debt financing is always inefficient, Section 5 demonstrated that equity financing could result in an efficient auction if the equity market knew the underlying valuations of the bidders. As we have argued, this may not be verifiable, a more realistic assumption is that the equity market must learn the type of the bidder from the bidder's actions in the auction. As a consequence, the bidder uses the bid to influence the pricing of equity. Next, we examine an equity financed cash auction where the stock price is set after the bids are revealed (but the bidder knows before the auction how the price of equity will be set). This approach closely follows our argument with debt and shows that competitive equilibria do not yield efficiency.

Assuming that the bidder still needs to finance a portion of their bid,  $b_i > \bar{c} \forall i$  (which must be checked in equilibrium), the bidder's problem is

$$\max_{b_i} \left[ \{H\lambda_i + (1 - \lambda_i)L - c_i - \phi(b_i, c_i)(b_i - c_i)(H\lambda_i + (1 - \lambda_i)L)\} Prob[b_i > \max_{\forall j \neq i} b_j] \right]. \quad (57)$$

As with ex-post debt, we cannot solve for the general solution. However, we can use Lemmas 2 and 3 to characterize the efficient auction and ask whether it is consistent with a competitive equilibrium (see Table 2 for a summary).

*Proposition 6.* If the equity provider does not know  $\lambda_i$  or  $c_i$  but can commit to set a market capitalization that depends on the bid and the amount of cash the bidder chooses to reveal<sup>21</sup>, then the only  $\phi$  which results in an efficient auction is  $\phi(b_i, c_i) =$

$$\left[ 1 - \frac{c_i}{Hb_\phi^{-1}(b_i) + (1 - b_\phi^{-1}(b_i))L} - \int_{\underline{\lambda}}^{b_\phi^{-1}(b_i)} \frac{c_i(H - L)F^{n-1}(x)}{(Hx + (1 - x)L)^2 F^{n-1}(b_\phi^{-1}(b_i))} dx \right] \frac{1}{b_i - c_i}, \quad (58)$$

where  $b_i$  is the bid of bidder  $i$ ,  $c_i$  is the amount of cash the bidder claims to have and where  $b_\phi(\cdot)$  is any function chosen by the equity provider which satisfies,  $b_\phi(\underline{\lambda}) \geq c_i$ ,  $b'_\phi(\lambda_i) > 0$ . Furthermore,  $b_\phi(\cdot)$  is the resulting equilibrium bid function.

*Proof.* See the Appendix.

With equity financing as with debt financing there exist efficient auctions. However any efficient auction with equity requires that bidders with higher cash positions (keeping valuation fixed) receive higher information rents. Next, we use Lemmas 2 and 3 to demonstrate that with ex-post financing the competitive equilibrium in the equity market is not efficient.

*Corollary to Proposition 6* The stock price that is a function of the bid and results in an efficient auction is not the result of perfect competition in the equity market.

*Proof.* We use Lemma 2 and note that

$$a(\lambda_i, c_i) = \frac{1}{\lambda_i H + (1 - \lambda_i)L}, \quad (59)$$

$$p(\lambda_i, c_i) = \phi(b(\lambda_i), c_i)(b(\lambda_i) - c_i), \quad (60)$$

$$\nu(\lambda_i, c_i) = 1 - \frac{c_i}{H\lambda_i + (1 - \lambda_i)L} \quad (61)$$

Since

$$\frac{1}{a(\lambda_i, c_i)} [\nu_1(x, c_i) - p_2(x, x, c_i)] = \frac{c_i(H\lambda_i + (1 - \lambda_i)L)(H - L)}{(Hx + (1 - x)L)^2} \quad (62)$$

it follows from Lemma 2 that competitive equity financing cannot be efficient. Q.E.D.

Again there is a tension between the efficient auction and competitive equilibrium. Incentive compatibility (higher information rent for greater cash position  $c$ ) conflicts with zero profit equity markets (equal information rent for all cash positions  $c$ ).

Our general conclusion is that with standard securities like debt and equity, perfectly competitive financial markets do not yield the efficient outcomes in the auction unless bidders use only equity financing and the equity market has ex-ante perfect information.

<sup>21</sup>The bidder's revelation must be  $c \in [\underline{c}, \bar{c}]$ .

□ **Post Auction State-Contingent Financing**

When bidders obtain competitive financing with either debt or equity, and the financing depends on their bids (or the amount that they borrow), we have shown that the auction will not be efficient. However, this raises the question as whether there exist contingent claims that can yield efficient auctions. We allow for contingent claims and find that if every bidder is cash constrained and if ex-post payments from the lender to the winning bidder are allowed, then proper security design can yield an efficient auction. However if we allow for the possibility that some bidders do not need to get financing, then the auction will not be efficient.

Since there are only two states, we consider state contingent payment functions, where  $h(b_i, c_i) \leq H$  is the payment in the high state and  $l(b_i, c_i) \leq L$  is the payment in the low state. Thus the expected payment  $q_i(b_i, \lambda_i, c_i) = \lambda_i h(b_i, c_i) + (1 - \lambda_i)l(b_i, c_i)$ . We do not impose the condition that  $h(b_i, c_i) \geq 0$  and  $l(b_i, c_i) \geq 0$ . Thus, at this stage, we allow for the possibility that the lender may pay the bidder in some states of the world. The bidder's problem becomes

$$\max_{b_i} \left[ \{H\lambda_i + (1 - \lambda_i)L - c_i - \lambda_i h(b_i, c_i) - (1 - \lambda_i)l(b_i, c_i)\} Prob[b_i > \max_{\forall j \neq i} b_j] \right]. \quad (63)$$

Remember that the seller is not directly concerned with how the bidders raise money; the seller just awards the firm to the bidder who pays him the most (has the highest bid). Using Lemmas 2 and 3, we can prove the following proposition (see Table 3 for a summary).

*Proposition 7.* If  $H\underline{\lambda} + (1 - \underline{\lambda})L > \bar{c}$  (all types borrow), then any competitive state-contingent financing scheme such that  $h(b_i, c_i) = w(b_i) - c_i \leq H$ ,  $l(b_i, c_i) = z(b_i) - c_i \leq L$ , with the restrictions that  $h_x(x, c_i) - l_x(x, c_i) \leq 0$  and  $H - L - [h(x, c_i) - l(x, c_i)] \geq 0$ , at least one inequality strict,  $\forall c_i$ , results in an efficient auction.

*Proof* Once again we use a direct revelation mechanism approach and write  $b_i$  as  $b(x)$  where  $x$  is a selection from  $[\underline{\lambda}, \bar{\lambda}]$ . Next we assume that the auction is efficient, which we will later verify. If the auction is efficient the probability of winning the auction is  $F^{n-1}(x)$ . Furthermore,  $h(b_i, c_i)$  and  $l(b_i, c_i)$  can be written as  $h(b(x), c_i)$  and  $l(b(x), c_i)$  or just  $h(x, c_i)$  and  $l(x, c_i)$ . We will show that  $H\underline{\lambda} + (1 - \underline{\lambda})L > \bar{c}$  ensures that every bidder needs to borrow, therefore, the bidder's problem is

$$\max_x [\{H\lambda_i + (1 - \lambda_i)L - c_i - \lambda_i h(x, c_i) - (1 - \lambda_i)l(x, c_i)\} F^{n-1}(x)]. \quad (64)$$

Define

$$\begin{aligned} \nu(\lambda_i, c_i) &= H\lambda_i + (1 - \lambda_i)L \\ p(x_i, \lambda_i, c_i) &= c_i + \lambda_i h(x, c_i) + (1 - \lambda_i)l(x, c_i), \end{aligned} \quad (65)$$

and define

$$f(x | \lambda_i) = [\nu(\lambda, c) - p(x, \lambda, c)] \frac{dF^{n-1}(x)}{dx} - p_1(x, \lambda, c) F^{n-1}(x) \quad (66)$$

The first order condition is that  $f(\lambda_i | \lambda_i) = 0$  and the sufficient condition is that  $f(x | \lambda_i)$  is increasing in  $\lambda_i$  for all  $x$ . This requires that,

$$h_x(x, c_i) - l_x(x, c_i) \leq 0, \quad (67)$$

and

$$H - L - h(\underline{\lambda}, c_i) + l(\underline{\lambda}, c_i) \geq 0, \quad \forall c_i, \quad (68)$$

with one of the two inequalities strict.

Using Equation(9) we obtain that the payment function is

$$p(\lambda_i, \lambda_i, c_i) = \nu(\lambda_i, c_i) - \int_{\underline{\lambda}}^{\lambda_i} [\nu_1(x, c_i) - p_2(x, x, c_i)] dx \quad (69)$$

$$= H\lambda_i + (1 - \lambda_i)L - \int_{\underline{\lambda}}^{\lambda_i} \frac{[H - L - [h(x, c_i) - l(x, c_i)]] F^{n-1}(x) dx}{F^{n-1}(\lambda_i)}. \quad (70)$$

If the financing is competitive then

$$b(\lambda_i) - c_i = \lambda_i h(\lambda_i, c_i) + (1 - \lambda_i) l(\lambda_i, c_i). \quad (71)$$

Therefore, the bid function is

$$b(\lambda_i) = H\lambda_i + (1 - \lambda_i)L - \int_{\underline{\lambda}}^{\lambda_i} \frac{[H - L - [h(x, c_i) - l(x, c_i)]] F^{n-1}(x) dx}{F^{n-1}(\lambda_i)}. \quad (72)$$

Since the bids are increasing in  $\lambda_i$ , the lowest bid is  $H\underline{\lambda} + (1 - \underline{\lambda})L$ , which must be greater than  $\bar{c}$  to ensure that every bidder needs to borrow.

From Lemma 2 we know that an efficient auction that is consistent with a competitive market must satisfy the requirement that

$$\nu_1(\lambda_i, c_i) - p_2(\lambda_i, \lambda_i, c_i) = (H - L) - [h(x, c_i) - l(x, c_i)] \quad (73)$$

is independent of  $c_i$ . This implies that,

$$\frac{\partial h(x, c_i)}{\partial c_i} = \frac{\partial l(x, c_i)}{\partial c_i}, \quad (74)$$

almost everywhere. Differentiating Equation (71), the zero profit requirement, w.r.t.  $c_i$  tells us that

$$\lambda_i \left[ \frac{\partial h(x, c_i)}{\partial c_i} - \frac{\partial l(x, c_i)}{\partial c_i} \right] + \frac{\partial l(x, c_i)}{\partial c_i} = -1. \quad (75)$$

Thus,

$$\frac{\partial l(x, c_i)}{\partial c_i} = -1 = \frac{\partial h(x, c_i)}{\partial c_i}. \quad (76)$$

Therefore, the only solutions that allow for an efficient auction with competitive state contingent financing are  $h(\lambda_i, c_i) = w(\lambda_i) - c_i$  and  $l(\lambda_i, c_i) = z(\lambda_i) - c_i$ .<sup>22</sup> Q.E.D.

<sup>22</sup>The bidder clearly has no incentive to lie about or reduce  $c_i$ , since his payment just goes up by the amount he reduces  $c_i$ . And the bidder cannot increase  $c_i$  since he is cash constrained.

The intuition behind Proposition 7 is as follows. First we have to make information rents independent of the cash position. The only way to do this is to make the bidder's ex-post payoff move identically in each state with the cash position (this is Equation (74)). With the zero profits constraint (71), this implies that one must rebate the cash position in every state of the world, Equation (76), leading to the solution that is obtained.

We note that the above scheme may require payments from the lender to the winning bidder in the low state. In particular, the maximum payment in the low state is  $L$ . Since  $l(\lambda_i, \underline{c}) = z(\lambda_i) - \underline{c} \leq L$ , we have that  $l(\lambda_i, \bar{c}) = l(\lambda_i, \underline{c}) + (\underline{c} - \bar{c}) \leq L + (\underline{c} - \bar{c}) \leq 0$  if  $L \leq (\bar{c} - \underline{c})$ . This is certainly true if  $L = 0$ . In this case, all such schemes require subsidies in the low state to some bidders. If subsidies are not allowed, then no competitive state contingent schemes exist.

While the above theorem provides the requirements for a contingent scheme that is consistent with competitive equilibrium, no scheme is actually shown. Given the conditions of the theorem, there exist many competitive contingent schemes. One example is the following. Let  $h(\lambda_i, c_i) = H + \delta(\lambda_i - \underline{\lambda}) - c_i$  and  $l(\lambda_i, c_i) = L + \nu(\lambda_i - \underline{\lambda}) - c_i$  where  $\delta < 0$  and  $\nu > 0$ . Also, the parameters satisfy  $\nu(\bar{\lambda} - \underline{\lambda}) < \underline{c}$ . This ensures that the total payment in the low state is less than  $L$ . For higher types, the scheme reduces payments in the high state and increases payments in the low state. This makes it incentive compatible. Thus, there exist many contingent payment schemes that satisfy the conditions of Proposition 7.

So we find that with state contingent payoffs, a form of competitive financing results in an efficient auction. The intuition for this result is straightforward. To ensure that the efficient auction is competitive, we need to make information rents independent of cash position. The only way to do this is to ensure that changes in the cash position affect the payoff in each state equally. This requirement with the zero-profit condition implies that one must rebate the cash position in every state of the world and then the bidder is charged an amount that only depends on his type. We note that such contingent schemes are very different from standard financing where the contingent payment does not involve rebating of the cash paid.

## 7 Variations

■ We now consider some variations on our basic model. First we allow for some bidders to be unconstrained, i.e., to have sufficient capital. This makes it difficult to obtain efficiency as contingent securities that ensure that constrained bidders and unconstrained bidders bid identically in equilibrium (for a given valuation) do not exist. Then we ask whether a government that restricts access to capital markets can ensure efficiency and whether the presence of capital markets helps or hurts.

### □ Some Bidders Unconstrained

It seems that with appropriate contingent claims the auction will be efficient. The above result relied on a particular assumption that will not in general be true: all bidders needed to obtain financing. Further the efficient auction relied on securities that rebated the cash back in all states of the world and essentially made

all bidders make contingent payments. If a bidder does not obtain financing then his cash is not rebated back and he does not make a contingent payment. This implies that his incentives to affect the lender disappear. In this more general case, the auction will not be efficient if some bidders do not require financing.

*Proposition 8.* If some bidders require financing and other bidders do not, and some of the bidders who require financing bid such that  $b(\bar{\lambda}) > L + \underline{c}$  (financing entails risk), then competitive financing will not result in an efficient auction.

*Proof.* If the auction is efficient then the probability that a particular bidder with type  $x$  wins is  $F^{n-1}(x)$ , regardless of whether he needs to borrow. Therefore, the maximization problem of a bidder who needs financing is Equation (64) and his resulting bid is Equation (72). Simultaneously, a bidder that does not need to borrow faces

$$\max_x [\{H\lambda_i + (1 - \lambda_i)L - b(x)\}F^{n-1}(x)]. \quad (77)$$

Remember, this is true assuming the auction is efficient. Thus, a bidder who does not need to borrow bids

$$b(\lambda_i) = H\lambda_i + (1 - \lambda_i)L - \int_{\underline{\lambda}}^{\lambda_i} \frac{[H - L] F^{n-1}(x) dx}{F^{n-1}(\lambda_i)}. \quad (78)$$

This bid function is only equal to Equation (72) if  $h(x, c_i) = l(x, c_i)$ , bidders make the same payment in both states of the world. If bidders make the same payment in both states then this payment must be less than  $L$  plus  $c$  or the bidders cannot pay. This payment must also be larger than or equal to what they ‘borrowed’, or else the financier does not break even. Therefore, the largest amount borrowed must be less than or equal to  $L$ ,  $b(\bar{\lambda}) - \underline{c} \leq L$ . In other words, the financing must not entail any risk. If instead, the financing requires risk, as assumed by  $b(\bar{\lambda}) > L + \underline{c}$ , then the auction will not be efficient. Q.E.D.

In order to get complete efficiency, the amount financed would need to be so low that the ‘borrower’ would never default. Thus, auctions with bidders who obtain competitive financing are not efficient if some of the bidders do not need financing.

#### □ **Efficiency With Restricted Capital Markets**

When governments hold auctions they are often very interested in efficiency. Our discussion in the previous sections suggests that a cash auction is unlikely to be efficient when bidders finance their bids in competitive financial markets. To obtain efficiency, the government might offer financing. We analyze this under two different assumptions. First, the seller is the only source of financing. Second, a competitive financial market exists (the outside option).

In some parts of the world (such as Eastern Europe) or for some objects, financing is difficult to acquire. In these situations the seller may be the only avenue for financing. Aghion, Hart and Moore (1992) and others have suggested that when bidders cannot get financing the seller allow bids in the form of securities such as debt. Rhodes-Kropf and Viswanathan (2000) examine this idea for the case when bidders have identical amounts of cash. This section analyzes efficiency for the seller when the seller is the only source of financing for bidders.

It is uninteresting to consider a seller who can provide pre-auction efficient financing, because a seller with this much information about the bidders would not need an auction. More reasonably, the seller provides financing after seeing the bid.

*Proposition 9.* There exists an efficient seller financed auction with either debt or equity even if some bidders do not require financing and other bidders will never default.

*Proof.* The possibility of no default or no borrowing means that for any amount of cash  $c_i$  the bid could be in one of three regions (some possibly empty): with  $\lambda_i \in [\underline{\lambda}, \underline{x}(c_i)]$  then  $b_c(\lambda_i) < c_i$ , with  $\lambda_i \in [\underline{x}(c_i), \bar{x}(c_i)]$  then  $c_i < b_c(\lambda_i) < c_i + L$ , with  $\lambda_i \in [\bar{x}(c_i), \bar{\lambda}]$  then  $c_i + L < b_c(\lambda_i)$ . Here  $b_c(\lambda_i)$  is the cash auction bid. Since the auction is efficient, then any bidder in the first region (no borrowing) will bid the standard cash bid, Equation (78). In the second region (no default) the bidder's problem is

$$\max_x [\{H\lambda_i + (1 - \lambda_i)L - p(x, \lambda_i, c_i)\}F^{n-1}(x)], \quad (79)$$

where  $p(x, \lambda_i, c_i) = c_i + \lambda_i(1 + r(b(x), c_i))(b(x) - c_i)$ . Using standard techniques we find that in equilibrium  $p(\lambda_i, \lambda_i, c_i) = b_c(\lambda_i)$  as long as  $r(b(x), c_i) = 0$  (there is no default in this region, so we are back to the cash auction). Therefore, region 1 can be merged with region 2 into  $\lambda_i \in [\underline{\lambda}, \bar{x}(c_i)]$ . In Proposition 5 if we choose  $b_r(\lambda_i) = b_c(\lambda_i)$  we will obtain the cash bid, the ensures that the bid in the last region will also be the cash bid. The sufficient conditions can be verified directly (due to the changes in these regions there are more cases to consider). A similar argument holds for the equity auction. Q.E.D.

Therefore, if the government has the appropriate information about the distribution of bidder types and they can require bidders to use the seller financing (or the seller is the only source of financing), then they can ensure efficiency. This is an important result for privatizations. It tells us that if governments require contingent payments of a particular form then they can ensure efficiency even if bidders vary in their access to cash.

#### □ **Efficiency With Outside Access to Financial Markets**

If the auction allows cash bids, then the requirement that the seller is the only source of financing is a strong assumption. If the auction is efficient then the bidder could use the bid and his contractible cash to go to a competitive financial market. Under such circumstances, seller financing will only be used if it offers terms better than a competitive financial market. If there is one bidder who does not require financing, the seller cannot attain efficiency. Under the conditions of Proposition 10, seller financing cannot improve upon the competitive financial market (in terms of achieving efficiency).

*Proposition 10.* Suppose some bidders require financing and other bidders do not, and some of the bidders who require financing bid such that  $b(\underline{\lambda}) > L + \underline{c}$  (there is risk in lending to the lowest type with the lowest cash). Further suppose the seller is constrained to offer financing at equal or better terms than a competitive financial market (the outside option). Then there is no efficient auction.

*Proof.* Assume the auction is efficient. Since there is one bidder who is not financially constrained, Proposition 8 demonstrates that all bidders must make the same bid as their respective cash auction bid. With competitive

outside financing the bidders who borrow from the seller should expect to pay back no more than what they borrowed. Thus, in total, the bidders should expect to pay no more than the cash bid. Hence, with seller financing the expected payment made by any bidder must be less than or equal to the cash auction (otherwise this bidder will obtain financing on better terms in the competitive financial market, the outside option). Therefore, the cash bid, Equation (78), minus the contingent auction expected payment, Equation (70), must be less than or equal to zero, or

$$\int_{\underline{\lambda}}^{\lambda_i} \frac{[h(x, c_i) - l(x, c_i)] F^{n-1}(x) dx}{F^{n-1}(\lambda_i)} \leq 0. \quad (80)$$

However, from Equation (70) we know that for the lowest type,  $\underline{\lambda}h(\underline{\lambda}, c_i) + (1 - \underline{\lambda})l(\underline{\lambda}, c_i) + c_i = H\underline{\lambda} + (1 - \underline{\lambda})L$ . Rearranging this equality yields  $\underline{\lambda}[h(\underline{\lambda}, c_i) - l(\underline{\lambda}, c_i)] = H\underline{\lambda} + (1 - \underline{\lambda})L - [l(\underline{\lambda}, c_i) + c_i] > H\underline{\lambda} + (1 - \underline{\lambda})L - [L + c_i] > 0$ . By assumption (that there is risk in lending) the last equation is true at least for the lowest type with the lowest cash. Thus,  $h(\underline{\lambda}, c_i) - l(\underline{\lambda}, c_i) > 0$  and hence we cannot satisfy the integral inequality in Equation (80) locally around  $\underline{\lambda}$  (we are using the continuity of the bid function here). Hence the financing provided by the seller must be more expensive for these types than competitive financing and thus, no efficient equilibrium exists. Q.E.D.

The intuition for Proposition 10 is as follows. Since financing involves contingent payments, locally around the lowest type, the expected payment increases with type faster than the cash bid payment. Since the bidder only bids the cash bid (a consequence of the presence of a bidder who does not need financing), this implies that the expected payment exceeds the bid. Under such circumstances, the bidder would go to the competitive market for financing and thus, the seller is unable to implement the efficient auction. This suggests that even when financing schemes exist that yield the efficient auction, the presence of a competitive financial market imposes constraints on the seller that make it impossible to attain efficiency.

## 8 Conclusion

■ Cash auctions are used worldwide to sell assets of significant unknown value. Since the bidder with the best use for the object for sale is often not the bidder with the most cash, financing is a regular occurrence. For example, in the FCC bandwidth auctions, in which the government sold sections of the radio spectrum, many bidders obtained financing. For a segment of the auction (the C Block) the government actually provided the financing. Many winners of the European 3G bandwidth auctions financed their bids. Bidders who buy property often acquire asset based financing. Firms sold in bankruptcy regularly have financed bids, as the current management team is often a bidder. In privatization auctions, bidders attempt to acquire financing. And mergers and acquisitions of all types have financed bidders.

Standard intuition is that as long as bidders have cash in the auction, the auction will be efficient; the bidder with the highest value will win. Or, at least if there are competitive liquid capital markets, then the

auction will be efficient. Baird (1986) and Jackson (1986) have argued that a cash auction is the efficient procedure for selling a bankrupt firm. Aghion, Hart, and Moore (1992) seem to agree with this view but argue that the issue of seller financing (allowing non-cash bids) is important if capital markets are not efficient. Work by Bolton and Roland (1992) and Rhodes-Kropf and Viswanathan (2000) focuses on non-cash auctions for privatizations and Eastern European bankruptcies because of the lack of access to cash. Our paper suggests that while market imperfections may be a source of auction inefficiencies, financed auctions of all types will be inefficient, even with competitive liquid capital markets.

We consider bidders who have private values and different amounts of available cash. However, they all have access to capital markets. Because we consider financial securities, we explicitly model the ex post uncertainty of the valuation of the object for sale. This innovation is critical for the examination of financing since the payoffs of financial claims depend differently on the final realizations. Capital markets can be reasonably modelled as having either no information about individual bidders, full information but incomplete contracting, or information about the actions of the bidders. We consider each possibility to show that it is unlikely that the auction will be efficient.

We characterize efficient auctions and show that there is a tension between efficient auctions and competitive markets. In an efficient auction, while the bid is independent of the cash position, the information rent is not. For example, with debt, a bidder with more cash will make a lower ex-post contingent payment. Thus the marginal change in a bidder's payoff for a change in valuation is higher for such a bidder, i.e., a bidder with more cash will have a higher information rent (keeping valuation fixed).

In contrast a competitive equilibrium that is efficient requires that information rents do not depend on cash positions. This tension between information rents in a efficient auction and zero profits in a competitive equilibrium implies that most often, competitive financing is not efficient.

Auctions around the world have bidders who attain financing based on project risk, or by demonstrating they are a good type, or after showing the amount they need to borrow. We have shown that the efficiency of the auction design cannot be considered separately from the way bidders finance their bids even if the financial markets are competitive. The adverse selection in the securities market affects the valuation of the object in the auction through the terms of financing. We also emphasize that it is this distortion in the valuation of the object that causes problems for efficiency and changing the auction format from first price to second price to Vickrey does not change the underlying rationale for this problem that stems from the financial market. Competitive financial markets cannot provide the incentives which make bids independent of the bidders' cash positions. Thus, an auction with financed bids will be inefficient.

## Appendix

■ Proofs of Lemma 4, Proposition 1, Proposition 2, part of Proposition 5 and Proposition 6 follow.

*Proof of Lemma 4.*

Assume that bidder  $i$  borrows an amount  $b_i - c_i + s_i$ . Since the bidder cannot steal, if he wins the auction then he expects to earn

$$H\lambda_i + (1 - \lambda_i)L - (c_i - s_i) - \lambda_i(1 + r)(b_i - c_i + s_i) - (1 - \lambda_i)(L + s_i), \quad (\text{A1})$$

as long as  $(b_i - \bar{c} + s)(1 + r) > L + s$ . Since  $(b_i - \bar{c})(1 + r) > L$  then this condition is met as long as the interest rate is positive. The derivative w.r.t.  $s_i$  is

$$1 - \lambda_i(1 + r) - (1 - \lambda_i) = -\lambda_i r < 0. \quad (\text{A2})$$

Therefore, as long as  $r > 0$  borrowing more than  $b_i - c_i$  is detrimental to the bidder. Q.E.D.

*Proof of Proposition 1.*

Define

$$f(x | \nu) = [\nu - b(x)] \frac{dG^{n-1}(x)}{dx} - b'(x)G^{n-1}(x). \quad (\text{A3})$$

The FOC for a type  $\nu$  bidder to tell the truth is  $f(\nu | \nu) = 0$  for all  $\nu$ . Note that  $f(x | \nu)$  is strictly increasing in  $\nu$ , hence by standard arguments  $f(x | \nu)$  is a single-peaked function with the peak obtained at  $x = \nu$ . By Lemma 1 we immediately know that Equation (19) is the unique symmetric equilibrium.

Individual rationality is easily shown since  $c_i < H\lambda_i + (1 - \lambda_i)L - \lambda_i(1 + r)(b_i - c_i) - (1 - \lambda_i)L$  as long as  $b_i < \frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i$ ; i.e. bidders bid less than their “value”. We need to show that all bidders default in the low state. The possibility of default requires

$$(b(\nu_i) - c_i)(1 + r) > L, \quad \forall i. \quad (\text{A4})$$

A necessary and sufficient condition that ensures default is

$$(b(\underline{\nu}) - \bar{c})(1 + r) > L. \quad (\text{A5})$$

If  $1 + r < 1/\underline{\lambda}$  and  $r > 0$  (the first part of condition A), then  $\nu$  is strictly decreasing in  $c$ . In this case

$$b(\underline{\nu}) = \frac{H}{1+r} - \frac{\bar{c}}{\underline{\lambda}(1+r)} + \bar{c}, \quad (\text{A6})$$

and the condition, (A5), can be reduced to

$$(H - L)\underline{\lambda} > \bar{c}, \quad (\text{A7})$$

which is the second part of condition (A).

If  $\underline{\lambda} > 1/(1 + r)$  then  $\nu$  is strictly increasing in  $c$ . Thus,

$$b(\underline{\nu}) = \frac{H}{1+r} - \frac{\underline{c}}{\underline{\lambda}(1+r)} + \underline{c}, \quad (\text{A8})$$

and the condition, (A5), becomes

$$1 + r < \frac{H - L}{\bar{c} - \underline{c}} - \frac{\underline{c}}{(\bar{c} - \underline{c})\underline{\lambda}}. \quad (\text{A9})$$

which is the second part of condition (B).

Finally we show that if the lending market is perfectly competitive, then there exists an equilibrium competitive rate  $r$  such that  $1 < 1 + r < \frac{1}{\lambda}$  (Case A).

Under perfect competition the lender must expect to be paid an amount equal to what he lent. In equilibrium the lender expects to lend

$$n \int_{\underline{\lambda}}^{\bar{\lambda}} \int_{\underline{c}}^{\bar{c}} (b(\lambda, c) - c) P^{n-1}(\lambda, c) \Psi'(c) F'(\lambda) dc d\lambda, \quad (\text{A10})$$

where  $P(\lambda, c) = \int_{\underline{c}}^{\bar{c}} F\left(\frac{\hat{c}}{H+(1+r)(\hat{c}-\nu(\lambda, c))}\right) \Psi'(\hat{c}) d\hat{c}$  is the probability of winning for type  $(\lambda, c)$ . The lender then expects to be paid

$$n \int_{\underline{\lambda}}^{\bar{\lambda}} \int_{\underline{c}}^{\bar{c}} [\lambda(b(\lambda, c) - c)(1 + r) + (1 - \lambda)L] P^{n-1}(\lambda, c) \Psi'(c) F'(\lambda) dc d\lambda. \quad (\text{A11})$$

A competitive  $r$  is a rate such that Equation (A10) = Equation (A11). If  $r = 0$  then Equation (A10)  $\geq$  Equation (A11) since  $\lambda \leq 1$ . If  $r = \frac{1}{\lambda} - 1$  then  $\lambda(1 + r) \geq 1$ . Therefore, Equation (A10)  $\leq$  Equation (A11). Since the relevant functions are continuous, the competitive rate exists such that  $1 < 1 + r < \frac{1}{\lambda}$ . Q.E.D.

*Proof of Proposition 2.* The FOC for a type  $\nu$  bidder when deciding which type to report is

$$f(x | \nu) = [\nu - b(x)] \frac{dG^{n-1}(x)}{dx} - b'(x) G^{n-1}(x) \quad (\text{A12})$$

Truth telling requires  $f(\nu | \nu) = 0$  for all  $\nu$ . Standard arguments show that  $f(x | \nu)$  is a single-peaked function with the peak obtained at  $x = \nu$ .

Individual rationality is easily shown since  $c_i < H\lambda_i + (1 - \lambda_i)L - \phi(b_i - c_i)(H\lambda_i + (1 - \lambda_i)L)$  as long as  $b_i < \frac{1}{\phi} - \frac{c_i}{\phi(H\lambda_i + (1 - \lambda_i)L)} + c_i$ ; i.e. bidders bid less than their ‘‘value’’. Finally, Lemma 1 yields Equation (26), as the unique symmetric equilibrium.

In a manner very similar to the Proof of Proposition 1 for debt financing, we can obtain the sufficient conditions for all bidders to raise equity and for the lowest type to be increasing or decreasing in cash. Q.E.D.

*Completion of Proof of Proposition 5.* To complete the proof of Proposition 5, we have to show that bidders truthfully report their cash positions. If bidders under-report their cash by an amount  $s$ , then their objective function becomes

$$\max_{x, s} [\{H\lambda_i - (c_i - s) - \lambda_i(1 + r(b(x), c_i - s))(b(x) - (c_i - s)) - (1 - \lambda_i)s\} F^{n-1}(x)]. \quad (\text{A13})$$

The term  $-(1 - \lambda_i)s$  now enters the objective because the borrower cannot steal and thus must pay  $s$  if they are in default as long as  $p(\lambda_i, c_i - s) > L$ ; which is still true as long as  $(H - L)\underline{\lambda} > \bar{c}$ . Substituting in the efficient interest rate function and taking the derivative w.r.t  $s$  yields

$$\left\{1 - \frac{1}{x}\right\} F^{n-1}(x) - \int_{\underline{\lambda}}^x \frac{F^{n-1}(w)}{w^2} dw < 0. \quad (\text{A14})$$

Therefore, for any true  $\lambda_i$  and any report  $x$ , bidders want to make the largest report possible about  $c_i$ . However, if they report cash greater than  $c_i$  then they will not receive enough money to cover their bid (since the lender will only lend  $b_i$  minus their reported cash). Therefore, they will not pay their bid and thus lose the object and the auction. Thus,  $s = 0$  and bidders will never under nor over report. Q.E.D.

*Proof of Proposition 6.* To prove this proposition we will assume that there is a stock price that depends on the bid and the revealed cash and results in an efficient auction. Furthermore, we will assume that because bidders cannot steal

they truthfully reveal their cash. Then we will determine what stock price function is required, and show that with this stock price the auction is efficient and that bidders do indeed truthfully reveal their cash. To begin we will use the direct revelation mechanism and write  $b_i$  as  $b(x)$  where  $x$  is a selection from  $[\underline{\lambda}, \bar{\lambda}]$ . Since we have assumed the auction is efficient (which we will later verify) the probability of winning the auction is  $F^{n-1}(x)$ . Furthermore,  $\phi(b_i, c_i)$  can be written as  $\phi(b(x), c_i)$ , where  $c_i$  is the assumed truthfully revealed cash. Therefore, the bidder's problem is

$$\max_x [H\lambda_i + (1 - \lambda_i)L - c_i - \phi(x, c_i)(b(x) - c_i)(H\lambda_i + (1 - \lambda_i)L)]F^{n-1}(x). \quad (\text{A15})$$

Set

$$a(\lambda_i, c_i) = \frac{1}{\lambda_i H + (1 - \lambda_i)L}, \quad (\text{A16})$$

$$p(x, c_i) = \phi(x, c_i)(b(x) - c_i), \quad (\text{A17})$$

$$\nu(\lambda_i, c_i) = 1 - \frac{c_i}{\lambda_i H + (1 - \lambda_i)L} \quad (\text{A18})$$

and use Equation (9). This immediately yields that

$$p(\lambda_i, c_i) = 1 - \frac{c_i}{H\lambda_i + (1 - \lambda_i)L} - \int_{\underline{\lambda}}^{\lambda_i} \frac{(H - L)c_i F^{n-1}(x)}{(Hx + (1 - x)L)^2 F^{n-1}(\lambda_i)} dx. \quad (\text{A19})$$

Standard techniques (similar to that in Proposition 5) show this is the unique symmetric equilibrium.

Individual rationality is easily shown since  $0 \leq H\lambda_i + (1 - \lambda_i)L - c_i - \phi(x, c_i)(b(x) - c_i)(H\lambda_i + (1 - \lambda_i)L)$  as long as  $p(\lambda_i, c_i) \leq 1 - \frac{c_i}{H\lambda_i + (1 - \lambda_i)L}$ . It must also be the case that the bidders need to finance and therefore bid more than their cash, therefore,  $p(\lambda_i, c_i) > 0$ . Since  $p_1(\lambda_i, c_i) > 0$  and  $p_2(\lambda_i, c_i) < 0$  the financing condition can be reduced to the sufficient and necessary condition  $H\underline{\lambda} - L(1 - \underline{\lambda}) > \bar{c}$ .

We need to show that there is a  $\phi$  function that implements the payment function in Equation (A19). Using the definition of  $p(\lambda_i, c_i)$  we obtain that in equilibrium

$$\phi(b(\lambda_i), c_i) = [1 - \frac{c_i}{H\lambda_i + (1 - \lambda_i)L} - \int_{\underline{\lambda}}^{\lambda_i} \frac{(H - L)c_i F^{n-1}(x)}{(Hx + (1 - x)L)^2 F^{n-1}(\lambda_i)} dx] \frac{1}{b(\lambda_i) - c_i}. \quad (\text{A20})$$

Two final steps in this proof remain. First we need to substitute this equity price function into the bidder's objective (with the function  $b_\phi^{-1}(b_i)$  being used to invert the bid) and to show that the maximization yields the bid function  $b_\phi(\cdot)$  and that the first order condition is sufficient. Second we need to show that bidders will truthfully report  $c_i$  to the equity market. This proof is very similar to that in Proposition 5 for debt and is omitted. Q.E.D.

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Table 1: Debt Securities

Table 1 shows our results for debt securities under three different contracting assumptions: (1) a fixed interest rate  $r$ , (2) an interest rate that depends on the valuation  $\lambda$  and cash position  $c$  but not the bid  $b$  and (3) an interest rate that depends on the bid  $b$  and the cash position  $c$ .

	Fixed Interest Rate	Pre-Auction Financing	Post-Auction Financing
	$r$	$r(\lambda_i, c_i)$	$r(b, c_i)$
$\nu(\lambda_i, c_i)$	$\frac{H}{1+r} - \frac{c_i}{\lambda_i(1+r)} + c_i$	$\frac{H}{1+r(\lambda_i, c_i)} - \frac{c_i}{\lambda_i(1+r(\lambda_i, c_i))} + c_i$	$H - \frac{c_i}{\lambda_i}$
$p(b, \lambda_i, c_i)$	$b$	$b$	$(1 + r(b, c_i))(b - c_i)$
Default Condition	$(H - L)\lambda > \bar{c}$ (Case A)	$\nu(\lambda) > L + \bar{c}$ (under efficiency)	$(H - L)\lambda > \bar{c}$ (under efficiency)
Efficient Auction Possible	No	Yes	Yes
Efficient Auction With Competitive Market	No	No	No

Table 2: Equity Securities

Table 2 shows our results for equity securities under three different contracting assumptions: (1) a fixed price for equity, (2) an price for equity that depends on the valuation  $\lambda$  and cash position  $c$  but not the bid  $b$  and (3) an interest rate that depends on the bid  $b$  and the cash position  $c$ .

	Fixed Price of Equity	Pre-Auction Financing	Post-Auction Financing
$\nu(\lambda_i, c_i)$	$\frac{1}{\phi} - \frac{c_i}{\phi(H\lambda_i + (1-\lambda_i)L)} + c_i$	$\frac{1}{\phi(\lambda_i, c_i)} - \frac{c_i}{\phi(\lambda_i, c_i)(H\lambda_i + (1-\lambda_i)L)} + c_i$	$\phi(b, c_i)$
$p(b, \lambda_i, c_i)$	$b$	$b$	$1 - \frac{c_i}{\lambda_i H + (1-\lambda_i)L}$ $\phi(x, c_i)(b(x) - c_i)$
Default Condition	$1/\phi > H\lambda + (1-\lambda)L > \bar{c}$ (Case A)	$\nu(\lambda) > \bar{c}$ (under efficiency)	$H\lambda - L(1-\lambda) > \bar{c}$ (under efficiency)
Efficient Auction Possible	No	Yes	Yes
Efficient Auction With Competitive Markets	No	Yes	No

Table 3: Contingent Securities

Table 3 shows our results with contingent securities.

	Contingent Securities $h(b, c_i) \leq H, l(b, c_i) \leq L$
$\nu(\lambda_i, c_i)$	$H\lambda_i + (1 - \lambda_i)L$
$p(b, \lambda_i, c_i)$	$c_i + \lambda_i h(x, c_i) + (1 - \lambda_i)l(x, c_i)$
Bankruptcy Condition	$H\underline{\lambda} + (1 - \underline{\lambda})L > \bar{c}$
Efficiency With Competitive Capital Markets	Yes
Efficiency With $l(b, c_i) \geq 0$ (no lender payments) and Competitive Capital Markets	Not if $L \leq (\bar{c} - \underline{c})$
Efficiency With Some Bidders Unconstrained, Competitive Capital Markets	No
Efficiency with Some Bidders Unconstrained, Monopoly Lender	Yes
Efficiency with Monopoly Lender, Outside Access to Competitive Capital Markets	No