*(1) Externalities and Public Goods*

Three firms—Once-ler, Twice-ler, and Thrice-ler—manufacture Thneeds. In the process, they emit Gluppity Glupp which hinders the business of two ecotourism firms—The Lazy Lorax and The Purple Bar-Ba-Loot.

The ecotourism firms (just call them (e) and (f)) have managed to estimate the costs to their business of Gluppity Glupp emissions. As such, the estimated benefits (B) to each ecotourism firm for a given reduction G in emissions are as follows:

$$B_e = \frac{-G^2}{28} + \frac{10}{7}G \qquad B_f = \frac{-3G^2}{28} + \frac{10}{7}G$$

Meanwhile, the Thneed producers (just call them 1, 2, and 3) have estimated their own costs for abating Gluppity Glupp as:

$$C_1 = \frac{(G_1)^2}{2} \qquad C_2 = \frac{(G_2)^2}{4} \qquad C_3 = (G_3)^2$$

where $G_1$, $G_2$, and $G_3$ are the respective amounts of abatement for each firm:

$$G_1 + G_2 + G_3 = G$$

Answer the following:

a -- For a given level of abatement G, what is the efficient level of abatement for each firm?

b -- Assuming abatement is efficiently distributed among the three firms, what is the Thneed industry's marginal cost of a given reduction, G? (Hint: This is the Thneed industry's supply curve for abatement. There are two ways to get it—a short way and a long way.)

c -- What is the efficient level of total abatement, G*? If the government were to implement a permit system with the Thneed producers to achieve the efficient level of abatement G*, what would be the market clearing price of a permit?

d -- Come up with at least two kinds of market failures that could exist in this situation, if the ecotourism firms and the Thneed producers were left to their own devices. If the ecotourism firms could pay the Thneed producers to reduce their emissions, how much abatement would firms e and f purchase, assuming the market for abatement was competitive among the Thneed producers? How does this compare to the efficient level G* obtained in (c)?

*(2) Equilibrium between Two Abatement Technologies (Tietenberg, 6th Ed. Ch. 15)*

Note: I'm recommending this because it's graph-friendly; you can solve for the optimal mix of abatement between two firms graphically.

Two firms can control emissions of a pollutant at the following marginal costs:

$MC_1 = 200q_1$ $\qquad\qquad$ $MC_2 = 100q_2$

where $q_1$ and $q_2$ are the respective amounts of abatement by each firm.

Assume that with no control at all, each firm would be emitting 20 units (40 total).

(a) Using a graph, compute the cost-effective allocation of control responsibility if a total *reduction* of 21 units of emissions is necessary.

(b) What tax would achieve this allocation? What would be the tax revenues? What would be the total costs (total abatement costs plus taxes) to each firm?

(c) Using your graph in (a), compute the allocation that would result if 10 emission permits were given to the second source and 9 were given to the first source. What would the net permit expenditure be for each source after trading?

(d) Suppose a new source of emissions entered the area with a constant marginal cost equal to $1600 per unit of abatement. Assume further that it would add 10 units of emissions in the absence of any control. What would be the resulting allocation of control responsibility? How much would each firm clean up? What would happen to the permit price? What trades would take place?

*(3) Present & Future Value*

If you were guaranteed an interest rate of 10%, approximately what amount of money received yearly for 40 years, starting a year from now, would be worth the same as receiving $500 today?

*(4) Property Rights*

Suppose the government implemented a cap system with firms that emitted a particular pollutant. The system allowed each firm to emit a fixed amount of the pollutant but no more. What criteria, if any, does this allowance lack in order to be a well-defined property right?

**Suggested Solution Set – Quantitative Exercise #1**
**Problems by Professor Martin Smith, solutions by Zachary Brown**
**Sustainability & Renewable Resource Economics, Spring 2007**

*Question 1*

A – Suppose *you only like to eat pasta, hamburgers, and pizza at the cafeteria, you like them equally well (and they cost the same), you eat at the cafeteria every weekday (Monday through Friday), and you never like to eat the same thing two days in a row. It's Monday, and you learn that all three entrees will be offered every day. You can decide what to eat day-by-day, or you can plan it all out in advance*

[Heuristic Discussion]

The state variable can be thought of as the way things are when we wake up in the morning, in this case, what we've eaten for lunch on previous days. For our objective—not to eat the same thing two days in a row—the only part of this state that matters is what we ate yesterday. That is, by simply not eating the same thing today as we ate yesterday, we can achieve our objective: We needn't worry about tomorrow at all.
So this problem *is* backward-looking and in the sense of being dependent on time (in the past) can be viewed as dynamic, but it is *not* forward-looking.

[A Formal Set-Up]

Some folks might view this as unnecessarily complicated, but it might help to see this set-up as a way to relate this problem to a typical dynamic optimization one.

For food, denote hamburgers with a 0, pasta with a 1, and pizza with a 2.

The time variable $t$ here indicates which day of the week it is (*e.g.* $t = 0$ means a Monday, *etc.*).

Define the function of time $u(t)$ as what we decide to eat at time $t$. So $u(1) = 2$ means that on Tuesday we eat pizza. This is our control.

Define $x(t)$ as what we ate on day $t$-1. This is our state variable.

Finally, define the function $g(u(t),x(t))$ so that:

$$g(u(t), x(t)) \equiv \begin{cases} 1 & \text{if } x(t) = u(t) \\ 0 & \text{otherwise} \end{cases}$$

Then our problem can be written as:

$$\underset{u(t)}{\text{Min}} \left\{ \sum_{t=0}^{4} g(u(t), x(t)) \right\}$$

such that (1) $x(t + 1) = u(t)$ (state equation)
(2) $x(0) = -1$ (initial condition)

Note that there could be any value for x(0) such that u(0) could never equal x(0): We just want g(u(0),x(0)) = 0. Also note that, though we our minimizing the number of days in which we eat the same thing as the day before, if there is a plan in which we never have to eat the same thing in a row, then we necessarily achieve the minimum. That is, if our ideal plan exists, then the above program will find it.

B – *As in scenario a, you only like to eat pasta, hamburgers, and pizza at the cafeteria, you like them equally well (and they cost the same), you eat at the cafeteria every weekday, and you never like to eat the same thing two days in a row. However, this week pasta is only offered on Wednesday, and pizza is not offered on Tuesday or Thursday. You can decide what to eat day-by-day, or you can plan it all out in advance.*

[Heuristic Discussion]

The problem is the same as (A), just more constrained. We now have to account for information we know about the future (*e.g.* we have to eat hamburgers—or nothing—on Tuesday and Thursday) and hence we have to be forward-looking. This problem is certainly dynamic. In some loose sense, it is more so than in (A) in the fact that we have to be *more* conscious of how we navigate our gastronomical activities across the week…

[Formal Argument]

To formally program this problem, we can use the program from (A) and just add the following constraints:

(3) $x(t) \neq 1$ if $t \neq 2$
(4) $x(t) \neq 2$ if $(t = 1)$ or $(t = 3)$